## Vehicle Routing Problems with Simultaneous Pickup and Delivery Soft Time Window

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Abstract: A mixed integer linear programing (MILP) model is presented to formulate the vehicle routing problems with simultaneous pickup and delivery soft time window where products' delivery or collecting of EOL products are performed simultaneously and early and late time intervals have to be violated (Soft) with adding some penalty costs. The presented a model is more generic that obtain a large solution space where more feasible solutions could be found for larger number of problems depending on its characteristics. The model is solved and tested with exact approach using standard benchmark problems. 85% of tested problems with large travel time got a feasible solution with the proposed VRPSPD-STW model while it didn't have a feasible solution with the HTW model.

Keywords: Vehicle Routing Problem, simultaneous pick and delivery, soft time window, hard time window

#### 1. INTRODUCTION

The vehicle routing problem (VRP) is an important problem in the fields of transportation that was first proposed by Dantzig and Ramser in 1960 [1]. It is a combinatorial optimization problem [2]seeking to serve a number of customers with a fleet of vehicles.

VRP has a wide range of applications with the objective of determining the minimum cost for all routs resulting from serving those customers and minimum number of vehicles used. Each of these optimal routs is performed by a single vehicle that start and end at the central node, each customer is visited exactly once by one vehicle. A reverse logistics networks where delivering of a demanded goods and collecting end of life products (EOL) [3] [4] within an illustrated time frame are called Vehicle Routing Problems with Simultaneous Pickup and Delivery with time window (VRPSPD-TW) its proposed by Kassem, S. and Chen, M. (2012) [5], this a hard time window constraint couldn't be violated.

In a real life environment vehicles are operated in a network with different traffic levels of congestion base on the day and time. Being restricted with the predefined time frame is not valid for some applications of VRPSPD such as food industries where delivery of products is a must even if violating the time interval. In this paper our routing problem has a simultaneous pickup and delivery with a soft time window where violating a time window constraint is allowable with adding a Penalty cost in case of arriving at the customer before the early or after the late allowable arrival time. This is called a Vehicle Routing Problems with Simultaneous Pick and Delivery Soft Time Window (VRPSPDSTW). VRPSPDSTW is first proposed by Bhusri, N., Qureshi, A.G., Taniguchi, E. (2013) [6]. They developed a mathematical model and a genetic algorithm metaheuristic-based procedure to solve this kind of problems. In their model they present a distribution and routing system for Convenience Store chains for a food retail business in Japan. One of their objective is to attaining satisfaction of their customers so they allow violating time window [a, b], (a) represent an early allowable arrival time and (b) represent the late allowable arrival time for vehicles at each franchise convenience store, with adding a penalty cost if violation happened. A tradeoff is considered between if adding an early or late penalty cost or using new vehicle with its vehicle cost.

This violation is also limited by a new early and late time window denoted by [a', b'] where vehicle is not allowed to arrive beyond this new time window, arriving after the allowed b time and before b' a tradeoff will considered between if will serving this customer and adding a late arrival penalty cost or using a new. vehicle with its fixed cost. In case of vehicle arrived before the allowed early time a, and after the new limit a', vehicle will be wait and start the service at the original allowed early time and at same time an early penalty cost will be added.

In our paper the soft time window is obtained by allowing a time violation if vehicle is not arrived within

the defined time interval with taking a penalty cost into account.

If vehicle is arrived with in the allowed time interval customer will be served without adding any penalty costs, if vehicle is arrived before the allowable early time, will serve the customer immediately adding an early penalty cost same as if vehicle is arrived after the allowable late time, will be served adding a late penalty cost. In Bhusri, N., Qureshi, A.G., Taniguchi, E. paper, they are limiting the solution space as no feasible solution will be found if vehicle arrived beyond the allowed [a', b'] time in our model a large solution space is provided so problem is more generic and mostly has a feasible solution. Also if the result of the tradeoff is using new vehicle with its fixed cost practically is always will be more costly than adding a penalty cost and will cause more traffic congestion.

The developed mathematical module in our paper is formulate to get optimal solution to obtain the following 1) Minimum transportation cost overall the rout 2) Minimum Penalty costs if happened 3) Minimum number of vehicle used. Benchmark Solomon examples are solved by our proposed mathematical model using an exact approach to get optimal solutions. The remainder of this paper is organized as the follows. Section 2, a literature review is presented where different types of VRP and VRPTW are illustrated. Section 3, a brief description of vehicle routing problem with simultaneous pickup and delivery a soft time window is explained and a formulated mixed integer linear programing (MILP) mathematical model is presented. Section 4, modified VRP Solomon problems is tested using LINGO and experimental results and analysis is clarified. Finally, conclusion and future work is represented in section 5.

#### 2. LITERATURE REVIEW

Many publications had covered vehicle routing problems with all of its extensions, e.g. Laporte,G. (1992)[7] [8], also Baldacci, R., Mingozzi, A. and Roberti, R.(2012) [9] describe the vehicle routing problem (VRP) and algorithms are developed. A Green Vehicle Routing Problem (G-VRP) was studied and solution techniques are proposed by Erdog an,S. and Miller-Hooks, E. (2012) [10]. Another extension is Dynamic vehicle routing problem (DVRP) where new orders arrive when the working day plan is in progress, its studied by Khouadjia, M.R., Sarasola, B., Alba, E., Jourdan, L. and Talbi, E. (2012) [11] where a solution method based on particle swarm optimization and variable neighborhood search paradigms is proposed and Hong,L. (2012) [12]proposed a large neighborhood search (LNS) algorithm to solve a dynamic vehicle routing problem with hard time windows (DVRPTW). Capacitated vehicle routing problem and Distance-Capacitated vehicle routing problems mentioned in Bortfeldt, A.(2012) [13] and in Kek, A.G.H., Cheu, R.L. and Meng, Q. (2008) [14], while Heterogeneous Fleet Vehicle Routing Problem is studied by Penna, P.H.V,

Subramanian, A. and Ochi, L.S. (2013) [15] and Subramanian, A., Penna, P.H.V., Uchoa, E. and Ochi, L.S. (2012) [16].

Belmecheri, F., Prins, C., Yalaoui, F. and Amodeo, L. (2013) [17] studied a complex VRP called HVRPMBTW (the Vehicle Routing Problem with Heterogeneous fleet, mixed backhauls, and Time Windows) and proposed a Particle Swarm Optimization (PSO) approach with Local Search (LS) to solve this problem. Vehicle routing problems with time window has also studied by several researchers e.g. Vidal, T., Crainic, T.G., Gendreau, M. and ChristianPrins (2013) [18] presented an efficient Hybrid Genetic Search with Advanced Diversity Control for a large class of time-constrained vehicle routing problems. Also Kok, A.L., Hans, E.W. and Schutten, J.M.J. (2012) [19] compared four strategies for avoiding traffic congestion through developing better vehicle route plans and proposed a speed model on real road networks that reflects the key elements of peak hour traffic congestion and used this speed model to generate a set of realistic VRP instances for testing the impact of the different strategies. VRP-TW was improved to involve a simultaneous pick-up and delivery which is called Vehicle routing problem with simultaneous pickup and delivery and its studied by several researchers e.g. Kassem, S. and Chen, M. (2013), they developed a heuristic solution methodology to solve vehicle routing problems combining simultaneous pick-up and delivery with time windows problems, and proposed a solution methodology which consists of a heuristic method to generate high-quality initial solutions. Then a simulatedannealing-based search process is used to improve the initial solutions. Also Tasan, S. and Gen, M. (2012) [20] provided efficient and effective genetic algorithm based approaches that provide feasible routes for VRPSPD, computational example is presented with parameter settings in order to illustrate the proposed approach and performance of the proposed approach is evaluated by solving several test problems.

Due to a wide range of applications found in VRPSPD, soft time window is a potential in a real life applications as strictly following the Time Window interval is not practically preferred in many applications and industries like food industries where products delivery is necessary even if time interval is violated. The following are studies obtained in the VRPSTW:

Tas,D., Dellaert,N., Woensel,T.V. and Kok,T.,D. (2013) [21] described the vehicle routing problem having a soft time window and a stochastic travel time which leading to stochastic travel time, they proposed the first model that differentiate between transportation cost and service cost. Another contribution they had in their paper, they proposed a solution approach to solve this problem where first, construct an initial feasible solution taking into consideration adding penalty costs to the total transportation cost. Second, a Tabu search algorithm is proposed to improve this initial solution and finally proposed a post-optimization method to enhance the solution provided by the tabu search algorithm. Due to the important of the travel time stochasticity, they continued their work in (2014) [22] [23] to build efficient and reliable routs for the medium and large sized problems. They proposed a branch and price method and an exact solution approach based on column generation algorithm.

Jiuping, Xu., Fang, Y., Steven, L. (2011) [24] studied a vehicle routing problems soft time window in Fuzzy systems where they found some stochastic factors in VRPSTW environment, such as arrival time at customer node was hardly obtained due to some reasons like whether, season, traffic congestion of some days, customer change his arrival time due to the day, more than one person is in charge at the customer site which cause a stochasticity in choosing a respondents. In their paper they proposed a multi-objective programming model for the VRPSTW in a fuzzy random environment with two objectives, minimizing the total transportation cost and maximizing the level of customers' satisfaction. They proposed a Global Local Neighbor Particle Swarm Optimization (GLNPSO-ep) to solve their problem and a case study that contains uncertain information to illustrate the effectiveness of the proposed GLNPSO-ep.

Beheshti, A.K., Hejazi, S.R. (2015) [25] found that VRPSTW has more advantage due to its flexibility, however due to the nature of such problems is strongly NP-Hard, the difficulty is provided from the complexity of the delivery time cost function. To solve these complex problems they proposed a mathematical model and an efficient hybridization column generation metahearistic approach. The proposed model and approach are performed in two modes, integrative and collaborative, in the integrative phase to solve a column generation sub problems they used a quantum inspired evolutionary algorithm while in the collaborative mode they paralyzed column generation and electromagnetism algorithm then exchange the information obtained to find better solution. They use a modified benchmark problems to test the proposed approaches. A Combination between two extensions of the VRP the VRPSTW and VRPSPD is a VRPSTWSPD. It's firstly introduced by Bhusri, N., Qureshi, A.G., Taniguchi, E. (2013) paper where they proposed the VRPSTWSPD model for convenience store chains in Japan. A mathematical model is formulated in their paper, then a branch and price exact based approach has been also proposed to solve small problems and a genetic algorithm metaheuristic based approach has been developed to solve large scale problems. They test their work using a modified benchmark problems.

#### 3. Problem Description and Model

We present a below a Mathematical model developed for the VRPSPDSTW after defining the problem description.

#### 3.1. Problem Description

VRWSPDSTW is an updated model from the VRPSPD-TW where networks of customers are served using a fleet of vehicles. A determined amount of new and EOL products will be delivered and picked up from those customers simultaneously.

Vehicles departure from a one central point with a limited predefined capacity going to serve those customer only for one time where delivering of a needed demand and collecting a defined pickup returning to the central depot.

A time window interval is defined for each customer with early and late limits. If vehicle arrived not within this known time interval, customers will be served immediately with considering an addition penalty costs in account. We consider a predefined early and late penalty cost in case of violating the time window. To simplify our model we set costs (Transportation cost, early penalty cost and late penalty cost) for the central depot equals to zero and it doesn't have a time window limits or a capacity limit.

The solution of the problem is to build an optimal route for each vehicle with considering the following, minimum transportation cost, minimum additional penalty costs and minimum number of vehicles used. The mathematical model of our VRPSPDSTW model is defined below after defining the notations and variables.

#### 3.2. Mathematical Model

According to the above description given for the problem, a MILP model for the VRPSPDSTW is formulated and given below

#### • Parameters

N: Number of customers

K: Maximum number of vehicles to employ

VC: Vehicle capacity

 $\label{eq:constraint} \begin{array}{l} C &: Transportation \mbox{ cost between node } i \mbox{ and node } j, \\ \mbox{where node } 0 \mbox{ represents the site of the central depot; } i, j \\ = 0, 1, 2... \ N \mbox{ and } C_{ii} = 0. \end{array}$ 

\_C : Early penalty cost for customer i, if vehicle arrive before the earliest allowed start time at node i, the service will start with considering this penalty cost; i= 0, 1, 2... N and  $PC_E_0 = 0$  where 0 is a central depot.

\_C : Late penalty cost for customer i, if vehicle arrive before the earliest allowed start time at node i, the service will start with considering this penalty cost; i=0, 1, 2... N and PC\_L<sub>0</sub>=0 where 0 is a central depot.

T<sub>ij</sub>: Travel time between node i and node j, i, j = 0, 1, 2... N and T<sub>ii</sub> = 0.

 $E_j$ : Earliest allowed start time of service for the customer at node, j = 0, 1, 2...N.

 $L_j$ : Latest allowed start time of service for the customer at node, j = 0, 1, 2...N.

 $Sr_j$ : Service time for the customer at node, j = 0, 1, 2...N

 $D_j$ : Amount of new products to deliver to the customer at node, j = 0, 1, 2....N.

 $P_j$ : Amount of EOL products to collect from the customer at node, j = 0, 1, 2...N.

M: A positive large number

#### • Decision Variables

 $L_{0v}$ : Load of vehicle v when it leaves the depot, v = 1,2...K

 $L_{jv}$ : Load of vehicle v after it serves the customer at node j, j = 0, 1, 2...N

 $St_j$ : Service starting time for the customer at node, j = 0, 1, 2...N

 $X_{ijv} \begin{cases} 1, if vehicle v travels from node i to node j, i, j = 0, ..., N\\ 0, otherwise \end{cases}$ 

 $\text{PE}_{T_{iv}} \begin{cases} 1, \text{if early start time for customer } i, \text{St}_i < \text{E}_i \text{ , } i \ \notin [\text{E}_i \text{ , } \text{L}_i] \\ 0, \text{otherwise} \end{cases}$ 

 $\begin{array}{l} \text{PL}_{T_{\mathbf{iv}}} \begin{cases} 1, \text{ if Late start time for customer } i, \text{St}_i > L_i \text{ , } i \ \notin \ [E_i \text{ , } L_i] \\ \\ 0, \text{ otherwise} \\ i = 1, \dots, N \text{ And } v \ = \ 1, \dots, K \end{cases}$ 

The objective function of the model is to minimize the total transportation cost and minimize the additional Penalty early and late costs:

 $\begin{array}{l} \text{Minimize } z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{v=1}^{K} C_{ijv} * X_{ijv} + \\ \sum_{i=1}^{N} \sum_{v=1}^{k} \text{PE}_{-}C_{i} * \text{PE}_{-}T_{iv} + \sum_{i=1}^{N} \sum_{v=1}^{k} \text{PL}_{-}C_{i} * \text{PL}_{-}T_{iv} \\ (1) \end{array}$ 

Subject to the following constraint functions:

 $\sum_{i=1}^{N} x_{0jv} = 1 \qquad v = 1, 2...k \quad (2)$ 

 $\sum_{i=0}^{N} \sum_{v=1}^{k} x_{ijv} = 1 \qquad j = 1, 2...N \qquad (3)$ 

 $\sum_{i=0}^{N} x_{ibv} = \sum_{i=0}^{N} x_{biv}$  b = 1, 2...N, v = 1, 2...k (4)

$$L_{0v} = \sum_{i=0}^{N} \sum_{j=1}^{N} D_{j} * x_{ijv} \quad v = 1, 2...k$$
 (5)

$$\begin{split} & L_{jv} \geq L_{av} - D_j + P_j - M \left[ 1 - \sum_{v=1}^{K} x_{ijv} \right] \\ & a = 0, 1..N, \, i, j = 1..N, \, v = 1..K, \, a \neq j \, , \, i \neq j \end{split}$$

$$L_{jv} \le Vc$$
  $j = 0,1...,N, v = 1,2...K$  (7)

$$\begin{array}{ll} \operatorname{St}_{i}+\operatorname{T}_{ij}+\operatorname{Sr}_{i}-\operatorname{M}\times\left[1-\operatorname{x}_{ijv}\right]\leq\operatorname{St}_{j}\\ i=0...\mathrm{N},\,j=1...\mathrm{N},\,v=1...\mathrm{K} \end{array} \tag{8}$$

$$M \times PE_{T_{iv}} + St_i \ge E_j, \tag{9}$$

$$-M \times (1 - PE_T_{iv}) + St_i < E_j$$
, (10)

$$-M \times PL_{-}T_{iv} + St_{i} \leq L_{j}, \qquad (11)$$

$$M \times (1 - PL_T_{iv}) + St_i > L_j,$$
(12)  
i = 1....N, v = 1, 2...K

 $X_{ijv} = \{0 \text{ , } 1\} \qquad i,j = 0,1....N, v = 1,2...K \qquad (13)$ 

 $PE_{-}T_{iv} = \{0, 1\} \quad i = 0, 1..., N, v = 1, 2...K$  (14)

$$PL_T_{iv} = \{0, 1\}$$
  $i = 0, 1..., N, v = 1, 2...K$  (15)

The objective function (1) in the above developed model is to get an optimal set of routes with minimize the total route cost (transportation cost, early penalty cost and late penalty cost). While solving the model using an exact approach, trials are tested for each problem to use a minimum number of vehicles. Constraint no. 2 is to make sure that each vehicle starts from the depot where in constraint no. 3 we make sure that each customer is visited only one time using one vehicle. Constraint no.4 ensure that vehicle arrived at any node must leave that node. Constraint no. 5 is to make sure that load of vehicle at the central depot is equals to the demand needed for all customers and in constraint no. 6 calculate vehicle load at each node and in no. 7 we make sure that load at each node not exceed the vehicle capacity. In constraint no. 8 we calculate the vehicle arrival time at each node in the established route.

Soft time window constraints are presented in sets from 9 to 12 where in constraints no. 9 and no. 10, allow vehicle to start service for customer i if arrived after the early allowable time (within time interval) or earlier (violating time interval).

And in constrains no. 11 and no. 12, allow vehicle to start service for customer i if arrived before the latest allowed time (within time interval) or after (violating time interval). Constraints 13, 14 and 15 are binary and integrality constraints.

#### 4. Testing Problems

The known 56 Solomon's benchmark problems introduced by (Solomon 1987) for the VRPTW, where modified by (S. Kassem and M. Chen in 2013) to add a pickup amount for each customer to solve their VRPSPD-TW model. Those Solomon examples have 1 depot and 100 customers with different characteristics; they are divided into 6 groups (C1, C2, R1, R2, RC1, and RC2). Customers' nodes in C1 and C2 groups are clusters and in groups R1 and R2 are scattered while in RC1 and R C2 some customers are clusters and others are scattered. Time window intervals are small for customers in group C1, R1 and RC1 and vehicles' capacity used is 200 while fleet size is large, on the other hand rime window intervals are bigger in groups C2,R2 and RC2 while the used vehicles capacities are 700 and 1000 for C2 and (R2, RC2) respectively and fleet size is small.

Demand  $D_j$ , Pickup  $P_j$  and Service Time  $Sr_j$  for each customer j, are given in the modified Solomon examples used for the VRPSPD-TW model. Transportation Cost  $C_{ij}$  and Travel time  $T_{ij}$  for customers from I to j are also given in the modified examples base on the calculated distance between node i and node j where x and y coordinates are determined.

For testing our VRPSPD-STW model we use those modified problems, we got a same solution for almost problems solved by VRPSPD-TW in Kassem and chin model. Some modifications need to be added to those problems so we can test the soft time window model, such that new travel time is used in the VRPSPD-STW model, and for simplicity penalty costs used are considered same as the calculated costs for each customer j. The new travel time is calculated as follows: T ij = t ij \* r where r is a random number between [1,10] range. r was increased until no feasible solution is found by the VRPSPD-TW model, then these modified T ij values are used to be solved by the VRPSPD-STW model, we got an optimal solution for almost problems tested with our VRPSPD-STW model by Lingo.

#### 5. Results and Analysis

The formulated model was coded and solved on an exact approach, LINGO. Runs were performed on Intel(R) Core i7-4600M CPU 2.90 GHz, and 8 GB RAM-OS Windows10 64 bit.

Problems' naming convention are the same as problems' names used in S. Kassem and M. Chen paper, where problem P10-C101 is means that original Solomon problem C101 with 10 customers data.

In Table 1, Table 2, Table 3 and Table 4 we present the obtained results for the used modified

benchmark problems on Lingo. Each table illustrate the used Solomon benchmark problem name and number of customers tested, the optimal solution result for the not modified Travel Time  $T_{ij}$  problems on both VRPSPD-TW and VRPSPD-STW models in columns HTW, STW respectively and result for the modified T problems whereas described before used travel time is increased so we couldn't get feasible solution using the Hard Time Window model and optimal feasible solution found by our Soft Time Window model.

**Table 1**: HTW and STW models comparison for 10

 Customers Problems

Problem	HTW	STW	HTW - modified	STW - modified T <sub>ii</sub>
			T <sub>ij</sub>	mounicu 1 <sub>1j</sub>
P10-C101	84	84	No Feasible S.	99.61810
P10-C107	51.7212	51.7212	No Feasible S.	66.85420
P10-C201	No Feasible So.	199.495	No Feasible S.	250.1970
P10-C202	No Feasible So.	149.000	No Feasible S.	161.0000
P10-R101	242.8430	256.241	No Feasible S.	343.6440
P10-R102	201.5840	201.796	NO Feasible S.	338.2340
P10-R105	242.8415	235.0279	NO Feasible S.	235.0279
P10-R205	207.0730	200.4810	NO Feasible S	200.7760
P10-R211	167.0090	164.367	NO Feasible S	167.0090
P10-R208	164.3673	164.3673	NO Feasible S	164.3673
P10-RC106	150.9023	152.811	NO Feasible S	207.3952
P10-RC108	126.4870	126.487	NO Feasible S	126.487
P10-RC204	142.5002	142.5002	NO Feasible S	175.1282
P10-RC207	51.7212	51.7212	NO Feasible S	175.1282

In the above Table 1, we present comparison to the tested problems for 10 customers, optimal routes are established and optimal solution is found in maximum 2 minutes of running time. Maximum number of vehicles used is 3 as recorded in the below table. For problem P10-C201 and P10-C202 in Hard Time window model and original Travel time no feasible solution could be found.

Same comparison is applied for benchmark problems with 15 customers in Table 2 and for problems with 20 customers in Table 3.

For the modified problem where increasing the travel time between nodes i and j, as seen no feasible solution found in the HTW model and optimal solutions are found with using the STW model.

Almost of all small sized problem could be solved and optimal solution could be reached at a reasonable running time using Lingo. We could not reach optimal solution for problems with size more than 20 customers.

Table 4 illustrate the lower bounds and best feasible cost values of the cost objective function could be reached by LINGO after more than 20 hours of computing running time for problems P50-C101, P50-C107 and P50-R105 where in problem P50-R105 we got no feasible solution after 2 hours of running time. No feasible solution could be found by LINGO, for the other tested problems with 35 and 50 customers.

**Table 2**: HTW and STW models comparison for 15Customers Problems

Problem	HTW	STW	HTW -	STW -
			modified T <sub>ii</sub>	modified
			,	T <sub>ij</sub>
P15-C101	141.374	146.5001	NO Feasible S	185.4475
P15-C107	141.374	146.5001	NO Feasible S	185.4475
P15-C201	NO Feasible S.	264.777	NO Feasible S.	393.4360
P15-C202	NO Feasible S.	193.000	NO Feasible S.	193.000
P15-R105	330.9376	317.4438	NO Feasible S	330.9738
P15-R205	266.5391	266.5391	NO Feasible S	302.4771
P15-R211	263.3911	256.5760	NO Feasible S	266.5391
P15-R208	256.5786	256.5786	NO Feasible S	262.0026
P15-RC106	269.3679	259.4476	NO Feasible S	313.6926

**Table 3**: HTW and STW models comparison for 20Customers Problems

Problem	HTW	STW	HTW - modified T <sub>ij</sub>	STW - modified T <sub>ij</sub>
P20-C201	No feasible	486.85	NO Feasible S	486.8599
P20-C202	No feasible	289.00	NO Feasible S	289.0000
P20-R205	325.8871	321.09	NO Feasible S.	322.0942
P20-R211	297.7662	297.76	NO Feasible S.	300.9142
P20-R208	287.5046	287.50	NO Feasible S	296.1653
P20-R105	415.7298	284.8950	NO Feasible S	562.9846

**Table 4**: HTW and STW models comparison for 50Customers Problems

Problem	HTW	STW	HTW - modified T <sub>ij</sub>	STW - modified T <sub>ij</sub>
P50-C101	358.713	358.713	NO Feasible S	584.18
<b>P50-C107</b>	395.253	273.853	NO Feasible S	365.476
P35-R105	608.88	608.88	NO Feasible S.	684.18

In problems P50-C101, P50-C107 and P50-R105 number of used cars are 3, 5 and 5 respectively and this lower bound solution found after 45, 20 and 20 hours respectively from a CPU running time.

#### 6. Conclusion and future work

Due to a wide range applications found in VRPSPD-STW area especially in food industries where in real life environment violating a time window limits is a normal due to traffic congestion per day. A generalized VRPSPD-STW model is proposed in this paper, where a generic model is formulated to accept any time window violation without adding another limits on time. The objective of the developed model is to establish a set of optimal routes with minimum transportation cost, minimum penalty cost and minimum number of vehicles used. In this work a modified benchmark problems are used and results indicates that the proposed model works well and obtains an optimal feasible solution for the modified problems with large travel time comparing with VRPSPD-HTW model's results and for the small travel time STW model has at most times better solution than the HTW.

Although Impotency of the exact approach but it does respected only to small scale problems, as it takes very long computational time trying to get optimal solution for a large scale problems. In our future work we are planning to develop a Heuristic based approach so we can get optimal solution for problems with large scale customers' nodes.

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