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The Relation Between Rough Sets And Fuzzy Sets Via Topological Spaces

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Abstract: *Theories of rough sets and fuzzy sets are related and complementary methodologies to handle uncertainty of vagueness and coarseness, respectively. They are generalizations of classical set theory for modeling vagueness and uncertainty. A fundamental question concerning both theories is their connections and differences. There have been many studies on this topic. Topology is a branch of mathematics, whose ideas exist not only in almost all branches of mathematics but also in many real life applications. The topological structure on an abstract set is used as the base, which used to extract knowledge from data. In this paper: topological structure is used to study the relation between rough sets and fuzzy sets. Membership function is used to convert from rough set to fuzzy set and vice versa. This conversion will achieve the advantages of two theories. Some examples and theories are introduced to indicate the importance of using general binary relations in the construction of rough set concepts, and indicate the relation between rough sets and fuzzy sets according to the topological spaces.*

Keywords: Rough Sets; Fuzzy Sets; Topological Spaces.

1. Introduction

Fuzzy set theory appeared for the first time in 1965, in a famous paper by Zadeh [1]. Since then a lot of fuzzy mathematics has been created and developed. Nevertheless, concepts like fuzzy set, fuzzy subset, and fuzzy equality (between two fuzzy sets) usually depend on the concept of grade of membership. The fuzzy set theory deals with the ill-definition of the boundary of a class through a continuous generalization of set characteristic functions. The indiscernibility between objects is not used in fuzzy set theory. A fuzzy set may be viewed as a class with unsharp boundaries, whereas a rough set is a crisp set that is coarsely described [2].

Rough set theory is a mathematical tool for approximation reasoning for decision support. Pawlak introduced it in 1982 [2]. The concept is particularly well suited for classification of objects. To put the theory in a wider perspective, it belongs to the new branch of research in data mining and knowledge discovery [3-8]. The rough set theory takes into consideration the indiscernibility between objects. The indiscernibility is typically characterized by an equivalence relation. Rough sets are the results of approximating crisp sets using equivalence classes. All equivalence classes form a partition of a universe of discourse. However, an equivalence relation imposes restrictions and limitations on many applications [9,10]. Hence, many extensions have been made in recent years by replacing equivalence relation or partition by notions such as binary relations [8,11,12], neighborhood systems and Boolean algebras [3,5,13], and coverings of the universe of discourse [14,15].

Nowadays, rough set theory and fuzzy set theory are the two main tools used to process uncertainty and incomplete information in the information systems. The two theories are related but distinct and complementary [16,17,18]. In the past twenty years, research on the connection between rough sets and fuzzy sets has attracted much attention. Intentions on combining rough set theory and fuzzy set theory can be found in different mathematical fields [19-23].

Topology is a branch of mathematics, whose ideas exist not only in almost all branches of mathematics but also in many real life applications [24,25]. The topological structure on an abstract set is now used as the base used to extract knowledge from data. The relation between fuzzy set and rough set has been investigated in [26,27,28]. In [27], basic concepts of rough sets are applied to define a set called granular fuzzy set. The purpose of present work is to give the relation between rough sets and fuzzy sets via topological spaces. The article is organized as follows: In the next section, we review research on basic concepts of rough sets, fuzzy sets, and topology. Rough sets in topology are presented in section 3. In section 4, we discuss the conversion method of fuzzy set from rough set. We present our proposed method for converting rough set from fuzzy set in section 5. Rough set by general relation will be discussed in section 6. Section 7 presents the Summary of this article. The conclusion will be presented at the last section.

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2. Basic Concepts:

2.1 Rough Sets:

The motivation for rough set theory has come from the need to represent subsets of a universe; in terms of elements of a partition of that universe. The partition is derived from an equivalence relation on elements of the universe, the indiscernibility relation [2,7].

2.1.1 Approximation Spaces:

The partition mentioned above characterizes the approximation space, a pair K = (U, R), where U is a set called the universe and R is the indiscernibility relation.
The indiscernibility relation is a subset of U ×U, where a pair (x, y) $\in$ R should be read, "x and y are indistinguishable in K".
Since R is an equivalence relation on U, we can consider the equivalence classes of R. These equivalence classes are known as the atoms of K, or elementary sets, notation $R_x \subseteq U$ for the equivalence class of x $\in$ U.
We associate with these approximation spaces two operators, upper and lower approximation of a subset of universe. These operators are formalized as follows: If X $\subseteq$ U, then,

$$\bar{R}X = \{x \in U : R_x \cap X \neq \Phi\} \quad (1)$$

$$\underline{R}X = \{x \in U : R_x \subseteq X\} \quad (2)$$

$$BN_R(X) = \bar{R}X - \underline{R}X \quad (3)$$

$$POS_R(X) = \underline{R}X \quad (4)$$

$$NEG_R(X) = U - \bar{R}X \quad (5)$$

See Fig. 1:

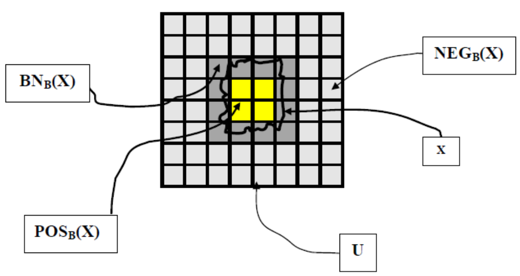

Fig. 1: Rough set concepts

2.1.2 Rough Sets:

Essentially, the class of rough sets is the collection of subsets of the universe, which are indistinguishable in their upper and lower approximations. See Fig. 2:

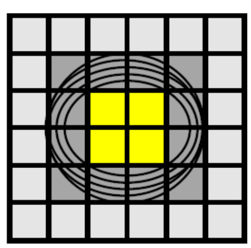

Fig. 2: Class of equivalent rough sets

Using the notions of upper and lower approximations, we can define the equivalence relation rough equality on subsets of U, which will be used to define the equivalence classes of rough sets.

ROUGH EQUALITY: LET K = (U, R) BE AN APPROXIMATION SPACE, AND X, Y $\subset$ U. THEN,

1. X, Y are roughly bottom-equal in K, $X \eqsim_R Y$, iff $\underline{R}X=\underline{R}Y$.

2. X, Y are roughly top-equal in K, $X \simeq_R Y$, iff $\overline{R}X=\overline{R}Y$.

3. X, Y are roughly equal in K, $X \approx_R Y$, iff $\underline{R}X=\underline{R}Y$ and $\overline{R}X=\overline{R}Y$.

We introduce the following example to indicate the above notion.

Example (2.1):
Let K=(U,R), U={1,2,3,4,5,6,7}, U/R={{1,2},{3,4},{5,6},{7}},
And X_1={1,2,3}, X_2={1,2,5}, X_3={2,3,4}, X_4={1,2,4}
Then we get to:
$X_1$and X_2 are roughly bottom-equal in K.
$X_1$and X_3 are roughly top-equal in K.
$X_1$and X_4 are roughly equal in K.

2.1.3 Accuracy Measure:
Rough set can be also characterized numerically by the coefficient defined in the following:
Let K=(U,R) be an approximation space, and X $\subseteq$ U, then

$$\alpha_R(X) = \frac{|\underline{R}X|}{|\overline{R}X|}, \qquad X \neq \phi \qquad (6)$$

Where $|\underline{R}X|$ denotes cardinality of X. Obviously $0 \leq \alpha_R(X) \leq 1$. If $\alpha_R(X)=1$ then X is exact set with respect to R, and otherwise X is Rough set.

2.1.4 Rough membership function:

Rough set can be also introduced using Rough membership function, defined by:
Let k=(U,R) be an approximation space, and X $\subseteq$ U, then

$$\eta_X^R(x) = \frac{|[x]_R \cap X|}{|[x]_R|}, \qquad x \in U \qquad (7)$$

$[x]_R$ is the equivalence class of x. All members of the same equivalence class has the same grade.

$0 \leq \eta^R_X(x) \leq 1$

The membership function is a kind of conditional probability and its value can be interpreted as a degree of certainty to which x belongs to X.

2.2 Fuzzy Sets:

Fuzzy set is a way to represent populations that set theory can't describe definitely, fuzzy sets use a many (usually infinite) valued membership function, unlike classical set theory which uses a two valued membership function (i.e. an element is either in a set or it isn't).

2.2.1 Membership Functions:

This section looks at the technical details of set membership in fuzzy set theory.

The Membership Function: Let U denotes a (universal) set and $A \subseteq U$.

Then, a membership function on U, μ_A, is a function;

$$\mu_A: U \rightarrow L \text{ for some partially ordered set } L$$

It is usually the case that L is a lattice.

The membership function, μ_A, gives the degree to which an element $x \in U$ is in the fuzzy set A.

It can be seen that a fuzzy subset A of U is not a set in the classical sense, i.e. a selection of elements of U, but simply as this membership function giving the degree to which each element $x \in U$ satisfies a property (such as "in a forest"). Note that a set, A, in the classical sense can be thought of as a fuzzy set, by considering it as a function $\mu_A: U \rightarrow L$, defined by:

$$\forall x \in U, \mu_A(x) = \begin{cases} T & if \quad x \in A \\ \\ \perp & otherwise \end{cases} \qquad (8)$$

(T,⊥ are the top and bottom elements of the lattice L.)

In this paper, we shall consider fuzzy sets in this function notation, and crisp sets as special cases of these fuzzy sets.

Equality and Subsets of Fuzzy Sets: We say that two fuzzy sets are equal, $\mu_A=\mu_B$,
iff $\forall x \in U, \mu_A(x) = \mu_B(x)$

Finally, we say that one fuzzy set is a subset of another, $\mu_A \subseteq \mu_B$ iff $\forall x \in U, \mu_A(x) \leq \mu_B(x)$

The Standard Fuzzy Set Theory: uses the closed interval [0, 1] of the real line as the lattice the membership function takes its values from. All of the notions in Section 2.2.1 are applicable when replacing L by [0,1].

2.3 Topological Spaces:

A topological space is a pair (U,τ) consisting of a set U and family τ of subset of U satisfying the following conditions [8]:

(T1) $\Phi \in \tau$ and $U \in \tau$.

(T2) τ is closed under arbitrary union.
(T3) τ is closed under finite intersection.

The set U is called a space, the elements of U are called points of the space and the subsets of U belonging to τ are called open in the space; the family τ of open subsets of U is also called a topology for U.

2.3.1 Bases and Subbases:

It often happens that the open sets of space can be very complicated and yet they can all be described using a selection of fairly simple special ones. When this happens, the set of simple open sets is called a base or subbase (depending on how the description is to done). In addition, it is fortunate that many topological concepts can be characterized in terms of these simpler base or subbase elements.

Base for topology: let (U, τ) be a topological space. A family $\beta \subseteq \tau$ is called a base for (U, τ) iff every non_empty open subset of U can be represented as a union of subfamiliy of β.

Clearly, a topological space can have many bases.

Subbase for topology: let (U, τ) be a topological space. A family $S \subseteq \tau$ is called a subbase for (U, τ) iff the family of all finite intersections is a base for (U, τ).

2.3.2 Closure of Sets:

If (U, τ) is a topological space and $A \subseteq U$, then

$\overline{A} = \cap \{F \subseteq U: A \subseteq F \text{ and } F \text{ is closed}\}$ (9)

is called the τ-closure of A.
Evidently, $\overline{A}$ is the smallest closed subset of U which contains A. Note that A is closed iff $A = \overline{A}$.

2.3.3. Interior of Sets:

If (U, τ) is a topological space and $A \subseteq U$, then

$A^o = \cup\{G \subseteq U: G \subseteq A \text{ and } G \text{ is open}\}$ (10)

is called the τ-interior of A.
Evidently, A^o is the union of all open subsets of U which containing in A. Note that A is open iff $A = A^o$.

3. Rough Sets in Topological Spaces:

The reference space in rough set theory is the approximation space based on the equivalence classes that form a base for a topology. This topology belongs to a special class known by quasi-discrete topology, that is, in which every open set is closed. Yao [29] wrote, "The condition of equivalence limits the application of Rough set". A lot of work investigated the process of generalizing equivalence relations to a general binary relation, but without using topological tools.
Our aim in this work to generalize the reference space to a general topological space.

Definition (3.1): If (U, τ) is a topological space , $X \subseteq U$, X is exact set if $X^b = \Phi$, otherwise X is Rough set.
In the following example, we indicate the above definition.

Example (3.1):
Let U={a,b,c,d}, τ={U,Φ,{a,b,c},{b,c,d},{d},{b,c}},
Then we get to X_1={a,b,c}, X_2={d} are exact sets.
And X is Rough set $\forall X \in P(U)$, $X \neq X_1$, $X \neq X_2$.

Proposition (3.1): If τ is the quasi-discrete topology, then the above definition concedes with Pawlak space.

Proof: In quasi discrete space every open set is closed and thus the base of τ is a partition that yields an equivalence relation which is basic tools in Pawlak space.

Remark (3.1): If τ is a general space, not quasi-discrete, then $\overline{X}=X \rightarrow X^o=X$ is not generally true. The following example ensures this fact.

Example (3.2):
Let U={1,2,3,4,5}, τ={U,Φ,{1,2},{2,3,4},{5},{2},{1,2,3,4},{1,2,5},{2,3,4,5}},
And X={1,2}, then we get to $\overline{X}$ = {1,2,3,4}, and X^o = {1,2}, then $X = X^o$, $X \neq \overline{X}$.

According to remark (3.1) and example (3.2), our approach is a generalization of Pawlak approach.

Proposition (3.2): If (U,τ) is a topological space, $X \subseteq U$. The following are equivalent
(1) X is exact (2) $\overline{X}=X^o$
Proof:
1 to 2, Since X is exact, then $X^b = \Phi$, then $\overline{X}=X^o$.
2 to 1, Since $\overline{X}=X^o$, then $X^b = \Phi$, then X is exact.

In Pawlak space k=(U,R) a subset $X\subseteq U$ has two possibilities rough or exact. The following remark (3.2) indicates that subset $X\subseteq U$ has four possibilities.

Remark (3.2): If (U,τ) is a topological space, $X \subseteq U$. X has the following types of definability's

(1) X is totally definable if X is exact set " $\overline{X} = X = X^o$ ".
(2) X is internally definable if $X = X^o$, $X \neq \overline{X}$
(3) X is externally definable if $X \neq X^o$, $X = \overline{X}$
(4) X is undefinable if $X \neq X^o$, $X \neq \overline{X}$

Example (3.3):

Let U={1,2,3,4,5}, τ={U,Φ,{1,2},{2,3,4},{5},{2},{1,2,3,4},{1,2,5},{2,3,4,5}},
And X_1={1,2,3,4}, X_2={1,2}, X_3={1,5}, X_4={1,3}, then we find that:

X_1 is exact, X_2 is internally definable, X_3 is externally definable, and X_4 is undefinable.

Proposition (3.3): If A is an exact set in (U,τ) and $\tau\subset\tau`$ then A is exact with respect to τ`.
Proof:
Since $BND_{\tau`}A \subset BND_{\tau}A$ and $BND_{\tau}A=\Phi$. Then $BND_{\tau`}A=\Phi$ and A is exact with respect to τ`. In other words if A is τ_exact then A is τ_cloopen and consequently τ`_cloopen. Hence A is τ`_exact.

It is easy to have examples for a τ`_exact set which is not τ_exact.

Example (3.4):

Let U={a,b,c,d}, τ`={U,Φ,{a},{b},{b,c,d},{a,b}} and τ={U,Φ,{a},{a,b}}.
Where $\tau\subset \tau`$ then {a} and {b,c,d} are τ`_exact but not τ_exact.
The following proposition gives the condition for τ`_exact sets to be τ_exact sets.

Proposition (3.4): If (U,τ) is a space and $\tau\subset \tau`$ then each exact A in τ` is exact in τ iff $cl_{\tau}A=cl_{\tau`}A$.
Proof:
If A is τ`_exact then $A\in \tau`$ and $cl_{\tau`}A=A$ and $cl_{\tau}A=A$, hence $cl_{\tau}A=cl_{\tau`}A$.
Conversely: if $cl_{\tau}A=cl_{\tau`}A$ and A is τ`_exact Then A is τ_exact.

3.1. The Rough Membership Function By τ:

In $Eq^n(7)$ the rough membership function is defined using equivalence classes. In the following, we introduce this membership function using the base for topological structure.

If τ is a topology on a finite set U, where its base is β, then the Rough membership function is

$$\mu_X^{\tau}(x)=\frac{|\{\cap B_x\}\cap X|}{|\cap B_x|},\qquad B_x\in\beta, x\in U \tag{11}$$

Where B_x is any member of β containing X.

Example (3.5):

Let U={0,1,2,3,4,5}, β={{2},{3},{0,1,2},{2,3,4}{3,5}}, X={2,4,5}
Then we get:

$$\therefore \mu_X^{\tau}(0)=\frac{|\{0,1,2\}\cap\{2,4,5\}|}{|\{0,1,2\}|}=1/3$$

, $\mu^{\tau}_X(3)=0$

$\mu^{\tau}_X(1)=1/3$, $\mu^{\tau}_X(4)=2/3$

$\mu^{\tau}_{X}(2)=1$, $\mu^{\tau}_{X}(5)=1/2$

4. Fuzzy Set From Rough Set:

Let (U,τ) be a topological space, then any subset $X \subseteq U$ can be expressed as:

$$\underset{\sim}{X} = \{(x, \mu^{\tau} X(x)) : \forall x \in U\} \qquad (12)$$

That is, each subset of the total space U can be interpreted as a fuzzy set by using the rough membership function.

Example (4.1):
Let U={0,1,2,3,4,5}, U/R={{0,1,2},{3,4},{5}}
Since β is a partition ⇒ β ={{0,1,2},{3,4},{5}},
which is U/R by proposition(3.1).
⇒ τ ={U,Φ,{0,1,2},{3,4},{5},{0,1,2,3,4},
{0,1,2,5},{3,4,5}}
Let X ={0,1,2,3} ⊆ U. We find that:

$$\eta_X^R(x) = \frac{|[x]_R \cap X|}{|[x]_R|}$$

$$= \mu_X^{\tau}(x) = \frac{|\{\cap B_x\} \cap X|}{|\cap B_x|}, \qquad (13)$$

$$x \in B_x, B_x \in \beta$$

$$\therefore \mu_X^{\tau}(0) = \frac{|\{0,1,2\} \cap \{0,1,2,3\}|}{|\{0,1,2\}|} = 1$$

$\mu^{\tau}_{X}(1)=1$,

$\mu^{\tau}_{X}(2)=1$, $\mu^{\tau}_{X}(4)=1/2$

$\mu^{\tau}_{X}(3)=1/2$, $\mu^{\tau}_{X}(5)=0$

Then $\underset{\sim}{X}$={(0,1),(1,1),(2,1),(3,1/2),(4,1/2),(5,0)}

5. Rough sets from fuzzy sets:

Let (U,τ) be a topological space, and X⊆U. Then From the fuzzy set we find that:
The lower approximation is all elements of X which have membership function 1, i.e. $\mu^{\tau}_{X}(x)=1$. The upper approximation is all elements of X which have membership function not equal to zero, i.e. $\mu^{\tau}_{X}(x) \neq 0$. The boundary region is all elements of X which have membership function $0<\mu^{\tau}_{X}(x)< 1$. The negative region is all element of X which have $\mu^{\tau}_{X}(x)=0$. Then we can write the upper, lower approximation, boundary and negative region of set $X \subseteq U$ as follows:

$\overline{R}X = \{x \in U: \mu^{\tau}_{X}(x) >0\}$ (14)
$\underline{R}X = \{x \in U: \mu^{\tau}_{X}(x) =1\}$ (15)
$BND_R(X) = \{x \in U: 0<\mu^{\tau}_{X}(x)<1\}$ (16)
$NEG_R(X) = \{x \in U: \mu^{\tau}_{X}(x) =0\}$ (17)

Example (5.1):
Let $\underset{\sim}{X}$={(0,1),(1,1),(2,1),(3,1/2),(4,1/2),(5,0)}

Then from fuzzy set we get to:
$\underline{R}X=X^{o}=\{0,1,2\}$, $\overline{R}X=\overline{X}=\{0,1,2,3,4\}$,

$NEG_R(X)=\{5\}$, $BN_R(X)=\{3,4\}$
Then X is a rough set.

Classical set operation: induces fuzzy set theoretical operation by using the τ-membership function $\{\cap B_x\}$, union and intersection are defined as follows:

$$\mu_X^{\tau} \cap \mu_Y^{\tau}(x) = \frac{|\{\cap B_x\} \cap X \cap Y|}{|\{\cap B_x\}|},$$

$$\mu_X^{\tau} \cup \mu_Y^{\tau}(x) = \frac{|\{\cap B_x\} \cap \{X \cup Y\}|}{|\{\cap B_x\}|}$$

The membership function $\mu^{\tau}_X(x)$ is the (unique) fuzzy representation of X.

6. ROUGH SET BY GENERAL RELATION:

Consider a binary relation as a general relation and by using the group of sets which is formed by this relation R as a subbase S for topological space τ.
If U is a finite universe and R is a binary relation on U, then:
The after set: can be defined as xR={y: xRy}
To get the topology on U needed for the construction of rough set we start by the class S={xR: x∈U} as a subbase of τ which can form the base β for τ, and a subset $S_x \in S$ where $S_x=\{G\in S: x\in U\}$. According the above result we find that:

$$\because \mu_X^{\tau}(x) = \frac{|\{\cap B_x\} \cap X|}{|\cap B_x|},$$

$$x \in B_x, B_x \in \beta$$

$$\therefore \mu_X^{\tau}(x) = \frac{|\{\cap S_x\} \cap X|}{|\cap S_x|},$$

$$x \in S_x, S_x \in S$$

Example (6.1):
Let U={0,1,2,3,4,5}, 0R=1R={0,1,2}, 2R=3R={2,3}, 4R={3,4}, 5R={5}
Then S={{0,1,2},{2,3},{3,4},{5}}
⇒β={{0,1,2,},{2,3},{3,4},{5},{2},{3}}
⇒τ={U,Φ,{0,1,2},{2,3},{3,4},{5},{2},{3},{0,1,2,3},{0,1,2,3,4},{0,1,2,3},{2,3,4},{2,3,5},{3,4,5},{2,5},{3,5},{2,3,4,5},{2,3,5}

Let X={0,1,2,3}
⇒ $X^{o}=\{0,1,2,3\}$, $\overline{X}=\{0,1,2,3,4\}$

$$\therefore \mu_X^{\tau}(0) = \frac{|\{0,1,2\} \cap \{0,1,2,3\}|}{|\{0,1,2\}|} = 1$$

$\mu^{\tau}_{X}(1)=1$, $\mu^{\tau}_{X}(3)=1$

$\mu^{\tau}_{X}(2)=1$, $\mu^{\tau}_{X}(4)=1/2$

, $\mu^{\tau}_{X}(5)=0$

Then X={(0,1),(1,1),(2,1),(3,1),(4,1/2),(5,0)}
From fuzzy set X we get to:
$\underline{R}X=X^{o}=\{0,1,2,3\}$, $\overline{R}X=\overline{X}=\{0,1,2,3,4\}$,

$NEG_R(X)=\{5\}$, $BN_R(X)=\{4\}$

In the following example we indicate the relation between fuzzy sets with respect to more than one knowledge base.

Example (6.2):
Let U={0,1,2,3,4,5},
$0R_1=1R_1=\{0,1,2\}$, $2R_1=3R_1=\{2,3\}$, $4R_1=\{3,4\}$, $5R_1=\{5\}$,
$0R_2=1R_2=\{0,1\}$, $2R_2=3R_2=\{2,3\}$, $4R_2=\{3,4\}$, $5R_2=\{5\}$,
$0R_3=1R_3=\{0,1\}$, $2R_3=3R_3=\{2,3\}$, $4R_3=\{4\}$, $5R_3=\{5\}$
and X={0,1,2,3}. Where $X_{R3}\subset X_{R2}\subset X_{R1}$,
then we get to:

X_{R1}={(0,1),(1,1),(2,1),(3,1),(4,1/2),(5,0)}

X_{R2}={(0,1),(1,1),(2,1),(3,1),(4,1/2),(5,0)}

X_{R3}={(0,1),(1,1),(2,1),(3,1),(4,0),(5,0)}

From fuzzy sets X we get to:
$X_{R3}\subset X_{R2}\subseteq X_{R1}$

7. SUMMARY:

Pawlak space K=(U,R) is used to give us the lower and upper approximation for any subset $X\subseteq U$, which called rough iff $\underline{R}X \neq \overline{R}X$ and exact iff $\underline{R}X = \overline{R}X$.

From topological spaces we find that, every subset $X\subseteq U$ is either exact set if it is closed and open set in τ. Or Rough set if it is not closed and not open set in τ.

From fuzzy set X we get: the set X is exact set if all elements of X have not $0<\mu^{\tau}_{X}(x)\leq 1$, and X is called Rough set if at least one element of it has $0<\mu^{\tau}_{X}(x)<1$.

The work presented in this paper give an approach for connecting fuzzy sets and rough sets using topological structure. Also how to use general binary relation for connecting the two theories. We expect that this approach will help in transforming any collection of data to knowledge with small degree of approximation. Since rough sets are based on data without any human assumption and thus the resulted fuzzy sets will be more accurate. The use of topological concepts in knowledge base representation may help in the application of many fields.

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