Study on Feebly Open Set with Respect to an Ideal Topological Spaces

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Abstract:- The aim of this paper is to introduce the definition of feebly open set in ideal topological space, some relations and properties about this set will be study also.

Keywords— feebly open set, semi open set , I-open set . Feebly I-open set , local function , ideal topological spaces

1. INTRODUCTION

The concept of ideal topology in the classic text was introduced by kuratowski [5], D.Jankovie andT, R Hamlelt [2]. Introduced the concept of I – open set in ideal Topological space after that M.E.AbdEl.Monsef E. F.Iashien and A.A.Nasef [6] introduced a new study about the I – open set. S. N. Maheshwari and P. C. Jain [8] introduced the concept of feebly open set and feebly closed set in topological spaces, after that many Authors used the concept of feebly open set and feebly closed sets to study another concepts in topological space.

N.Ievin [7] introduced the concept of semi – open sets and semi – closed sets, also semi – open set was defined in ideal topological spaces by [3].

If I is an ideal on a topological space(X,T),then we can construct a topological space on X is called ∗ −**topology denoted by T**∗[2]

In our paper we will define the feebly open set in ideal topological space and study some properties and theorems about it .

II-Preliminaries

Definition 2.1:- [2]

Ideal is a non-empty collection I of subset of X is said to be an ideal on X if it satisfies the following two conditions:-

1- if $A \in I$ and $B \subseteq A$ then $B \in I$ [heredity]

2- if $A \in I$ and $B \in I$ then $A \cup B \in I$ [finite additivity]

Example 2.2 :-

Let $X = \{a, b, c, d\}$, $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$ then I is an ideal .

Example 2.3 :-

Let $X = \{a, b, c\}$, $I = \{\emptyset, \{a\}, \{c\}, \{a, b\}\}$ then I is not an ideal

Example 2.4 :-

Let $X = \{a, b, c, d, e\}$, $I = \{\emptyset\}$ then I is an ideal

Example2.5 :-

Let $X = \{e, f\}$, $I = \{\emptyset, X\}$ then I is not an ideal

Definition 2.6:-[2]

Let (X,T) be a space with an ideal I on X . Then

$A^*(I, T) = \{x \in X: A \cap u \notin I$ for every $u \in N_T(x)$ where $N_T(x)$ is open neighbourhoods of X$\}$

Example 2.7:- Let $X = \{a, b, c\}$, $I = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$T = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ Then $\{a, b\}^*=\{a, b\}$, $\{b, c\}^*=\emptyset$,$\{a\}^*=\{a, b, c\}$

Theorem 2.8:- [2]

Let (X,T) be a space with I is an ideal on X and let A and B be subsets of X Then :-

1- $A \subseteq B \rightarrow A^* \subseteq B^*$

2- $A^* = cl(A^*) \subseteq cl(A)$ (A^* is acloed subset to f cl(A))

3- $(A^*)^* \subseteq A^*$

4- $(A \cup B)^* = A^* \cup B^*$

Theorem 2.9:- [4]

Let (X,T,I) be a space and let $\{A_\alpha : \alpha \in D\}$ be any family of subsets of X . Then we have

1- $(\cup A_\alpha^* : \alpha \in D) \subseteq (\cup A_\alpha : \alpha \in \Delta)^*$

2- $(\cap A_\alpha : \alpha \in \Delta)^* \subseteq (\cap A_\alpha^* : \alpha \in \Delta)$.

Corollary 2.10:- [2]

For $A \subseteq (X, T, I)$ we have

1- $A^*(\{\emptyset\}) = cl(A)$, means that $x \in X$ is a closurepoint of A iff $x \in A^*(\{\emptyset\})$

2- $A^*(P(x)) = \emptyset$

Definition 2.11 :- [6]

Given a space (X,T,I) and $A \subseteq X$, A is said to be I – open if $A \subseteq int(A^*)$. We denote by $Io(X, T) = \{A \subseteq X : A \subseteq int(A^*)\}$ or simply write I.o for Io (X,T) when there is no chance for confusion .

Example 2.12:-

Let $X = \{a, b, c, d\}$ with a topology $= \{X, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$, $I = \{\emptyset, \{a\}\}$

Then $I.o = \{\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, c\}, \{a, b, c\}, X, \emptyset\}$

Remark 2.13 :-[6]

It is clearly that , I – open set and open set are independent concepts

Example 2.14 :-

Let $X = \{a, b, c, d\}$ with a topology $T = \{X, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$ and

$I = \{\emptyset, \{a\}\}$. Then $\{b, c, d\} \in Io(X, T)$ but $\{b, c, d\} \notin T$.

Example 2.15:-

Let X be as in Example 2.14 , $T = \{X, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$ it is clear that $\{a, c, d\} \notin T$, but $\{b, c, d\} \notin Io(X, T)$.

Remark 2.16:-[2]

Let $A \subseteq (X, T, I)$ then

1- if $A \subseteq A^*$ then A is called $*$ dense in it self

2- if $A^* \subseteq A$ then A is called $T^* -$ closed .

3- if $A = A^*$ then A is called $*$ - perfect .

Proposition 2.17 :-

Let (X, T, I) be an ideal topology space then every $T^* -$ closed I – open A subset of X is open set

Proof:-

Let A is I-open set then $A \subseteq int(A^*)$ and since A is T^*-closed set we have $A^* \subseteq A$ from that we get $int(A^*) \subseteq int(A)$ then $A \subseteq int(A^*) \subseteq int(A)$ then $A \subseteq int(A)$ and since $int(A) \subseteq A$ then A=int(A) and then for A is open set.

Theorem 2.18 :-

Let (X, T, I) be an ideal topology space then every $*$ dense open set is I – open set .

Proof :-

Let A is open set then A=int(A) and since $A \subseteq A^*$ **then** int(A)$\subseteq$ int(A*) **and from that we get** $A = int(A) \subseteq int(A^*)$ **and then A** is a open.

Theorem 2.19 :- [4]

Let (X, T, I) be a space and $A, B \subseteq X$ .Then

1- if $\{u_\alpha : \alpha \in \Delta\} \subseteq I.\, o\,(X, T)$ Then $\cup \{u_\alpha : \alpha \in \Delta\} \in I.\, o(X, T)$

2- if $A \in I.\, o(X, T)$ and $B \in T$. Then $A \cap B \in I.\, o(X, T)$.

Definition 2.20:- [6]

A subset $F \subseteq (X, T, I)$ is called I – closed set if its complement of I – open set .

Example 2.21:-

Let $X = \{a, b, c, d\}$ with a topology

$T = \{X, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$, $I = \{\emptyset, \{a\}\}$ then

$I.\, o = \{\{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, d\}, \{d\}, \emptyset, X\}$.

Definition 2.22:- [7]

A subset A of an ideal topological space (X,T,I) is said to be

1) Semi – open in (X,T) if $A \subseteq cl(int(A))$.

2) Semi-closed if X-A semi-open in (X,T)

Example 2.23:-

Let $X = \{a, b, c\}$, $T = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$.

S.O(X)=$\{\{a\}, \{b\}, \{a, b\}, X\}$

S.C(X)=$\{\{b,c\}, \{a,c\}, \{c\}, \phi\}$

Definition 2.24:-[3]

A subset A of an ideal topological space (X,T,I) is said to be semi-I-open if $A \subseteq cl^*(int(A))$.

Definition 2.26:- [8]

A subset A of a topological space X is said to be feebly open set if $A \subset scl(int(A))$

Lemmas 2.27:-[9]

In any topological space (X,T) Every feebly open set is semi – open set

III- Feebly open set in ideal topological space

Definition3.1

Let (X,T) be a topological space and I be an ideal defined on X the subset A of X is say to bye feebly I – open set in the ideal topological . Space (X,T,I) if $A \subseteq scl(int(A^*))$ and is denoted by F. I. O.

Example 3.2:-

Let $X = \{a, b, c, d\}$, $T = \{X, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$,

$I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$

then $S.\, O(X) = \{\emptyset, X, \{a, c, d\}, \{a, b, c\}, \{a, c\}, \{d\}\}$

$S.\, C(X) = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}, \{a, b, c\}\}$

$F.\, I.\, O(X) = \{\emptyset, \{a, c\}, \{d\}, \{a, b\}, \{a\}, \{a, b, c\}, \{a, d\}, \{a, c, d\}, \{a, b, d\}\}$.

Remark 3.3:-

The intersection of two feebly I – open need not to be feebly I – open since

$(\cap A_\alpha : \alpha \in \Delta)^* \subseteq (\cap A_\alpha^* : \alpha \in \Delta)$.

Theorem 3.4:-Let (X,T,I) be an IDT If A,B be a F. I. open sets then $\mathbf{A \cup B}$ is F. I. open

Proof:-

$\mathbf{A \subseteq scl(int(A^*))\,, B \subseteq scl(int(B^*))}$

$\mathbf{(A \cup B) \subseteq scl(int(A^*)) \cup scl(int(B^*))}$

$\mathbf{= scl(int(A) \cup int(B^*)) \subseteq scl(int(A^* \cup B^*)}$

$\mathbf{\subseteq scl(int(A \cup B)^*)}$

Proposition3.5:-

If (X,T) is ideal topological space then every feebly I-open T^*-closed subset A of X is semi open

Proof:-

Let A is feebly I-open set in X then

A$\subseteq$ SCL(int($\mathbf{A}^*$))$\subseteq$ CL(int($\mathbf{A}^*$)) and since A is $\mathbf{T}^*$-closed set then A$\subseteq$CL(int(A)) from that we get that A is semi-open.

Theorem 3.6:-[3]

1- Every open set is semi – I – open .

2- Every semi – I – open set is semi – open se.

Proposition 3.7:-

Let (X,T,I) be an ideal topological space then every I –open set is F – I – open set .

Proof:- Let A is I-open set in X then SCL(A)$\subseteq$SCL(int($\mathbf{A}^*$)

A$\subseteq$SCL(A)$\subseteq$SCL(int($\mathbf{A}^*$) then A$\subseteq$SCL(int ($\mathbf{A}^*$) then A is feebly I-open

Theorem 3.8 :-

Let (X,T,I) be ideal topological space such that A is $*$ dense in itself then every feebly open subset A of X is feebly I – open set .

Proof:-

if A is feebly open and $*$dense then int(A)$\subseteq$int($\mathbf{A}^*$) then A$\subseteq$SCL(int(A))$\subseteq$SCL(int($\mathbf{A}^*$) then Ais feebly –I-open.

Theorem 3.9 :-

Let (X,T,I) be an ideal topological space such that A is($\mathbf{T}^*$)– closed set then every feebly I – open subset A of X is feebly open .

Proof:-

Let A is feebly I-open and $\mathbf{A}^*\subseteq$A then int($\mathbf{A}^*$)$\subseteq$ int(A)

SCL(int($\mathbf{A}^*$))$\subseteq$ SCL(int(A)) since A is feebly I-open then A$\subseteq$SCL(int($\mathbf{A}^*$)) then A$\subseteq$ SCL(int(A)) then A is feebly open.
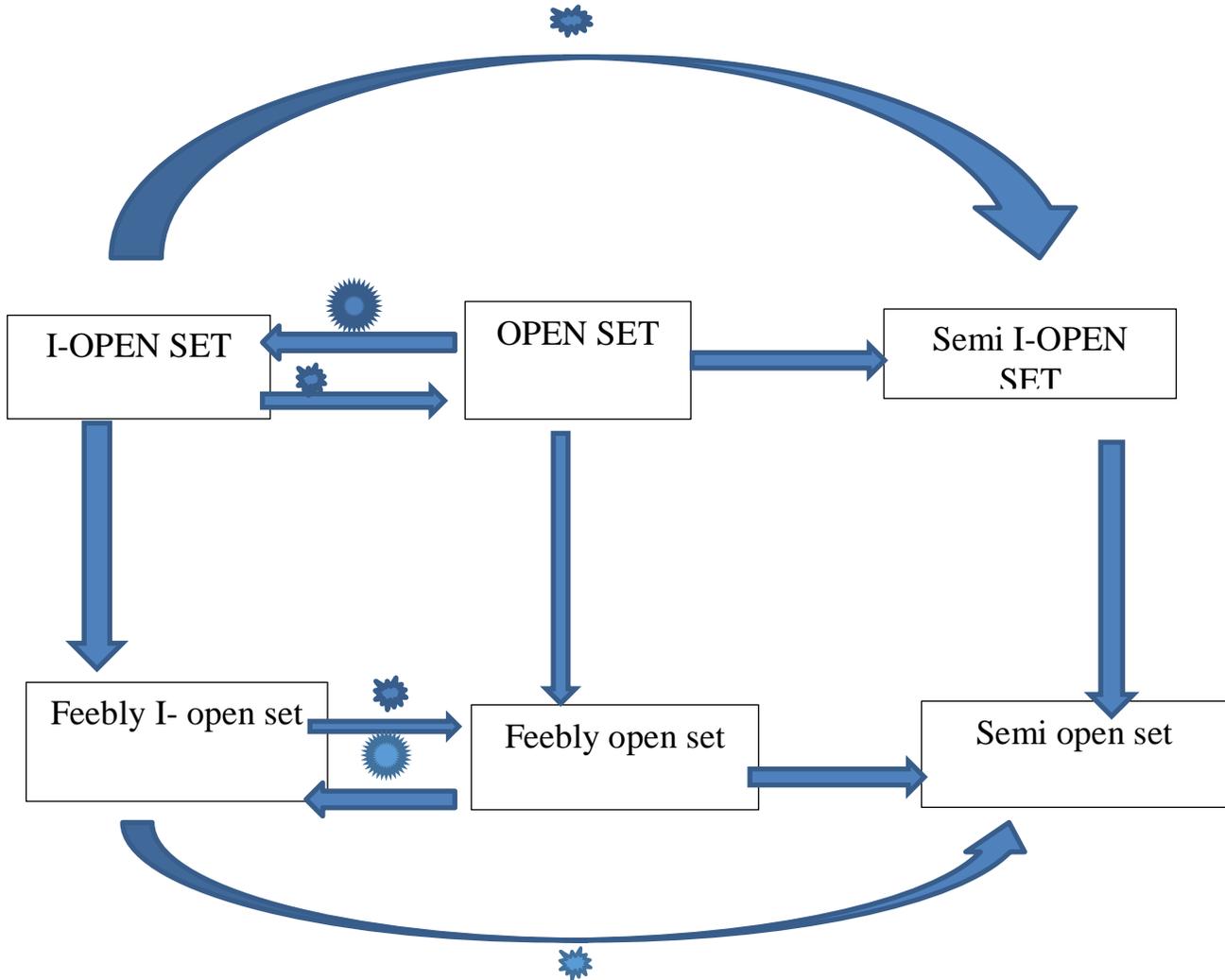
Theorem 3.10:-

if (X,T,I) be an ideal topological space then every I-open T^*-closed set is semi-I-open

Proof:

Let A is I-open set then A $\subseteq$(int($\mathbf{A}^*$).and since A is$\mathbf{T}^*$-closed

Then A$\subseteq$ int($\mathbf{A}^*$) $\subseteq$ int(A) $\rightarrow$A$\cup$($\mathbf{int(A)}$)$^*\subseteq$ int(A)$\cup$ ($\mathbf{int(A)}$)*=$\mathbf{cl}^*$(int(A)) and then A is semi- I open

The following diagram gives us all the above relations discussed

: mean that the set is T*-Closed

Mean that the set is *-dense

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