

# Another Results on Feebly Open Set With Respect to Ideal Topological Spaces

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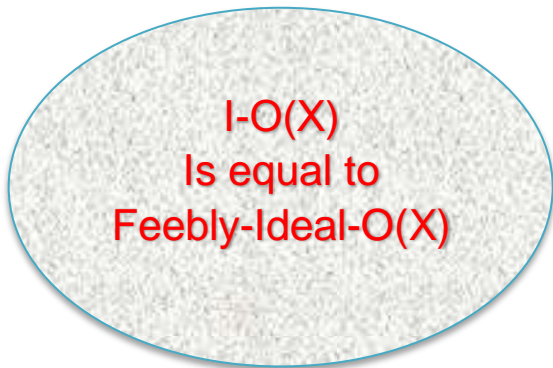
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**Abstract:** This paper introduce a new result about the definition of feebly open set in ideal topological spaces which is complementary to the results obtained in a previous research [1].

**Keywords:** feebly open set, I-open set. Feebly ideal -open set, feebly ideal  $T_i$ , ideal topological spaces

## 1. INTRODUCTION

In [1] we are defined the feebly open set with respect to ideal topological space as follow : subset  $A$  of an ideal topological space  $X$  is say to be feebly I – open set if  $A \subseteq \text{scl}(\text{int}(A^*))$  and since  $A^*$  is closed set we get that the set of all I-open set and feebly I-open set are equal , there for we use the set of feebly open set to expand the space of I-open sets to feebly –I-open sets .



Where  $A \subseteq \text{Scl}(\text{int}(A^*))$



Where  $A \subseteq \text{Scl}(\text{int}_f(A^*))$

Similar results were obtained by adding some conditions and also some other results introduced

In this paper  $\text{int}_f(A)$  mean the feebly interior of the set  $A$  where  $\text{int}(A) \subseteq \text{int}_f(A)$  and also  $\text{int}_f(A \cap B) \subseteq \text{int}_f(A) \cap \text{int}_f(B)$ .

### Definition (1-1)

Let  $(X, T)$  be a topological space and  $I$  be an ideal defined on  $X$ , a subset  $A$  of  $X$  is said to be feebly-ideal-open set in the ideal topological space  $(X, T, I)$  if  $A \subseteq \text{Scl}(\text{int}_f(A^*))$  and the family of all feebly-ideal-open sets denoted by  $F\text{-I-O}(X)$  .

### Example (1-2)

Let  $X = \{a, b, c, d, e\}$ ,  $T = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}$  and  $I = \{\emptyset, \{a\}\}$  then

$S.O(X) = \{X, \emptyset, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, b, d, e\}, \{b, c, d, e\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{a, d, e\}, \{a, c, d, e\}\}$ .

$F.O = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, c, d, e\}, \{a, b, d, e\}, \{b, c, d, e\}\}$

$$F-I-O(X) = \{\emptyset, \{d\}, \{b,d\}, \{c,d\}, \{d,e\}, \{b,c,d\}, \{b,d,e\}, \{c,d,e\}, \{b,c,d,e\}\}$$

**Definition (1-3)**

Let  $(X, T)$  be a topological space and  $I$  be an ideal defined on  $X$ , a subset  $A$  of  $X$  is said to be feebly-ideal-neighborhood of a point  $x \in X$  if there exist feebly-ideal-open set  $G$  in  $X$  such that  $x \in G \subseteq A$ .

**Theorem (1-4)**

A subset  $A$  of ideal topological  $(X, T, I)$  is Feebly-ideal-open set if and only if it is feebly-ideal-nbd. of each of its point.

**Proof**

Let  $A$  is Feebly-ideal-open then  $\forall x \in A \quad x \in A \subseteq A$  then  $A$  is Feebly-ideal-nbd for every  $x$  in  $A$ , Conversely let  $A$  is Feebly-ideal-nbd if  $A = \emptyset$  then  $A$  is open if  $A \neq \emptyset$  then  $\forall x \in A$  there exist  $U_x \in$  Feebly-ideal-open set such that  $x \in U_x \subseteq A$ ,  $A = \cup \{U_x : x \in A\}$  then  $A$  is Feebly-ideal-open.

**Definition (1-5)**

Feebly-ideal- interior of a subset  $A$  of An ideal topological space  $(X, T, I)$  is the union of all Feebly-ideal-open set containing in  $A$  and it is denoted by  $F-I-int(X)$ .

**Definition (1-6)**

Feebly-ideal-closure of a subset  $A$  of a topological space  $(X, T, I)$  is the intersection of all Feebly-ideal-closed subset of  $X$  containing  $A$ , and it is denoted by  $F-I-CL(X)$ .

**Proposition (1-7)**

Let  $(X, T, I)$  be an ideal topological space if  $\{A_\lambda; \lambda \in \Lambda\}$  is a family of feebly-ideal-open set, then  $\cup \{A_\lambda; \lambda \in \Lambda\}$  is feebly-ideal-open set.

**Proof**

Let  $A_\lambda$  is F-I-open set,  $\forall \lambda$  then  $A_\lambda \subseteq Scl(int_f(A^*_\lambda))$  then  $\cup A_\lambda \subseteq \cup Scl(int_f(A^*_\lambda))$  and then  $\cup A_\lambda \subseteq Scl(int_f(\cup A_\lambda)^*)$  therefor  $\cup A_\lambda$  is F-I-open set.

**Theorem (1-8)**

Let  $(X, T, I)$  be an ideal topological space, if  $A$  is T-open set and  $B$  is feebly-ideal-open set, then  $A \cup B$  and  $A \cap B$  is not feebly-ideal-open set.

**Example (1-9):**

Let  $X = \{a, b, c, d, e\}$   
 $T = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}$  and  $I = \{\emptyset\}$  then

$$F-I-O(X) = \{\emptyset, X, \{a\}, \{d\}, \{c, a\}, \{d, e\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, \{b, d, e\}, \{c, d, e\}, \{a, b, c, d\}, \{b, c, d, e\}, \{a, c, d, e\}, \{a, b, d, e\}\}$$

now  $\{a, c, d, e\}$  is feebly ideal open set and  $\{a, b, d, e\}$  is open set then

$\{a, c, d, e\} \cap \{a, b, d, e\} = \{a, d, e\}$  which is not feebly ideal open set and  $\{a\}$  is open set,  $\{d, e\}$  is feebly ideal open set then

$\{a\} \cup \{d, e\} = \{a, d, e\}$  which is not feebly ideal open set

**Theorem (1-10)**

Let  $(X, T, I)$  be an ideal topological space then if  $A$  is feebly-ideal-open set then  $A$  is \*-dence in itself.

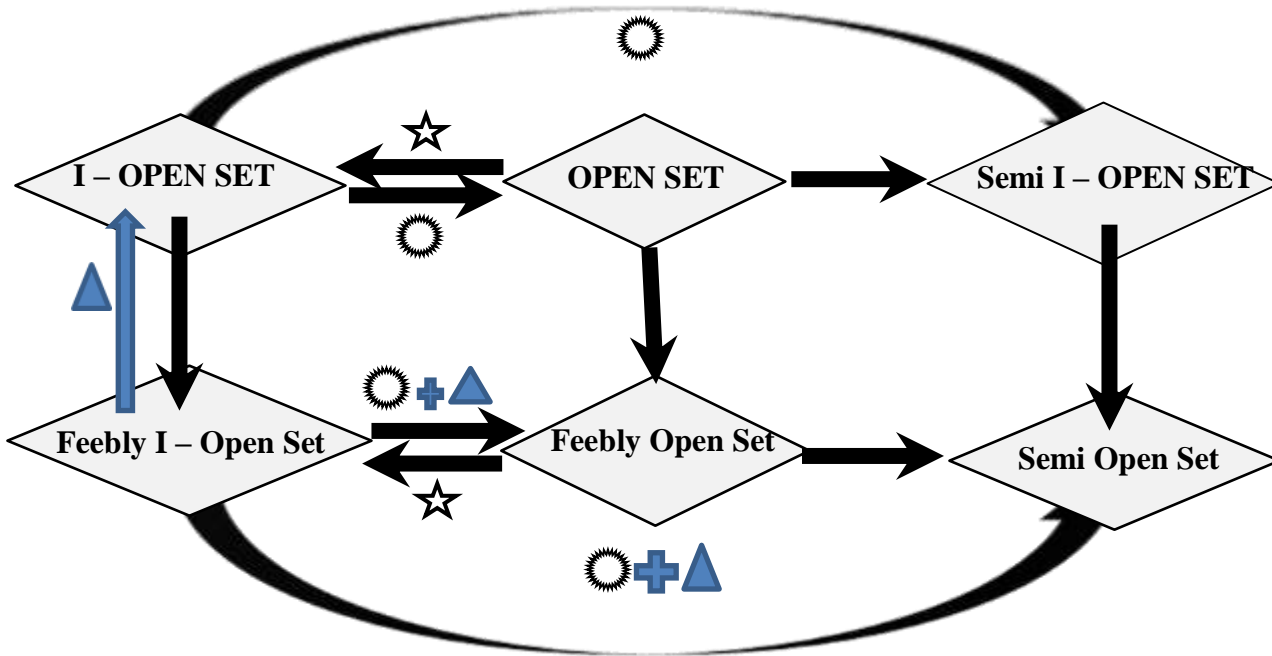
**Proof:** Exist by definition

The converse of the above theorem is not true as we show in the following example

**Example (1-11)**

By example (1.2) we get that  $\{b\}^* = \{b, e\}$  then  $\{b\} \subseteq \{b\}^*$  but  $\{b\}$  is not F-I-open.

The following diagram displays previous results with new conditions



☀ : mean that the set is  $T^*$ - closed

★ : Means that the set is  $*$ -dense in itself .

▲ : mean that every F-I-open set is open set.

**Definition (1-12)[ 2]**

Let  $(X, T, I)$  be an ideal topological space then :

- 1) **feebly-ideal- $T_0$  space** :- if for each pair of distinct point  $x$  and  $y$  in  $X$  there exist feebly-ideal-open set  $U$  in  $X$  such that  $x \in U$  and  $y \notin U$  or  $y \in U$  and  $x \notin U$  .
- 2) **feebly-ideal- $T_1$  space** :- if for each pair of distinct point  $x$  and  $y$  in  $X$  there exist feebly-ideal-open set  $U_1, U_2$  in  $X$ , such that  $x \in U_1, y \in U_2$  .
- 3) **feebly-ideal- $T_2$  space**:- if for each two pair of distinct point  $x$  and  $y$  in  $X$  there exist disjoint feebly-ideal-open set  $U_1, U_2$  in  $X$ , such that  $x \in U_1, y \in U_2$  .

4) **feebly- ideal-regular space**: if for each  $H$  closed set subset of  $X$  such that  $x \notin H$ , there exist disjoint feebly-ideal-open set  $U_1, U_2$  such that  $x \in U_1, H \subseteq U_2$  and if  $X$  is feebly-ideal-regular space and feebly-ideal- $T_1$  space, then  $X$  is called feebly - ideal- $T_3$

5) **feebly-ideal- normal space** : if for each  $U_1, U_2$  of disjoint closed subsets of  $X$ , there is a pair  $V_1, V_2$  of disjoint feebly-ideal-open sets of  $X$  such that  $U_1 \subseteq V_1$  and  $U_2 \subseteq V_2$ , and if  $X$  is feebly-ideal- $T_1$  space and feebly-ideal-normal space then  $X$  is feebly-ideal - $T_4$  space .

**Definition (1-13)**

Let  $(X, T, I)$  be an ideal topological space and let  $Y \subseteq X$  then  $(Y, T_Y, I_Y)$  is a subspace of the ideal topological space  $X$  were  $T_Y = \{G \cap Y : G \in T\}$   $I_Y = \{H \cap Y : H \in I\}$ .

**Remark (1-14)**

Let  $(X, T, I)$  be an ideal topological space than  $F-I-T_i$  – space is not hereditary property for each  $i$  where  $i=0, 1, 2, 3, 4$  .

**REFERENCES**

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