

Reliability-Based Approach for the Determination of the Required Compressive Strength of Concrete in Mix Design

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Abstract: Concrete is recognized as the second most consumed product in our modern life after water. The variability in concrete properties is inevitable. The concrete mix is designed for a compressive strength that is different from, typically higher than, the value specified by the structural designer. Ways to calculate the compressive strength to be used in the mix design are provided in building and structural codes. These ways are all based on criteria related purely and only to the statistical nature of the concrete production process. However, what really matters is the impact of the concrete properties on the soundness of the structure the concrete is intended to be built with. Structural reliability has never been used explicitly in concrete mix design. In this paper, an approach for the determination of the required compressive strength of concrete in mix design, based on the structural reliability of the reinforced concrete components being constructed with this concrete, is proposed. In addition, the results will be shown to be used for the optimization of the statistical parameters for quality control in the concrete mix process. The approach presented in this paper provides a practical platform to efficiently consider plant-specific variability the mix design process.

Keywords: mix design, concrete, structure, reliability, variability, optimization.

1. INTRODUCTION

The presence of variability in the properties of concrete requires that special measures are taken in the concrete mix design process to ensure its quality. Statistical procedures are used in the mix design process in order to provide assurance of satisfying the intended purposes of the designed concrete. One of the concrete properties that is given considerable attention is its compressive strength.

Concrete is recognized as the second most consumed product in our modern life after water. The variability in concrete properties is inevitable. This variability has been widely studied in the literature (Aichouni 2012, Laungrungrong *et al.*, 2010, Nowak and Szerszen 2003a,b, Ellingwood *et al.*, 1980, Mirza *et al.*, 1979). Some of these studies have considered controlling this variability as a criterion for the quality control of the concrete production process (Aichouni 2012, Laungrungrong *et al.*, 2010). Compressive strength, in most cases, is the most suitable and effective tool for the control of concrete quality (ACI 314R 2011), even when compressive strength is not the most important quality to be controlled because testing of the compressive strength gives the best reflection of the change not only in the average concrete quality but also in variability and in testing error (Day 2006).

In the concrete mix design process, the material engineer is provided a specified strength by the structural designer, f'_c . An optimum result of the concrete production is a batch with all tested specimens giving compressive strength exactly equal to f'_c . Realistically, the tested strength of concrete samples will differ from f'_c , some lower than f'_c and some higher. If the materials engineer provides a material with an average strength equal to f'_c then half of the concrete will have compressive strength less than f'_c (Mamlouk and Zaniewski 2011). In order to avoid such undesirable outcome, the materials engineer designs the concrete to have a required mean strength, f'_{cr} , greater than f'_c . Ways to calculate the value of f'_{cr} to be used in the mix design are provided in building and structural codes and standards. El-Reedy (2013) has compared the procedures for determining the required compressive strength of concrete for the mix design process in different international codes and standards.

The methods provided in codes and standards are based on criteria related purely and only to the statistical nature of the concrete production process. Under uncertainty, structural reliability theory and tools have proved to be powerful means of quantifying structural safety. Some of the code methods implicitly aim to achieve a given structural reliability level by providing parameters that define the required strength of the concrete, which depend on the available studies in the specific country concerning the variability on concrete material properties that vary from one country to another and even from one location to another in the same country (El-Reedy 2013). Structural reliability has never been used explicitly in the determination of f'_{cr} . Doing so can be very beneficial for a concrete production plant that exhibits variability in production different from code considered variability. In such a case, the mix design may be based on values tailored to fit its own production conditions while achieving the required reliability levels for the structural components being built.

In this paper, an approach for the determination of the required concrete compressive strength in mix design based on the structural reliability of the reinforced concrete components being constructed with this concrete is proposed. The approach is illustrated on examples of reinforced concrete columns and beams. Intuitively speaking, the structural reliability can be enhanced by either

increasing the mean of the concrete compressive strength or reducing its standard deviation or both. The cost of increasing the mean may be different from that of reducing the standard deviation of the produced concrete. In this paper, also, the results of the structural reliability analysis will be used to find an optimum set of values for the target mean and standard deviation of the concrete compressive strength to be produced at the lowest cost.

2. VARIABILITY IN THE COMPRESSIVE STRENGTH OF CONCRETE

Variations in the properties or proportions of the concrete ingredients, as well as variations in transporting, placing, and compaction of the concrete, lead to variability in the strength of the finished concrete (Aichouni 2012, Wight and MacGregor 2011, Laungrongrong *et al.*, 2010). According to the ASTM Standards C31 and C39, the standard test for measuring the strength of concrete involves a compression test on cylinders 15 cm in diameter and 30 cm high after they are made and cured for 28 days.

Any batch of concrete is produced based on a mix design aiming to achieve a specified strength value, f'_c , determined by the structural designer. An optimum result of the concrete production is a batch with all cylinders giving required compressive strength, f'_{cr} , exactly equal to f'_c . Realistically, the tested strength of concrete samples will differ from f'_c , some lower than f'_c and some higher. Concrete strength is believed to generally follow a normal distribution (Mirza *et al.*, 1979). Arafa (1997) showed that ready-mixed concrete types are well modeled by the normal distributions whereas site-mixed concrete is well represented by the log-normal distribution with low mean-to-nominal ratio and high coefficient of variation.

For batches of low variability, strength values of concrete will tend to cluster near to the average value; that is, the histogram of test results is tall and narrow. As the variability in the concrete compressive strength increases, the spread in the data increases and the normal distribution curve becomes lower and wider. The curve of this normal distribution is symmetrical about the mean value of the data, f'_{cr} , whereas the standard deviation, σ , measures the dispersion of the data. The mean is calculated as (ACI 214R-11)

$$f'_{cr} = \frac{\sum_{i=1}^n x_i}{n} \quad (1)$$

where x_i is the tested strength of cylinder i and n is the number of tested cylinders. The sample standard deviation is calculated as (ACI 214R-11)

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - f'_{cr})^2}{n-1}} \quad (2)$$

The coefficient of variation, V_R , is used to describe the degree of dispersion relative to the mean, and is calculated as (ACI 214R-11)

$$V_R = \frac{\sigma}{f'_{cr}} \quad (3)$$

3. ACI PROCEDURE FOR DETERMINING THE REQUIRED COMPRESSIVE STRENGTH OF CONCRETE

The ACI 318M (2011) lays out a procedure for the determination of the required compressive strength for mix design that is based on the criteria established by the ACI 214R (2011) and the ACI 301M (2010). This procedure starts with establishing a representing sample standard deviation. The calculation of the sample standard deviation depends on the number, nature and age of test records available at the production facility.

If a concrete production facility has at least 30 consecutive strength test records of concrete produced from the same specified class or within 7 MPa of f'_c , and these records are not more than 24 months old, then Equation (2) is used to calculate the sample standard deviation. However, if the concrete production facility has two groups of consecutive tests records totaling at least 30 tests produced from the same specified class or within 7 MPa of f'_c , and these records are not more than 24 months old, the sample standard deviation is calculated as (ACI 318M, 2011)

$$\sigma = \sqrt{\frac{(n_1-1)\sigma_1^2 + (n_2-1)\sigma_2^2}{n_1+n_2-2}} \quad (4)$$

where σ_1, σ_2 = sample standard deviations calculated from two test records, 1 and 2, respectively, and n_1, n_2 = number of tests in each test record, respectively.

Table 1: Modification factor for sample standard deviation when less than 30 tests are available.

Number of tests*	Coefficient of variation
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15	1.16
20	1.08
25	1.03
30 or more	1.00

* Interpolate for intermediate numbers of tests.

There may be a case where a concrete production facility does not have records of at least 30 consecutive strength tests or two groups of consecutive tests totaling at least 30 tests produced from the same specified class or within 7 MPa of f'_c . In such a case, if the production facility has test records not more than 24 months old based on 15 to 29 tests representing a single record of consecutive tests that span a period of not less than 45 calendar days consecutive tests, a sample standard deviation needs to be established as the product of the sample standard deviation calculated from Equation (3) or Equation (4) and a modification factor chosen from Table 1.

Once the sample standard deviation has been determined, the required mean compressive strength, f'_{cr} , is calculated as follows (ACI 318M, 2011)

$$f'_{cr} = \max \begin{cases} f'_c + 1.34\sigma & \text{for } f'_c \leq 35 \text{ MPa} \\ f'_c + 2.33\sigma - 3.5 & \text{for } f'_c \leq 35 \text{ MPa} \\ 0.9 f'_c + 2.33\sigma & \text{for } f'_c > 35 \text{ MPa} \end{cases} \quad (5)$$

However, if a production facility does not have test records that meet any of the conditions listed above, f'_{cr} , is determined as follows (ACI 318M, 2011)

$$f'_{cr} = \max \begin{cases} f'_c + 7.0 & \text{for } f'_c < 21 \text{ MPa} \\ f'_c + 8.3 & \text{for } 21 \leq f'_c \leq 35 \text{ MPa} \\ 1.1 f'_c + 5.0 & \text{for } f'_c > 35 \text{ MPa} \end{cases} \quad (6)$$

4. STRUCTURAL RELIABILITY

Uncertainties are present in the resistance of structural components, which are caused by the variability in the structural materials and constructed section properties. Uncertainties are also present in the loadings applied to these elements, especially the live loads and environmental loads due to wind, snow or earthquakes (Wight and MacGregor 2011). The topic of structural reliability (Thoft-Christensen and Baker 1982) offers a rational framework to quantify these uncertainties mathematically. This topic combines theories of probability, statistics and random processes with principles of structural mechanics and forms the basis on which modern structural design and assessment codes are developed and calibrated.

A safety margin, g , also known as the performance function, is defined as the difference between the resistance of a structural component and the load effect it is subjected to, and is given by

$$g = R - L \quad (7)$$

Because of the presence of uncertainties, R and L are treated as random variables. From Equation (7), g is also a random variable and its distribution is presented schematically in Figure 1. The value of zero separates the combinations of R and L that represent the safety of a structural component from those combinations that represent its failure. The distribution of g is used to find the probability of failure of the structural component.

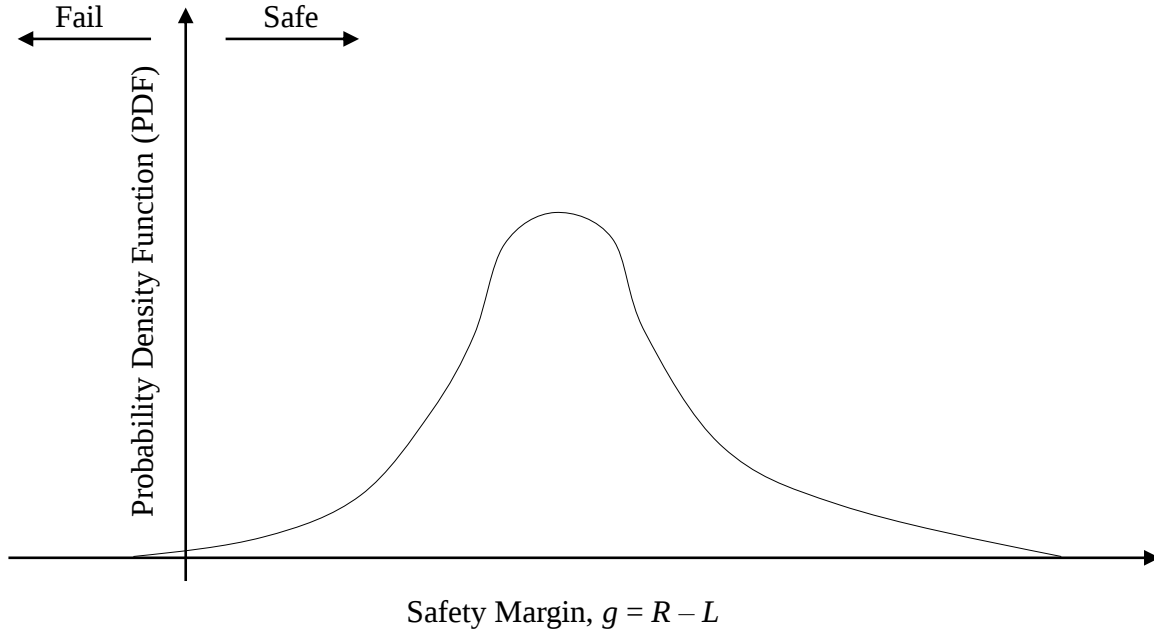


Figure 1. Schematic sketch of a typical probability distribution of the safety margin of a structural member. The probability of failure of a structural component is the chance that a particular combination of R and L will give a negative value of g , i.e., the load effect exceeds the resistance. This probability is equal to the ratio of the area in the fail region under the curve of g to the total area under the curve in Figure 1 (which is equal to 1.0). This can be expressed as (Ellingwood 2005)

$$P_f(t) = P[g < 0] = \int_0^{\infty} F_R(x, t) f_L(x, t) dx \quad (8)$$

where $F_R(x, t)$ is the instantaneous cumulative probability distribution function of the resistance and $f_L(x, t)$ is the instantaneous probability density function of the load effect.

The reliability of a structural component may also be represented by the reliability index, β . A typical assumption is that g is a Gaussian random variable. Accordingly, the reliability index β can be obtained from the probability of failure P_f by (Ang and Tang 1984)

$$\beta = \Phi^{-1}(1 - P_f) \quad (9)$$

where Φ is the standard normal distribution function.

Structural reliability has evolved over the past few decades such that numerous methods for the calculation of the reliability index and the corresponding probability of failure have become well established. In this study, First Order Reliability Method (FORM) is used to calculate the reliability index (Hasofer and Lind 1974). The software CALREL is used for such computations (Liu *et al.* 1989).

5. RELIABILITY BASED STRUCTURAL DESIGN

In order to account for the uncertainties in structural design, safety factors are established in design codes that in one hand magnify the design load effect, and in the other hand reduce the nominal resistance values. The general formula for deterministically ensuring structural safety with load and resistance safety factors is

$$\phi R_n \geq \sum \gamma_i L_i \quad (10)$$

where ϕ is the strength reduction factor, R_n is nominal resistance (strength), γ_i is the load factor for the i^{th} load effect and L_i is the i^{th} load effect. Load and resistance factors have been calibrated in structural codes using the concepts of structural reliability which

take into account the variability in load and resistance and ensure an acceptable level of reliability, called the target reliability, for the designed structural element.

An initial guess of the safety factors is made by solving an optimization problem where the objective is to minimize the difference between the reliability for a structural component considered and the target reliability designated for it. Then, the safety factors determined in this way are adjusted taking into account current engineering judgment and tradition.

6. SIMPLE COMPONENT EXAMPLE

Consider the fundamental structural reliability case with the linear performance function in Equation (1). Assume that this equation represents the limit state for the failure of the axially loaded tension element shown in Figure 2. If the random variables R and L are independent and normally distributed, the reliability index becomes (Ang and Tang 1984)

$$\beta = \frac{\mu_R - \mu_L}{\sqrt{\sigma_R^2 + \sigma_L^2}} \quad (11)$$

where μ_R , μ_L are the means of the resistance and load, respectively, and σ_R , σ_L are the standard deviations of the resistance and load, respectively.

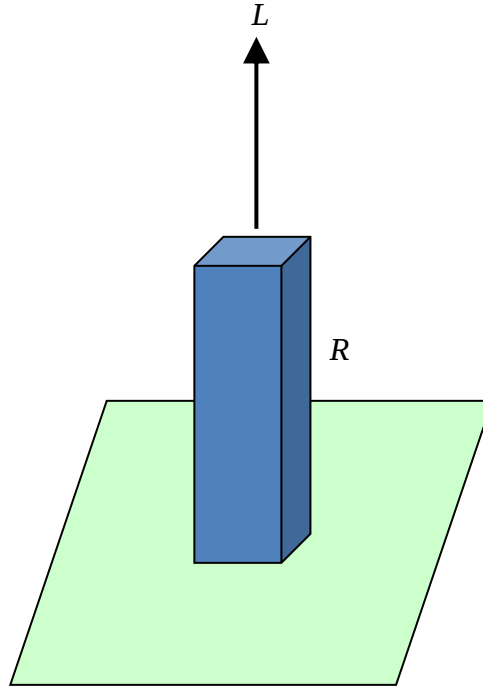


Figure 2. A structural element subjected to a tensile axial load.

The tension element in Figure 2 is assumed to be subjected to the tensile load L that is normally distributed with a mean of 20 kN and standard deviation of 2 kN. A specified mean of the resistance is assumed as $\mu_{Rs} = 25$ kN. The purpose of this example is to determine the required mean of the resistance that leads to a target reliability β_t . This is intended to simplistically resemble the case where a concrete production facility needs to determine the required mean compressive strength. Since the standard deviation is a function of the variability in the production of the resistance, and assuming that this variability is constant for any mean strength, the resistance coefficient of variation V_R is related to the standard deviation σ_R by Equation (3) as follows

$$\sigma_R = V_R \mu_R \quad (12)$$

The solution of this problem becomes one where μ_R is the root of the equation

$$\frac{\mu_R - \mu_L}{\sqrt{(V_R \mu_R)^2 + \sigma_L^2}} - \beta_t = 0 \quad (13)$$

The root of Equation (13) is found by the Optimization Toolbox of MATLAB (MathWorks 2014a) for different values of V_R and β_t and the results are shown in Figure 3. The figure shows the ratio of the required mean of strength, μ_R to the specified mean strength, μ_{Rs} as the coefficient of variation of the resistance V_R is changed for different values of target reliability indices, β_t . It is clear from the figure that the required mean strength is sensitive to both the value of the coefficient of variation of the mean resistance in addition to the target reliability.

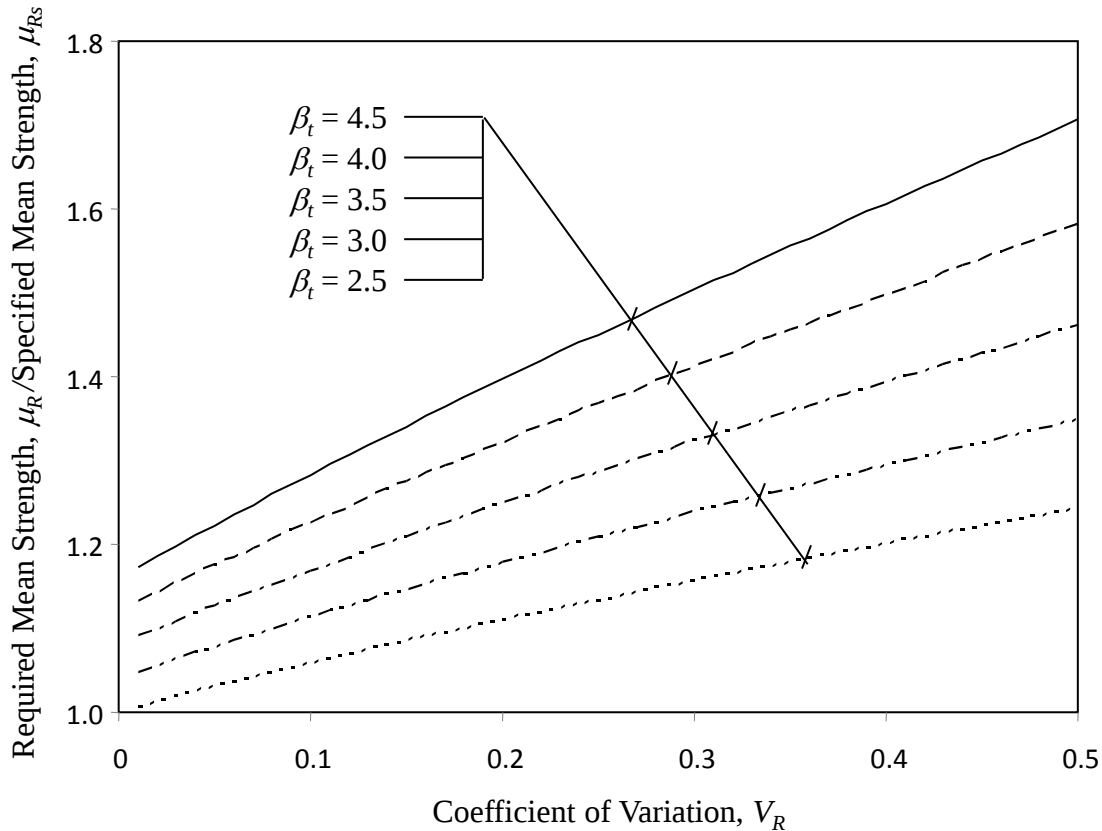


Figure 3. Variation of the ratio of the required mean strength, μ_R , to the specified mean strength, μ_{Rs} , as a function of the resistance coefficient of variation, V_R , for different target reliability indices, β_t , for the structural element subjected to a tensile axial load.

7. REQUIRED COMPRESSIVE CONCRETE STRENGTH FOR A COLUMN

In this analysis, a short concentrically loaded rectangular tied reinforced concrete column with gross cross sectional area A_g and reinforced with steel having a cross-sectional area of A_s and yield stress of f_y is considered. The design axial compressive strength of the column, ϕP_n , is calculated as (ACI 318M, 2011)

$$\phi P_n = \phi r [A_g 0.85 f'_c + A_{st} (f_y - 0.85 f'_c)] \quad (14)$$

where $r = 0.8$ is the factor accounting for accidental eccentricity in the concentrically designed tied column.

In this analysis, an arbitrary short concentric tied reinforced column is designed according to the ACI 318M-11 (2011). The column is designed which a specified concrete strength of $f'_c = 35$ MPa. The results of the design along with the associated statistical properties of the design variables are shown in Table 2 that are extracted from the literature (Nowak and Szerszen

2003a,b, Ellingwood *et al.* 1980, Mirza *et al.* 1979, Allen 1970, Cornell 1969, Ellingwood 1978, MacGregor *et al.* 1983, MacGregor 1976, 1983, Pier and Cornell 1973).

Table 2: Design variables and the associated statistical properties of the reinforced concrete column.

Quantity	Symbol	Unit	Deterministic design value	Distribution type	Bias factor	Coefficient of variation
Yield strength of steel	f_y	MPa	420	Normal	1.145	0.0500
Width	B	mm	300	Normal	1.005	0.0400
Breadth	H	Mm	400	Normal	1.005	0.0400
Area of steel reinforcement	A_s	mm ²	2512	Normal	1.000	0.0150
Distributed dead load	L_D	N/m ²	7	Normal	1.050	0.1000
Distributed live load	L_L	N/m ²	2	Normal	1.000	0.1800
Tributary area	A_T	m ²	200	Normal	1.000	0.2000

The performance function for the compressive strength limit state of a short concentric tied column constructed with concrete of required compressive strength F'_{cr} is given by

$$g_c = 0.8[A_g(0.85 F'_{cr}) + A_s(f_y - 0.85 F'_{cr})] - (L_D + L_L)A_T \quad (15)$$

The required compressive strength, F'_{cr} , is a normally distributed random variable with mean, f'_{cr} , and coefficient of variation, V_R . In order to ensure the target reliability for the column, β_{tc} , the following equation needs to be satisfied

$$\Phi^{-1}(1 - P[g_c < 0]) - \beta_{tc} = 0 \quad (16)$$

The target reliability index, β_{tc} , is set to a value of 4.0 for this column. The concrete mix design needs values for the statistical parameters of the required mean strength, F'_{cr} , that satisfies equation (16) for the target reliability, β_{tc} . The standard deviation is a function of the variability in the production of the concrete, and assuming that this variability is constant in a production facility for any mean strength, the resistance coefficient of variation V_R is related to the standard deviation by Equation (12).

For a given coefficient of variation, the problem becomes one of finding the root of Equation (16), that is the mean of F'_{cr} . A closed form solution does not exist for this problem. The evaluation of $P[g_c < 0]$ requires an iterative reliability technique. The technique used herein is FORM (Hasofer and Lind 1974). The software CALREL (Liu *et al.* 1989) is used for such computations. The iterative solution technique used for finding the root of Equation (16) is the MATLAB function `fzero` that uses a combination of bisection, secant, and inverse quadratic interpolation methods in the Optimization Toolbox in MATLAB (MathWorks 2014a). An interface between MATLAB and CALREL is established where a program written by the author modifies the input file for CALREL, runs CALREL, and extracts the reliability analysis results of CALREL from the output file in each iteration of the MATLAB root solving process. This is done for different values of the coefficient of variation, where for each value of the coefficient of variation, a required mean concrete compressive strength is determined. The results are plot in Figure 4.

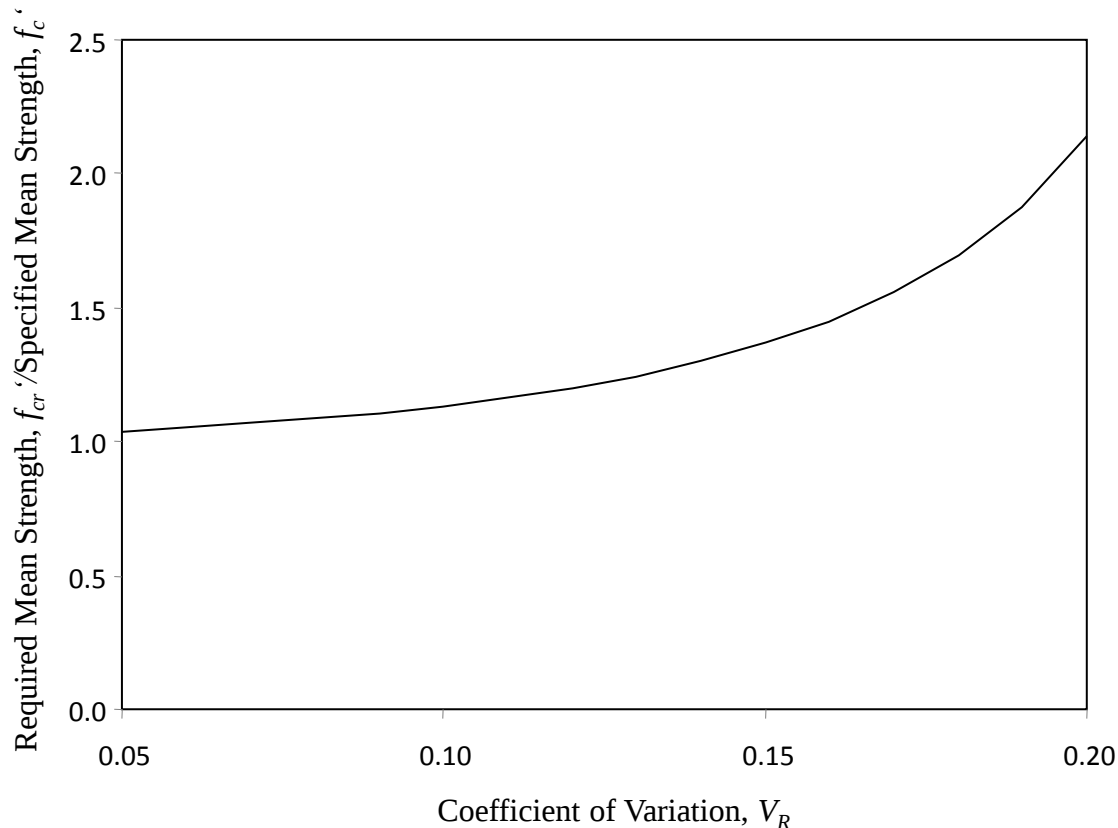


Figure 4. Variation of the ratio of the required mean strength, f'_{cr} , to the specified mean strength, f'_c , as a function of the resistance coefficient of variation, V_R , for the reinforced concrete column.

Figure 4 shows the ratio of the required mean compressive strength, f'_{cr} , to the specified mean strength, f'_c , as the efficient of variation of the resistance V_R is changed. Evidently, as the coefficient of variation increases, the ratio f'_{cr} / f'_c increases since this increase implies more variability and more uncertainty in the properties of the concrete material. This result underlines the importance of conducting a concrete mix design with a required compressive strength that takes into account the unique variability in the records of the concrete production facility.

The results obtained by the structural reliability-based analysis are compared with those obtained by the ACI equations. Figure 5 shows the required mean of strength, f'_{cr} , found from: (a) the structural reliability-based design, (b) from Equation (5) for the case where the concrete production facility has at least 30 consecutive strength test records, (c) from Equation (5) for the case where the concrete production facility has at least 15 consecutive strength test records but less than 30, and (d) from Equation (6) for the case where the concrete production facility does not have at least 15 consecutive strength test records.

Figure 5 shows that the reliability-based approach gives results that are less conservative than those of the ACI equations for low values of the coefficient of variation, and more conservative for higher values of the coefficient of variation. The results from all sources are about the same for moderate values of the coefficients of variation.

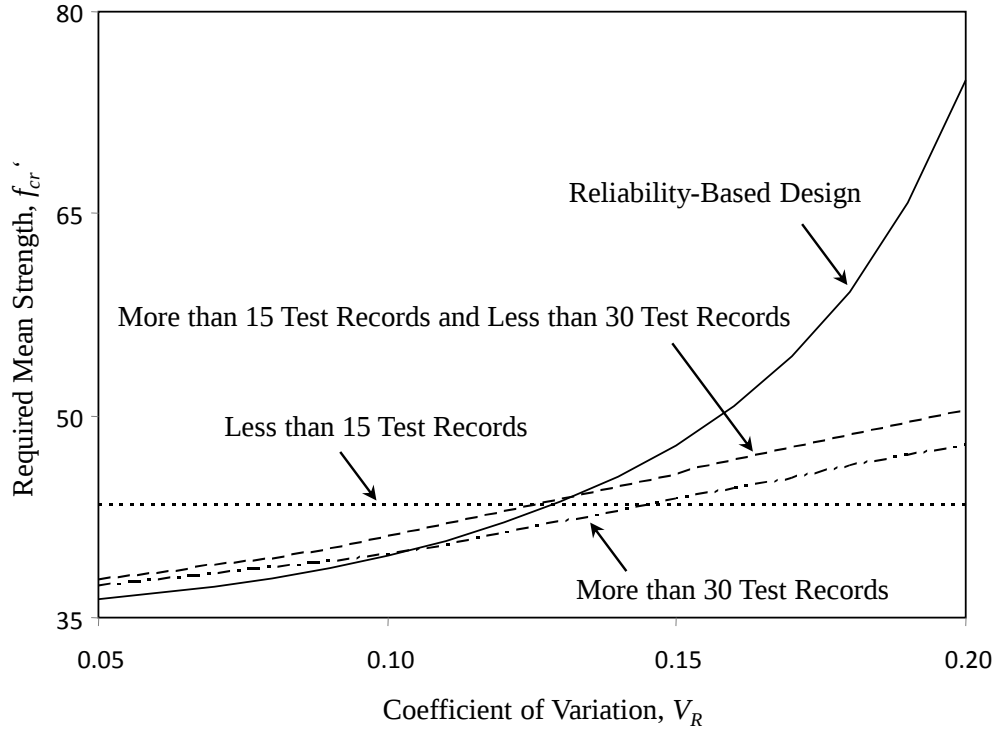


Figure 5. Variation of the required mean strength, f'_{cr} , as a function of resistance coefficient of the variation, V_R , for the tied reinforced concrete column compared with the ACI method results.

8. REQUIRED COMPRESSIVE CONCRETE STRENGTH FOR A BEAM

The design flexural strength of an under-reinforced concrete beam ϕM_n having a width b and reinforced with steel having a cross-sectional A_s located at an effective depth d is calculated as

$$\phi M_n = \phi A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b} \right) \quad (17)$$

The shear strength of this beam is given as

$$\phi V_n = \phi \left(0.17 \sqrt{f'_c} b d + \frac{A_v f_y d}{s} \right) \quad (18)$$

where A_v is the area of shear reinforcement crossing a shear crack and s is the spacing between the shear reinforcement stirrups.

Table 3: Design variables and the associated statistical properties of the reinforced concrete beam.

Quantity	Symbol	Unit	Deterministic design value	Distribution type	Bias factor	Coefficient of variation
Yield strength of steel	f_y	MPa	420	Normal	1.145	0.0500
Width	b	mm	400	Normal	1.01	0.0400
Effective depth	d	mm	739.5	Normal	0.99	0.0400
Span	L	m	8000	Normal	1.000	0.0500

Area of flexural reinforcement	A_s	mm^2	1473 (3 ϕ 25)	Normal	1.000	0.0150
Area of shear reinforcement	A_v	mm^2	79 (ϕ 8)	Normal	1.000	0.0150
Stirrup spacing	s	mm	300	Normal	1.000	0.04
Distributed dead load	L_D	N/m^2	7	Normal	1.050	0.1000
Distributed live load	L_L	N/m^2	2	Normal	1.000	0.1800
Tributary width	B_T	m	4	Normal	1.000	0.0700

In this analysis, an arbitrary simply supported beam section at midspan is designed according to the ACI 318M-11. The beam is designed with a specified concrete strength of $f'_c = 30$ MPa. The results of the design along with the associated statistical properties of the design variables are shown in Table 3 that are extracted from the literature (Nowak and Szerszen 2003a,b, Ellingwood *et al.* 1980, Mirza *et al.* 1979, Allen 1970, Cornell 1969, Ellingwood 1978, MacGregor *et al.* 1983, MacGregor 1976, 1983, Pier and Cornell 1973).

The performance function for the flexural failure limit state of a singly reinforced beam, g_m , is given by

$$g_m = A_s f_y \left(d - \frac{A_s f_y}{1.7 F_{cr}' b} \right) - \frac{(L_D + L_L) B_T L^2}{8} \quad (19)$$

and for the shear limit state, g_v , is given by

$$g_v = \left(\frac{\sqrt{F_{cr}'}}{6} b d + \frac{A_v f_y d}{s} \right) - \frac{(L_D + L_L) B_T L}{2} \quad (20)$$

In order to ensure that the target reliability associated with the flexural failure limit state, β_{tm} , and the shear limit state β_{tv} are achieved, the following equations need to be satisfied

$$\Phi^{-1}(1 - P[g_m < 0]) - \beta_{tm} \geq 0 \quad (21)$$

$$\Phi^{-1}(1 - P[g_v < 0]) - \beta_{tv} \geq 0 \quad (22)$$

The target reliability indices, β_{tm} and β_{tv} are set to the values of 3.0 and 3.5, respectively. Each of Equations (21) and (22) is separately turned into an equality and solved to find the required mean compressive strength, f'_{cr} , needed to satisfy the associated target reliabilities for a given coefficient of variation. The larger value of f'_{cr} obtained from solving both equations is considered for the mix design. The resistance coefficient of variation V_R is related to the standard deviation by Equation (12).

The software CALREL is used for the evaluation of $P[g_m < 0]$ and $P[g_v < 0]$ by FORM. An interface between MATLAB and CALREL is established by the program written by the author. The problem is solved for different values of the coefficient of variation, where for each value of the coefficient of variation, a required mean concrete strength is determined. The results are plot in Figure 6.

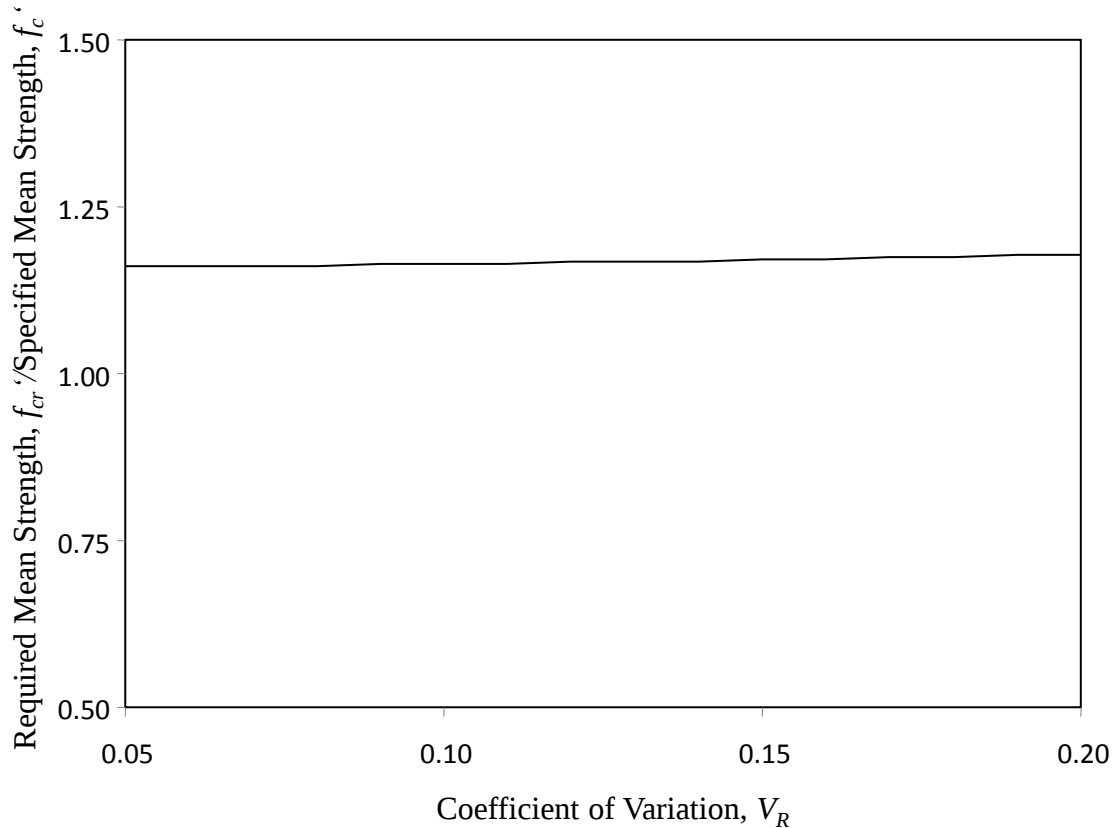


Figure 6. Variation of the ratio of the required mean strength, f'_{cr} , to the specified mean strength, f'_c , as a function of the resistance coefficient of variation, V_R , for the reinforced concrete beam.

Figure 6 shows the ratio of the required mean strength, f'_{cr} , to the specified mean strength, f'_c , as the efficient of variation of the resistance, V_R , is changed. Clearly, the ratio f'_{cr} / f'_c is insensitive to the change in the coefficient of variation. This result is consistent with the findings in Okasha and Aichouni (2015), where the reliability index was found in beams to be much less sensitive to the change in the coefficient of variation of concrete than it was found to be in columns.

9. OPTIMIZATION OF THE REQUIRED CONCRETE COMPRESSIVE STRENGTH STATISTICAL PARAMETERS

Quality control in the construction industry, particularly in the concrete production industry, has become an important topic for researchers and practitioners in the past few years (Okasha and Aichouni 2015, Aichouni 2012, ACI 121R-08, Day 2006). Statistical tools are used exclusively to classify the quality of concrete produced in ready-mixed concrete facilities. Reduction of the variability and uncertainty in the concrete properties has been the main goal in most quality control measures pursued or proposed thus far.

Any concrete production facility has its unique variability in the records of tests it has conducted over its past. This variability is typically represented by the coefficient of variation of concrete compressive strength. Due to this variability, the concrete produced must have a required compressive strength higher than the specified compressive strength as explained in this paper.

In one hand, quality control in concrete production aims to maintain the mean strength of the produced concrete to be as close as possible to the required compressive strength. On the other hand, quality control may study ways to reduce the variability, represented by the coefficient of variation, in the produced concrete. Either approach has its own associated cost. In lack of a rational approach for the selection, the decision of which approach to pursue may be subjective. Even if both parameters, i.e., the mean and coefficient of variation of the strength, are aimed for control, there is no clear indication of how much of each needs to be improved. It is shown herein by a simple optimization problem, which is based on the results of the structural reliability-based mix design, that an optimum solution can be established, where the solution entails the magnitudes for each of the mean and the coefficient of variation of the concrete strength to be targeted at the least possible cost.

Consider again the results of Figure 4. If a production plant had the option to select a value for the coefficient of variation for the concrete strength while achieving the target reliability, the required mean strength must take one of the values on the curve shown in the figure. Conversely speaking, if the plant had the option to select a value for the required concrete mean strength while achieving the target reliability, the coefficient of variation must also take one of the values on the curve shown in the figure. Thus, the problem is to determine which combination of the required mean and the coefficient of variation that satisfies the target reliability at minimum cost.

In order to find a solution to this problem, the cost associated with producing a concrete with compressive strength f'_{cr} must be known. Each production plant has its own unique cost function. For sake of illustration and maintaining generality of the approach, an example cost function is assumed where the cost of producing 1 m³ of concrete with f'_{cr} is a linear function of the value of f'_{cr} , and is given in USD currency by

$$\text{Cost of producing concrete with } f'_{cr} = 5(20 + f'_{cr}) \quad (23)$$

In addition, the cost associated with achieving a given coefficient of variation, V_R , must be known. It is also assumed that the cost of achieving V_R is a linear function of the value V_R , and is given in USD currency by

$$\text{Cost of achieving } V_R = 400(1 - 2.5V_R) \quad (24)$$

Equations (23) and (24) can be replaced with any plant-specific cost function while the same approach applies. Equations (23) and (24) are graphically shown in Figure 7. The main assumption in establishing the cost functions is that the cost proportionally increases with increasing f'_{cr} and with decreasing V_R . The total cost required for producing 1 m³ of concrete with a set of values of f'_{cr} and V_R is the sum of costs in Equations (23) and (24). The optimization problem can now be formulated as follows

Find: f'_{cr} and V_R

To Minimize:

$$5(20 + f'_{cr}) + 400(1 - 2.5V_R) \quad (25)$$

Subject to:

$$\Phi^{-1}(1 - P[g_c < 0]) - \beta_{tc} = 0 \quad (26)$$

Where:

$$g_c = 0.8[A_g 0.85 F'_{cr} + A_s(f_y - 0.85 F'_{cr})] - (L_D + L_L)A_T \quad (27)$$

In this optimization problem, the two design variables are linked. The value of f'_{cr} depends on V_R and is determined by solving the root finding problem in the equality constraint in Equation (26). The combinations of f'_{cr} and V_R that satisfy the equality constraint form the feasible space which contains the optimum solution. Accordingly, instead of solving this problem considering the two design variables as free variables, V_R is considered as the only design variable in the problem. In each iteration, the value of f'_{cr} is determined by solving the root finding problem in the equality constraint in Equation (26) and then the objective function in Equation (25) is calculated. Hence, the combination of f'_{cr} and V_R that gives the minimum total cost can be identified.

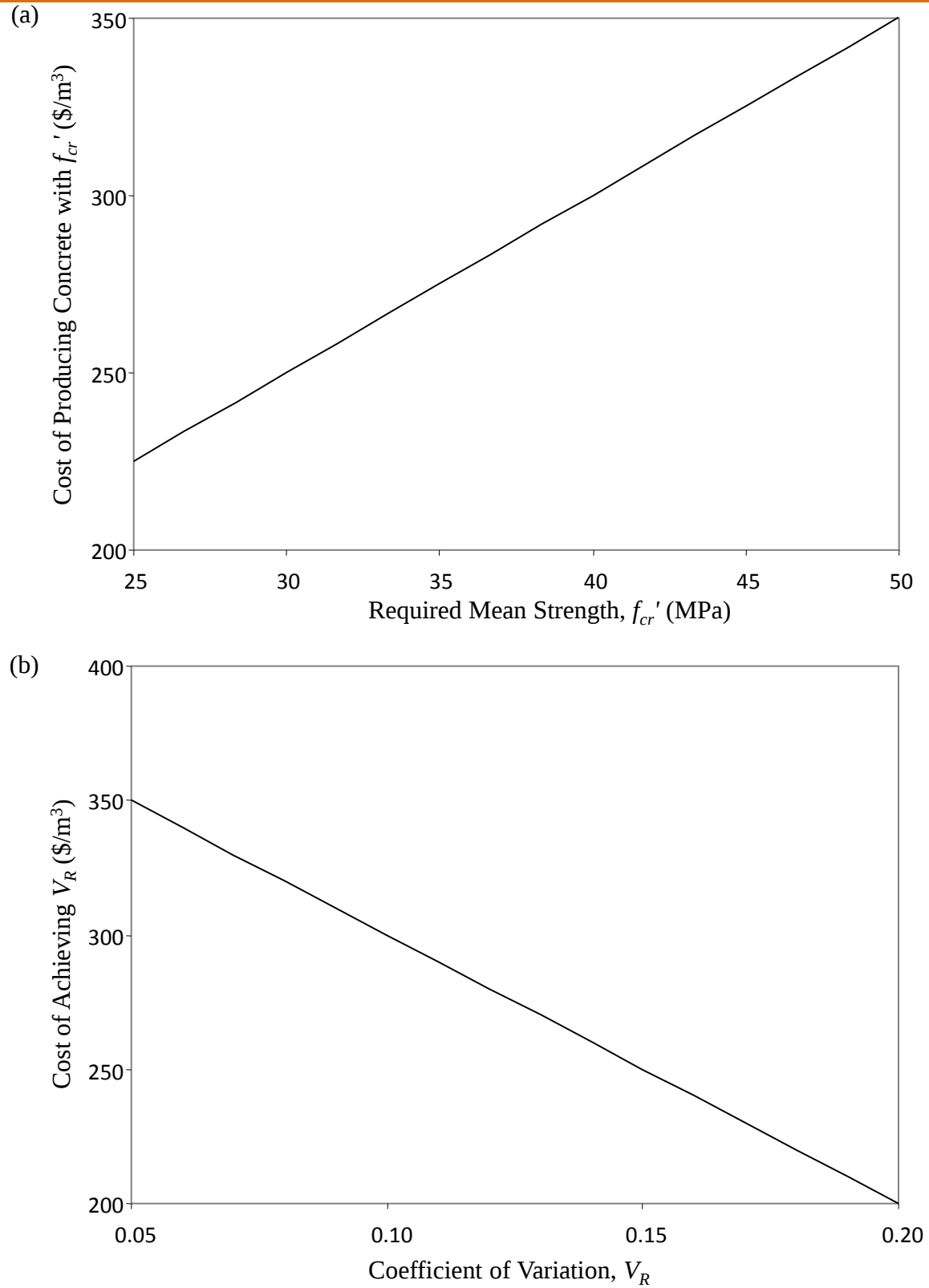


Figure 7. Cost function for (a) producing concrete with f'_{cr} and (b) achieving V_R .

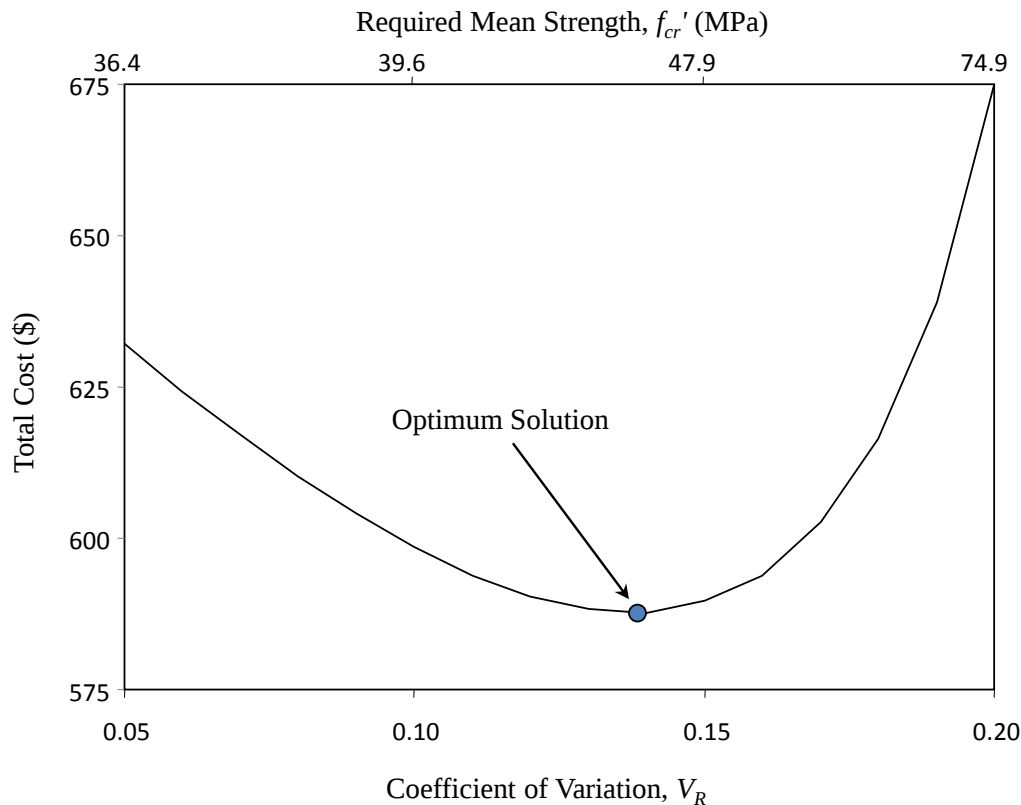


Figure 8. Optimization results for finding the best values of the required compressive strength and coefficient of variation to be targeted at a minimum total cost.

The optimization problem can be solved using numerous available techniques. Herein, it is solved using the Sequential Quadratic Programming method (SQP) in the Optimization Toolbox of MATLAB. The values of f'_{cr} are determined during the optimization process for each optimization iteration using the fzero function. An interface between MATLAB and CALREL is established where a program written by the author modifies the input file for CALREL, runs CALREL, and extracts the reliability analysis results of CALREL from the output file in each iteration of the MATLAB root solving process.

Figure 8 shows a graphical presentation of the feasible space of the combination of values of f'_{cr} and V_R that satisfy the equality constraint and the total cost of each combination for a target reliability index of 4.0 in the column case previously considered. The optimum solution is identified to be the combination where the required compressive strength to be targeted is 45.525 MPa with a coefficient of variation of 0.1402 leading to a total cost of about 587.425\$/m³. This solution depends on the cost function assumed, the target reliability considered, the limit state of the structural component and the statistical parameters used. Once these inputs are accurately established for a given concrete production facility, an optimum solution can be found following the same procedure.

10. CONCLUSIONS

This paper proposes a structural reliability-based approach for the mix design of concrete. The main focus of this paper is on determining the required compressive strength of the concrete in the mix design process. The approach is based on the structural reliability of the structures the concrete is used for constructing.

It can be concluded from the results of this paper that the required compressive concrete strength can be more accurately determined if the production facility's coefficient of variation of the compressive strength, the type of the structural element for which the concrete is used to construct and the target reliability are all considered. The approach presented in this paper provides a practical platform to efficiently consider these factors in the mix design process. It was also found in this paper that the influence of the variability of concrete on the structural reliability of beams under flexure or shear is relatively insignificant.

An optimization approach for finding the best values of the required compressive strength and coefficient of variation to be targeted at a minimum total cost was introduced. An example was provided for the case of a reinforced concrete tied column.

The practical importance of the proposed structural reliability-based approach is that ready-mixed concrete plant engineers and managers not only can decide on the degree of quality of the concrete they produce but also on the future safety of the structure being constructed using this concrete. The approach gives an ability for accurately determining the statistical properties of the required concrete strength giving into account the unique variability in the test records of the concrete production facility.

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