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Approximation Space Via Topological Structures

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Abstract- *Most granulation methods did not go deep in using topological structure. In this work we aim to use general topological structures as tools for approximation space in information systems. General relations to get granules that form subbase for topology. This topology is applied for obtaining lower and upper approximation. The suggested topological structure opens up the way for applying rich amount of topological facts and methods in the process of granular computing.*

Keywords: Rough set, topology and approximation space.

1. Introduction

Topology and its branches have become hot topics, not only for almost all fields of mathematics but also for many areas of science such as chemistry [2], physics [1], and information systems [13]. In the last decade of 20th century, the revolution of information has become in the focus of interest, topology has a significant place in this age; the age of information. The basic problem in this age is how to transform data to knowledge by using the available information.

The notion of rough sets was introduced by Pawlak in his seminal paper of 1982 [10]. It is a formal theory derived from fundamental research on logical properties of information systems. From the outset, rough set theory has been a methodology of database mining or knowledge discovery in relational databases. In its abstract form, it is a new area of uncertainty mathematics closely related to fuzzy theory. Rough sets and fuzzy sets are complementary generalizations of classical sets. In this work, we aim to use general topological structures as tools for decision making in information systems. General relations to get granules that form subbase for topology. This topology is applied for obtaining lower and upper approximation.

The approach we used depends on the topological concepts "interior and closure operators" which gives the lower and upper approximations.

2. Basic Concepts

2.1. Approximation Space

The approximation space [3,4,5,6,7,11] is a pair of (U,R), where U is a non-empty finite set of objects (states, patients, digits, cars, …….etc) called a universe and R is an equivalence relation over U which makes a partition for U, i.e. a family $C=\{X_1,X_2,X_3,\ldots\ldots,X_n\}$ such that $X_i \subseteq U$, $X_i \neq \phi$, $X_i \cap X_j=\phi$ for $i\neq j$, $i,j=1,2,3,\ldots\ldots,n$ and $\cup X_i=U$, the class C is called the knowledge base of (U,R).

The universe U of objects with relation R play an important role in converting data into knowledge which use R as a tool of a mathematical model for dealing with members and subsets of U. Thus we can say that R changes U from just being a set to a mathematical model. we will use $R_x \subseteq U$ to denote the equivalence class containing $x \in U$. In the approximation space, we consider two operators, the upper and lower approximations of subsets: Let $X \subseteq U$.

$$\overline{R}X = \{x \in U: R_x \cap X \neq \phi\}, \text{ "upper approximation"}$$

$$\underline{R}X = \{x \in U: R_x \subseteq X\}, \text{ "lower approximation"}$$

$$BN_R(X) = \overline{R}X - \underline{R}X, \text{ "boundary region".}$$

The Lower Approximation Interval [11] can be and must be defined inside of the Upper Approximation Interval. For instance, the definition of “Warm” in a temperature variable may be considered. Common sense and general knowledge can help defining its limits, for example:

– The temperature of the human body is about 37 °C. Nothing warmer will be considered “Cold”.

– In average, a human hand cannot hold an object, whose temperature is over 70°C, because it is “Hot”.

– If something is colder than the environment (let us say 17 °C), it cannot be considered “Warm” anymore.

A few statistical considerations, together with some sense of symmetry may assist an expert completing the definition of this Rough Interval "RI" and its neighbors. A set of two crisp intervals can represent the resulting RIs. The Rough Interval shown in Fig. 1 and Fig. 2 represents the qualitative value “Warm”. The key fact in this example was the use of precise concepts to define an imprecise one. They may be supported by verifiable knowledge, statistics or physical laws, which are in general measurable, trustworthy and easier to model than the original vague concept.

The Rough Set Theory reduces the vagueness of a concept to uncertainty areas at their borders.

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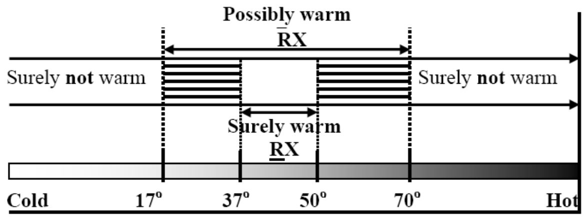

Fig. 1 "Warm" represented as a Rough Interval.

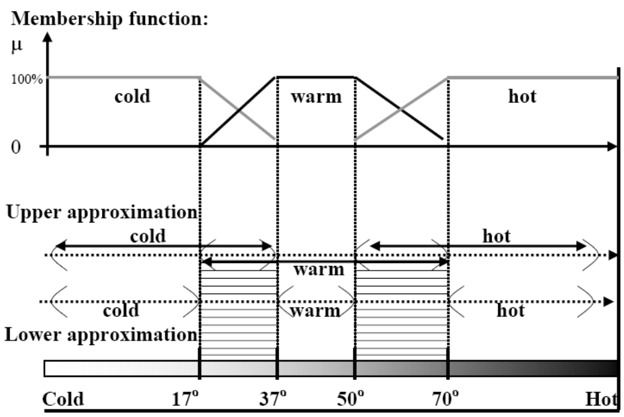

Fig. 2 The Rough interval notation

Let U be a finite set of elements called the universe and A be a non-empty finite set of attributes $a \in A$, such that a: $U \rightarrow V_a$ .The set V_a is called the range of the attribute a. For an element $x \in U$ and an attribute $a \in A$, the pair $(x,a(x)) \in U \times V_a$ indicates that x has the attribute value a(x). The pair (U,A) is called an information system [8] and is often referred to as a single-valued information system [9]. In a single-valued information system, attributes $a \in A$ map elements $x \in U$ to a single attribute value v=a(x) in the range V_a.

A multi-valued information system is a generalization of the idea of a single- valued information system. In a multi-valued information system, attribute functions are allowed to map elements to sets of attribute values [9]. More formally, we allow multi-valued attributes a such that a: $U \rightarrow 2^{V_a}$. A subset $a(x) \subseteq V_a$ may also be referred to as an attribute value.

In a multi-valued information system (U,A), each attribute $a \in A$ implies a relation $R_a \subseteq U \times V_a$ by setting $xR_a v \Leftrightarrow v \in a(x)$.

A single-valued information system is being a particular case of a multi-valued information system.

2.2. Topological Space

A topological space [12] is a pair (U,τ) consisting of a set U and family τ of subset of U satisfying the following conditions:

(T1) $\Phi \in \tau$ and $U \in \tau$.
(T2) τ is closed under arbitrary union.
(T3) τ is closed under finite intersection.

The pair (U,τ) is called a space, the elements of U are called points of the space, the subsets of U belonging to τ are called open set in the space, and the complement of the subsets of U belonging to τ are called closed set in the space; the family τ of open subsets of U is also called a topology for U.

It often happens that the open sets of space can be very complicated and yet they can all be described using a selection of fairly simple special ones. When this happens, the set of simple open sets is called a base or subbase (depending on how the description is to done). In addition, it is fortunate that many topological concepts can be characterized in terms of these simpler base or subbase elements. Formally, A family $\beta \subseteq \tau$ is called a base for (U, τ) iff every non_empty open subset of U can be represented as a union of subfamily of β. Clearly, a topological space can have many bases. A family $S \subseteq \tau$ is called a subbase iff the family of all finite intersections is a base for (U, τ).

$\overline{A} = \cap\{F \subseteq U: A \subseteq F$ and F is closed$\}$ is called the τ-closure of a subset $A \subseteq U$.
Evidently, $\overline{A}$ is the smallest closed subset of U which contains A. Note that A is closed iff $A = \overline{A}$.

$A^o = \cup\{G \subseteq U: G \subseteq A$ and G is open$\}$ is called the τ-interior of a subset $A \subseteq U$.
Evidently, A^o is the union of all open subsets of U which containing in A. Note that A is open iff $A = A^o$. And

$A^b = \overline{A} - A^o$ is called the τ-boundary of a subset $A \subseteq U$.

We will express rough set properties in terms of topological concepts. Let $X \subseteq U$,
$\overline{X}$, X^o and X^b be closure, interior, and boundary points respectively. X is exact if $X^b = \Phi$, otherwise X is rough. It is clear X is exact iff $\overline{X} = X^o$. In Pawlak space [9] a subset $X \subseteq U$ has two possibilities rough or exact. For a general topological space, $X \subseteq U$, X has four types of definability [4].

3. Topological Approximation Space "TAS"

The condition of equivalence relation in the approximation space limits the range of applications. Yao [14] introduced a method for generalization of approximation space depending on the right neighborhood as shown:

If U is a finite universe and R is a binary relation on U, then the class of right neighborhoods is:

$$(x)_R = \{y \in U \mid xRy\},$$

And the lower and upper approximations for a subset $X \subseteq U$ according to $(x)_R$ are shown as follows respectively:

$$\underline{X} = \bigcup_{(x)_R \subseteq X} (x)_R$$

$$\overline{X} = ((\underline{X^c})^c$$

The purpose of this article is to use a generalized

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approximation space (U,R) based on a general binary relation by using topological concepts, which is called topological approximation space TAS. Consider a binary relation as a general relation and by using the class of "after sets" (right neighborhood) and "for sets" which are formed by this relation R as a subbase for a topology τ on U.

If U is a finite universe and R is a binary relation on U, then we define:

1- "**After set**" as follows: **xR={y: xRy}.**

To construct the topology τ using 'after set", we consider the family $\mathbf{S_R=\{xR: x\in U\}}$ as a subbase., and we write $\mathbf{S_x=\{G\in S_R: x\in G\}}$.

2- "**For set**" as follows: **Ry={x: xRy}.**

To construct the topology τ, we consider the family $\mathbf{{}_RS=\{Rx: x\in U\}}$ as a subbase., and we write $\mathbf{{}_xS=\{G\in {}_RS: x\in G\}}$,

Since all finite intersections of members of a subbase form a base of topology τ.

In TAS method, we calculate the lower and upper approximations by using the interior and closure operators. We find that, TAS is preferred than Yao method where TAS method decrease the boundary region by increasing the lower approximation and decreasing the upper approximation.

There are tow methods for TAS depending on the using of "after set", or "for set" which make a subbase for a topology τ by the following definitions:

Definition (1)

Let U be a nonempty set of objects and R be a class of general binary relations on U, $R=\{r_1,r_2,r_3,\ldots\ldots.,r_n\}$, then (U,R) is called a topological approximation space TAS.

Definition (2)

Let U be a nonempty set of objects and R be a class of general binary relations on U, each $\mathbf{r\in R}$ yields a class $\mathbf{S_r=\{xr: x\in U\}}$ (if we use after set) or $\mathbf{{}_rS=\{rx: x\in U\}}$ (if we use for set) which called a subknowledge base.

Definition (3)

Let R be a class of general relations, then a subbase for $\boldsymbol{\tau}$ for all R is:

$$\mathbf{S_R}=\cup_{\mathbf{r\in R}}\, \mathbf{S_r} \quad \text{(if we use after set) or}$$

$$\mathbf{{}_RS}=\cup_{\mathbf{r\in R}}\, \mathbf{{}_rS} \quad \text{(if we use after set).}$$

Definition (4)

Let R be a class of general relations, then a base for $\boldsymbol{\tau}$ for all R is:

$\boldsymbol{\beta}_{\mathbf{R}}=\cap_{\mathbf{Sx\in SR}}\, \mathbf{S_x}$, $\forall$ x$\in$U. (if we use after set) or

${}_{\mathbf{R}}\boldsymbol{\beta}=\cap_{\mathbf{xS\in RS}}\, {}_{\mathbf{x}}\mathbf{S}$, $\forall$ x$\in$U. (if we use for set).

The following example indicates the comparison between TAS and Yao's method.

Example 1.

Let U={a,b,c,d} are persons , $A=\{A_1,A_2,A_3\}$ are languages, sports and skills as shown in the following table:

Table 1. Languages, sports and skills as information table

U/A	A_1	A_2	A_3
a	{E, F}	{T, F}	{S}
b	{E}	{B, F}	{R}
c	{E, A}	{T}	{R, F}
d	{A}	{T, B}	{S, F}

Where:

A_1=Languages = {English, French, Arabic}={E,F,A}

A_2=Sports = {Tennis, Football, Basketball}={T,F,B}

A_3=Skills = {Swimming, Running, Fishing}={S,R,F}

Let R be a general binary relation as follows:

$$xRy \text{ iff } A(x) \cap A(y) \neq \Phi$$

<u>For the first attribute A_1, we get:</u>

xA_1y={(a,a),(a,b),(a,c),(b,b),(b,a),(b,c),(c,c),(c,a),(c,b),(c,d),(d,d),(d,c)}

Then

aA_1={a,b,c}, bA1={a,b,c}, cA1={a,b,c,d}, dA1={c,d}

$(x)_{A1}$ ={{a,b,c,d},{a,b,c},{c,d}} as in Yao method [7]

S_{A1} ={{a,b,c,d},{a,b,c},{c,d}} as in TAS method.

In our method TAS "Topological Approximation Space", we get:

$$B_{A1}=\{\{a,b,c\},\{c,d\},\{c\}\}$$

$$\Rightarrow \tau_{A1}=\{\{U, \Phi,\{a,b,c\},\{c,d\},\{c\}\} \text{ and}$$
$$f_{A1}=\{\Phi,U,\{d\},\{a,b\},\{a,b,d\}\}$$

We find lower and upper approximation for all subset of U (2^4=16 subset) by using Yao's method [14] and our method TAS as shown in Table 2.

Table 2. Comparison between Yao and TAS Methods

X	X^c	Yao's method		TAS method	
		$\underline{X}$	$\overline{X}$	$\underline{X}$	$\overline{X}$
Φ	U	Φ	Φ	Φ	Φ
{a}	{b,c,d}	Φ	{a,b}	Φ	{a,b}
{b}	{a,c,d}	Φ	{a,b}	Φ	{a,b}
{c}	{a,b,d}	Φ	U	{c}	U
{d}	{a,b,c}	Φ	{d}	Φ	{d}
{a,b}	{c,d}	Φ	{a,b}	Φ	{a,b}
{a,c}	{b,d}	Φ	U	{c}	U
{a,d}	{b,c}	Φ	U	Φ	{a,b,d}
{b,c}	{a,d}	Φ	U	{c}	U
{b,d}	{a,c}	Φ	U	Φ	{a,b,d}

{c,d}	{a,b}	{c,d}	U	{c,d}	U
{a,b,c}	{d}	{a,b,c}	U	{a,b,c}	U
{a,b,d}	{c}	Φ	U	Φ	{a,b,d}
{b,c,d}	{a}	{c,d}	U	{c,d}	U
{a,c,d}	{b}	{c,d}	U	{c,d}	U
U	Φ	U	U	U	U

Note:

From the above table we find that our method reduces the boundary region by increasing the lower approximation (positive region) and decreasing the upper approximation with comparison to Yao's method [9].

For the second attribute A_2 we get:

S_{A2} ={{a,b,c,d},{a,b,d},{a,d}}

$\Rightarrow$ B_{A2}={{a,b,c,d},{a,b,d},{a,c,d},{a,d}}

$\Rightarrow$ τ_{A2} ={{U, Φ,{a,b,d},{a,c,d},{a,d}} and f_{A2}={Φ,U,{c},{b},{b,c}}

For the third attribute A_3 we get:

S_{A3} ={{a,d},{b,c},{b,c,d},{a,c,d}}

$\Rightarrow$$B_{A3}$={{a,d},{b,c},{b,c,d},{a,c,d},{c},{d},{c,d}},

τ_{A3}={{U,Φ,{a,d},{b,c},{b,c,d},{a,c,d},{c},{d},{c,d}} and

f_{A3}={Φ,U,{b,c},{a,d},{a},{b},{a,b,d},{a,b,c},{a,b}}

For all attributes, we get the following topology:

$\tau_A=\tau_{A1}\cup\tau_{A2}\cup\tau_{A3}$ ={U,Φ,{a,b,c},{c,d},{c},{a,b,d},{a,c,d},{a,d},{b,c},{b,c,d},{d}},

f_A={Φ,U,{d},{a,b},{a,b,d},{c},{b},{b,c},{a,d},{a},a,b,c}}

By using **for set**, we get the same result where the relation R which we selected is symmetric.

4. Conclusion

TAS method depends on the using of topological properties of the space such as interior and closure operators which are the lower and upper approximation as in Pawlak space respectively. By using TAS method, we get the minimal boundary region, where it increases the lower approximation and decreases the upper approximation better than Yao's method.

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