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Geographically Weighted Polynomial Regression: Application to Poverty Modeling in East Java Province, Indonesia

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Abstract: *Geographically weighted regression (GWR) is a regression method for exploring spatial nonstationarity by allowing different relationships at different locations. In the GWR, the response at each location is locally fitted by a linear function of a set of explanatory variables. In fact, it may have nonlinear relationship with one or more explanatory variables. Thus, the GWR model may not be able to accommodate the fact. In dealing with the problem, we attempt to introduce a geographically weighted polynomial regression (GWPolR) model. In this study, the GWPolR model is performed to explore spatially varying relationships between poverty and its factors in East Java Province, Indonesia. The factors studied here are the percentage of people educated less than elementary school, the percentage of people aged at least 10 years who cannot read and write, and the percentage of people working in the trading sector. Factually, the first and third factors tend to have nonlinear relationships with the poverty. Compared with the previous models in such condition, the GWPolR model gives a significant improvement and more complete understanding of how each explanatory variable was related to the poverty. This should allow improved planning of poverty alleviation strategies.*

Keywords: spatial variation, nonlinear relationship, poverty, polynomial function.

1. INTRODUCTION

According to some previous authors (e.g., see [2], [3], [4], [8], [9], [10], [11], [15] and [16]), geographically weighted regression (GWR) model can be expressed as

$$y_i = \beta_0(u_i, v_i) + \sum_{j=1}^{p} \beta_j(u_i, v_i)\, x_{ij} + \varepsilon_i, \qquad (1)$$

where $(y_i, x_{i1}, x_{i2}, \dots, x_{ip})$ are observations at a location (u_i, v_i) for $i = 1, 2, \dots, n,$, the regression coefficients $\beta_j(u_i, v_i)$, $j = 1, 2, \dots, p$ are unknown parameters at location (u_i, v_i), and ε_i is Normally distributed error with mean zero and common variance σ^2. Parameter estimation and related tests can be found in the references mentioned above.

The GWR model is an expansion of global linear regression (GLR) in spatial data. Its coefficients spatially vary over space. It is a powerful method in describing the spatial nonstationarity [11]. However, it is important to note that its response variable in each location is fitted as a linear function of a set of explanatory variables. This method was designed to model linear relationships between explanatory variables and the response in the spatial data. It may be unrealistic in some real-life situations. There are many possibility of nonlinearity cases in the relationships between one or more explanatory variables and the response. Reference [5] inspected the vulnerability modeling of dengue hemorrhagic fever (DHF) disease in Surabaya based on geographically regression. The results obtained have not been satisfactory. The existence of nonlinear relationships between one or more explanatory variables and the DHF level was suspected to be the cause. In other example, [6] showed that the influence of the convenience factor (access to public facilities) was nonlinear over the housing prices in Taipei, Taiwan.

As the nonlinear relationships existed in the real situation, the linear approach may be unrealistic to use [12]. Therefore, some approach models which accommodate the actual pattern of the real data are required to improve the basic GWR model. Some expansions of GWR have been proposed in recent years. One of the important expansions is geographically weighted generalised linear modeling (GWGLM) covering geographically weighted poisson regression (GWPR) and geographically weighted logistic regression (GWLR). Although there was GWGLM, an extension of GWR which accommodates response in continuous variable and has nonlinear relationships with the explanatory variables has not been found. Thus, an extension of the basic GWR model which can overcome the problems described above is required.

In dealing with the problem, we attempt to use a geographically weighted polynomial regression (GWPolR) model. It is a generalization of the basic GWR model. It will be a GWR model if there is no nonlinearity in relationships between response and its predictors. Here, we provide the weighted least square estimator which still depends on a bandwidth and several polynomial degrees. Next, this paper aims to provide an algorithm for selecting the optimum bandwidth and the optimum polynomial degree of each explanatory variable involved in the model, and apply the procedure to model the poverty rate in East Java Province, Indonesia.

There are several main factors of poverty including education and the job sector (e.g., see [1], [7]). In this study, we evaluate the relationship between poverty rate on the factors representating education and the job sector, i.e., people educated less than elementary school, people aged at least 10 years who cannot read and write, and people working in the trading sector. Factly, we found that there were two variables having nonlinear relationship with the poverty (see the explanation in the sub section 3.2). Hence, it was reasonable to analyze the poverty in East Java using the GWPolR model.

2. GEOGRAPHICALLY WEIGHTED POLYNOMIAL REGRESSION

2.1 Model and Estimation

Here, we expand the linear relationship of the GWR model in (1) by using polynomial function approach to be the following model:

$$y_i = \beta_0(u_i,v_i) + \sum_{k=1}^{p}\sum_{j=1}^{gmax_k} \beta_{k,j}(u_i,v_i)\, x_{ik}{}^{j} + \varepsilon_i \,, \quad (2)$$

where $\beta_{k,j}(u_i,v_i)$ is the regression coefficient of the k^{th} explanatory variable on j^{th} order of polynomial at location i. Then, we call the model with Geographically Weighted Polynomial Regression (GWPolR). It can be expressed in a matrix form as follows

$$y_i = \boldsymbol{x}_i^{*\mathrm{T}}\, \boldsymbol{\beta}_{pol}(u_i,v_i) + \varepsilon_i \,, \quad i = 1,2,\dots,n, \quad (3)$$

where

$$\boldsymbol{x}_i^{*\mathrm{T}} = \left(1 \;\; x_{i1} \;\; x_{i1}^2 \cdots x_{i1}^{gmax_1} \cdots x_{ip} \;\; x_{ip}^2 \cdots x_{ip}^{gmax_p}\right), \quad (4)$$

and

$$\begin{aligned}\boldsymbol{\beta}^{\mathrm{T}}_{Pol}(u_i,v_i) &= (\,\beta_0(u_i,v_i)\; \beta_{1,1}(u_i,v_i)\; \beta_{1,2}(u_i,v_i) \cdots \\ &\quad \beta_{1,gmax_1}(u_i,v_i) \dots \beta_{p,1}(u_i,v_i) \\ &\quad \beta_{p,2}(u_i,v_i) \cdots \beta_{p,gmax_p}(u_i,v_i)\,) \end{aligned} \quad (5)$$

For a given location (u_0,v_0), we can estimate $\boldsymbol{\beta}_{Pol}(u_0,v_0)$ by formulate the weighted least square problem. That is, minimize

$$\sum_{i=1}^{n}\left(y_i - \boldsymbol{x}_i^{*\mathrm{T}}\,\boldsymbol{\beta}_{pol}(u_0,v_0)\right)^2 K_h(d_{0i}), \quad (6)$$

with respect to each elements of $\boldsymbol{\beta}_{pol}(u_0,v_0)$. For this weighted least square problem, an explicit expression of the solution is

$$\hat{\boldsymbol{\beta}}_{pol}(u_0,v_0) = \left[\boldsymbol{X}^{\mathrm{T}}_{pol}\boldsymbol{W}(u_0,v_0)\boldsymbol{X}_{pol}\right]^{-1}\boldsymbol{X}^{\mathrm{T}}_{pol}\boldsymbol{W}(u_0,v_0)\boldsymbol{y} \quad (7)$$

where $\mathbf{y} = (y_1, y_2, \dots, y_n)^{\mathrm{T}}$,

$$\boldsymbol{X}_{pol} = \begin{bmatrix} 1 & x_{11} & x_{11}^2 & \cdots & x_{11}^{gmax_1} & \cdots x_{1p} & x_{1p}^2 & \cdots & x_{1p}^{gmax_p} \\ 1 & x_{21} & x_{21}^2 & \cdots & x_{21}^{gmax_1} & \cdots x_{2p} & x_{2p}^2 & \cdots & x_{2p}^{gmax_p} \\ & & & & \vdots & & & & \\ 1 & x_{n1} & x_{n1}^2 & \cdots & x_{n1}^{gmax_1} & \cdots x_{np} & x_{np}^2 & \cdots & x_{np}^{gmax_p} \end{bmatrix}, \quad (8)$$

and

$$\mathbf{W}(u_0,v_0) = \mathrm{diag}[K_h(d_{01}), K_h(d_{02}), \dots, K_h(d_{0n})\,] \quad (9)$$

is a diagonal weighting matrix with $K_h(\cdot) = K\left(\frac{\cdot}{h}\right)$ and $K(\cdot)$ is a kernel function for the bandwidth h. Furthermore, d_{ij} is the distance between location i and j in the form of

$$d_{ij} = \sqrt{\left(u_i - u_j\right)^2 + \left(v_i - v_j\right)^2}\,. \quad (10)$$

By taking (u_0,v_0) to be each of the designed locations (u_i,v_i), $i = 1,2,\dots,n$, we can obtain the vector of the fitted values for the response y at n designed locations as

$$\hat{\boldsymbol{y}}_{Pol} = (\hat{y}_1^*, \hat{y}_2^*, \dots, \hat{y}_n^*)^{\mathrm{T}} = \mathbf{G}\,\mathbf{y}\,, \quad (11)$$

where

$$\mathbf{G} = \begin{bmatrix} \boldsymbol{x}_1^{*T}\left[\mathbf{X}^T_{pol}\,\mathbf{W}(u_1,v_1)\,\mathbf{X}_{pol}\right]^{-1}\mathbf{X}^T_{pol}\;\mathbf{W}(u_1,v_1) \\ \boldsymbol{x}_2^{*T}\left[\mathbf{X}^T_{pol}\,\mathbf{W}(u_2,v_2)\,\mathbf{X}_{pol}\right]^{-1}\mathbf{X}^T_{pol}\;\mathbf{W}(u_2,v_2) \\ \vdots \\ \boldsymbol{x}_n^{*T}\left[\mathbf{X}^T_{pol}\;\mathbf{W}(u_n,v_n)\,\mathbf{X}_{pol}\right]^{-1}\mathbf{X}^T_{pol}\;\mathbf{W}(u_n,v_n) \end{bmatrix} \quad (12)$$

is called a hat matrix of GWPolR model, and $\boldsymbol{x}_i^{*\mathrm{T}}$ is the vector writed in (4). Based on the hat matrix $\mathbf{G}$, the residual vector of GWPolR is

$$\hat{\boldsymbol{\varepsilon}}_{Pol} = \boldsymbol{y} - \hat{\boldsymbol{y}}_{Pol} = (\mathbf{I} - \mathbf{G})\boldsymbol{y}\,, \quad (13)$$

and the residual sum of squares for GWPolR is

$$\mathrm{RSS}_{\mathrm{pol}} = \hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}_{pol}\hat{\boldsymbol{\varepsilon}}_{Pol} = \boldsymbol{y}^{\mathrm{T}}(\mathbf{I} - \mathbf{G})^{\mathrm{T}}(\mathbf{I} - \mathbf{G})\boldsymbol{y}\,, \quad (14)$$

where $\mathbf{I}$ is identity matrix of order n.

2.2 Choices of The Weighting Function

According to [8], the weighting matrix $\mathbf{W}(u_i,v_i)$ is a weighting design based on the proximity of the observation point i to the data points around i. For a fixed bandwidth h and the distance d_{ij}, the elements of the weighting matrix at location i can take one of the following functions:

1. $$w_j(i) = \begin{cases} 1, & if\ d_{ij} \le h, \quad j = 1,2,\dots,n, \\ 0, & if\ d_{ij} > h \end{cases} \quad (15)$$
2. Gaussian Kernel
$$w_j(i) = exp\left(-\tfrac{1}{2}\left(\tfrac{d_{ij}}{h}\right)^2\right), \; j = 1,2,\dots,n, \quad (16)$$
3. Bisquare Kernel
$$w_j(i) = \begin{cases} \left(1 - \left(\frac{d_{ij}}{h}\right)^2\right)^2, & if\ d_{ij} \le h \\ 0, & if\ d_{ij} > h. \; j = 1,2,\dots,n. \end{cases} \quad (17)$$

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2.3 Selection of Bandwidth and Polynomial Degrees Using Cross-Validation Criteria

Reference [8] stated that Cross-Validation (CV) procedure is commonly employed to select the optimum bandwidth h in the GWR modeling. In this paper, we adopt such procedure to find the optimum bandwidth integrated with the optimum polynomial degree for each explanatory variable. So, we have

$$\mathrm{CV}(h, \boldsymbol{g}) = \sum_{i=1}^{n}\left(y_i - \hat{y}_{(i)}(h, \boldsymbol{g})\right)^2, \tag{18}$$

as an objective function, where $\hat{y}_{(i)}(h, \boldsymbol{g})$ is the fitted value of y_i under bandwidth h and vector of polynomial degrees $\boldsymbol{g} = (g_1, g_2, \dots, g_p)$ with the observation at location (u_i, v_i) excluded from the estimation process. The quantity g_k is the polynomial degree of the k^{th} explanatory variable valued integer between 1 and $gmax_k$, for $k = 1, 2, \dots, p$. Find h_0, $d_{1o}, d_{2o}, \dots, d_{po}$ as the optimum values, such that (18) is minimum.

Here, we suggest an algorithm to find the optimum bandwidth and the optimum polynomial degrees based on the CV criterion as follows:

1) Specify the number of explanatory variables involved in the model, denoted by p.
2) Determine the maximum number of polynomial degree for each explanatory variable, denoted by $gmax_k$ for $k = 1, 2, \dots, p$.
3) Construct all arrays of numbers obtained from the existing polynomial degrees for all explanatory variables. Let s be the number of arrays, then $s = \prod_{k=1}^{p} gmax_k$.
4) Find the minimum CV value resulted by GWPolR modeling in each array.
5) Find the smallest CV value among the minimum CV values obtained from the entire arrays.
6) Choose bandwidth and polynomial degrees that produce the smallest CV value in step 5 as the optimum solution.

3. Application to Poverty Modeling

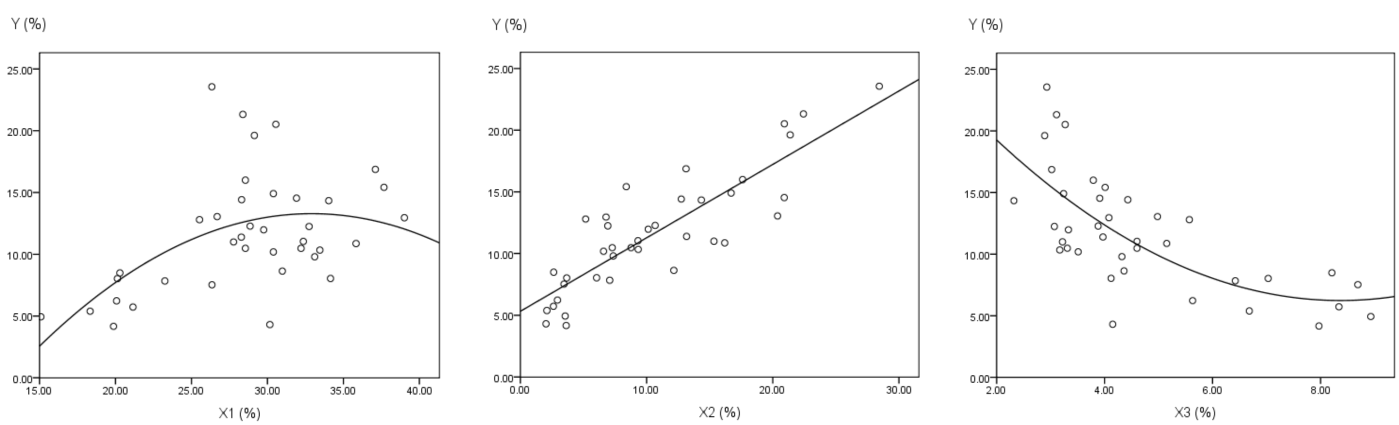
3.1 The Poverty Data

The poverty data used here were provided by Statistics of East Java for the year of 2017 ([13] and [14]). The data consist of 38 observation units of regencies or municipalities in East Java Province, Indonesia. The involved variables in this research are the percentage of poverty (Y), the percentage of people educated less than elementary school (X_1), the percentage of people aged at least 10 years who cannot read and write (X_2), and the percentage of people working in the trading sector (X_3). Here, each location is marked by a coordinate point consisting of latitude and longitude.

3.2 Global Linear Regression: A Preliminary Modeling

To detect possible relationships, the scatter plots with the closest fitted line are presented in Fig. 1. Visually, from Fig. 1 we can clearly see that X_2 variable has linear relationship with Y variable. Whereas, X_1 and X_3 variables tend to have nonlinear relationships with Y.

Global linear regression (GLR) model was performed using three explanatory variables mentioned above. The simultaneous test for parameters yielded F-statistic value of 39.25 with $df_1 = 3$ and $df_2 = 34$ (p-value = 0.000). It means that the parameters affect to the response variable simultaneously. This model yielded residual sum of squares of 184.67 and R^2 of 77.6%. Furthermore, the results of GLR estimation on the poverty data are listed in Table 1.



(a) (b) (c)

Fig. 1. *Scatter plot with the closest fitted line for (a)* X_1*, (b)* X_2*, and (c)* X_3 *variables versus* Y.

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Table 1: Summary of GLR results

Explanatory	Coef	SE Coef	T	*p*-value
Intercept	6.103	4.811	1.27	0.213
X_1	0.070	0.103	0.68	0.502
X_2	0.509	0.074	6.91	0.000
X_3	-0.400	0.385	-1.04	0.306

Based on partial test in Table 1 (with significance level of 0.05), only X_2 variable affects significantly to the poverty rate (p-value < 0.05). Whereas, X_1 and X_3 variables are not significant (p-value > 0.05). This might be caused by misspecification of function used in GLR, where they actually have nonlinear relationships with the response. We conclude that the GLR modeling is not feasible. Here, we do not continue with remodeling that uses significant explanatory variables. We want to compare the performance of GLR with that of spatial models (especially using polynomial approach) involving the same explanatory variables. In other words, if a model yielded better results while we didn't applied the same variables to all models in a comparison, we would not know if the improvement was due to the modeling approach or the explanatory variables that was used to build each model.

To clarify the nonlinearity allegation, we verify the misspecification function using Ramsey RESET test. Here, the null hypothesis states that there is no misspecification of function in the GLR. By using syntax of "`resettest`" in "lmtest" package on R-3.5.1 Software with optional input "`power=2`" (for testing the existence of polynomial degree of 2), we find the RESET statistic of 2.9541 with degrees of freedom 3 and 31 in the numerator and denominator, respectively (with p-value = 0.0478). If we take a significance level of 0.05, we reject the null hypothesis and conclude that there is misspecification function in GLR. This result supports the above suspicion that there are nonlinearity relationships in the poverty data. Hence, continuing with the GWPolR model is recommended.

3.3 The Results of Geographically Weighted Regression

This poverty data were drawn based on area, therefore proceeding with spatial model was warranted. Based on spatial regression models, we firstly implemented the GWR technique with the following model

$$y_i = \beta_0(u_i, v_i) + \beta_1(u_i, v_i)x_{i1} + \beta_2(u_i, v_i)x_{i2} + \beta_3(u_i, v_i)x_{i3} + \varepsilon_i \,. \qquad (19)$$

Here, GWR estimation was conducted by using the "GWmodel" package on R-3.5.1 Software. Based on model (19) and the weighting function in (16), we found that the R^2 of GWR was 84.14% with a bandwidth of 0.70° (or equals to 77.9254 km) when the minimum CV was 164.7028. Our preferred indicator of model fit, RSS, gave a value of 134.025. Here, we have four estimated parameters that vary in 38 locations. The summary of the GWR estimators is listed in Table 2. Positive and negative relationships were manifested in the results.

Table 2: Summary of GWR results

Coef. Of Explanatory	Min.	Q1	Median	Q3	Max.
Intercept	-8.712	-1.184	4.331	9.902	15.250
X_1	-0.211	-0.060	0.071	0.206	0.368
X_2	0.512	0.567	0.597	0.613	0.646
X_3	-1.038	-0.553	-0.166	0.201	0.730

3.4 The Results of Geographically Weighted Polynomial Regression

In this modeling we used the same explanatory variables with the previous one. Based on the algorithm outlined above, we had $p = 3$. For simplicity, we set the same value of the maximum polynomial degree for X_1, X_2, and X_3 variables, i.e., $d_1 = d_2 = d_3 = 2$. Based on the stipulation, we had eight arrays of polynomial degrees. Further, on the basis of GWPolR estimation procedure we selected the minimum CV value in each array. The minimum CV value and the corresponding optimum bandwidth are listed in Table 3.

Table 3: Optimum bandwidth and minimum CV for each array of polynomial degrees

Number	Array	Opt *h*	Minimum CV
1	(1, 1, 1)	0.68	164.4755
2	(1, 1, 2)	0.80	161.5843
3	(1, 2, 1)	0.68	162.2988
4	(1, 2, 2)	0.88	177.2904
5	(2, 1, 1)	0.83	180.4944
6	(2, 1, 2)	0.69	159.8442
7	(2, 2, 1)	0.76	179.3383
8	(2, 2, 2)	0.79	171.6931

From Table 3, the smallest CV value among eight minimum CV values is 159.8442. It is found in the row of number six, according to the optimum bandwidth of 0.69° (or equals to 76,8122 km) and array of (2, 1, 2). The array means that the optimum polynomial degree for X_1, X_2, and X_3 variables are 2, 1, and 2, respectively. So, the GWPolR model under the optimum condition can be expressed as follows

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$$y_i = \beta_0(u_i, v_i) + \beta_{1,1}(u_i, v_i)x_{i1} + \beta_{1,2}(u_i, v_i){x_{i1}}^2 + \beta_{2,1}(u_i, v_i)x_{i2} + \beta_{3,1}(u_i, v_i)x_{i3} + \beta_{3,2}(u_i, v_i){x_{i3}}^2 + \varepsilon_i \,. \quad (20)$$

Based on (20) and gaussian weighting function in (16) with optimum bandwidth $h_0 = 0.69°$, we found that the R^2 of GWPolR was 89.27%. A goodness of fit indicator, RSS, gave a value of 88.408. In this model, we have six parameters that vary in 38 locations. Furthermore, The summary of the GWPolR estimators is presented in Table 4.

From Table 4, we know that X_1 variable has positive sign in the first degree, but it has negative sign in the second degree. It means that X_1 variable has effect on increasing poverty, but with a deceleration. On the other hand, the X_3 variable has negative sign in the first degree, but it has positive sign in the second degree. It means that the X_3 variable has effect on decreasing poverty with a deceleration. So, X_1 and X_3 effects are not linear as in previous modeling.

Table 4: Summary of GWPolR results

Coef. Of Explanatory	Min.	Q1	Median	Q3	Max.
Intercept	-2.4910	2.3640	4.8340	11.1400	39.6500
X_1	-1.4150	-0.0752	0.1445	0.3452	0.6878
X_1^2	-0.0146	-0.0057	-0.0028	0.0056	0.0230
X_2	0.4640	0.5343	0.5929	0.6209	0.6849
X_3	-6.2710	-3.2790	-0.8943	0.8763	1.6880
X_3^2	-0.1276	-0.0644	0.0597	0.2372	0.5525

3.5 Model Comparison

In this sub section, we would like to compare the models based on some goodness of fit indicators (including the minimum CV, RSS, and R^2). The comparison is presented in Table 5.

Table 5: The model performance comparison

Indicator	Model		
	GLR	GWR	GWPolR
Minimum CV	-	164.703	159.844
RSS	184,67	134.025	88.408
R^2	77.60%	84.14%	89.27%

From Table 5, the GWPolR performance is the best. Furthermore, there are RSS decrease of 96.262 and 45.617 from GLR and GWR, respectively. There are R^2 increase of 11.67 and 5.13 from GLR and GWR, respectively. These are relatively strong evidence of an improvement in the GWPolR model fit to the data.

Another comparison, the prediction errors (i.e., residual) for the GLR, GWR and GWPolR models over the 38 districts in East Java are plotted in Fig. 2. It appears that the variation of the GWPolR residuals is lower than that of the GWR and GLR models.

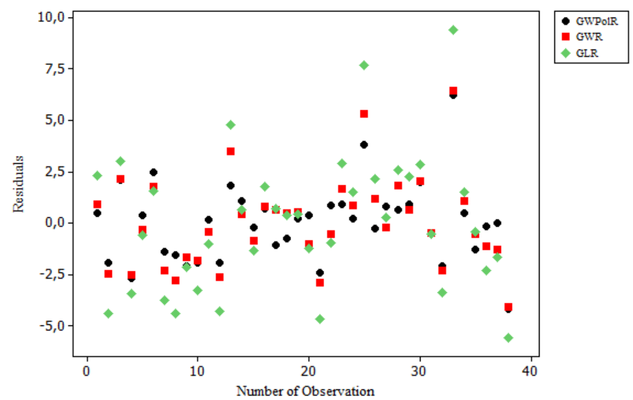
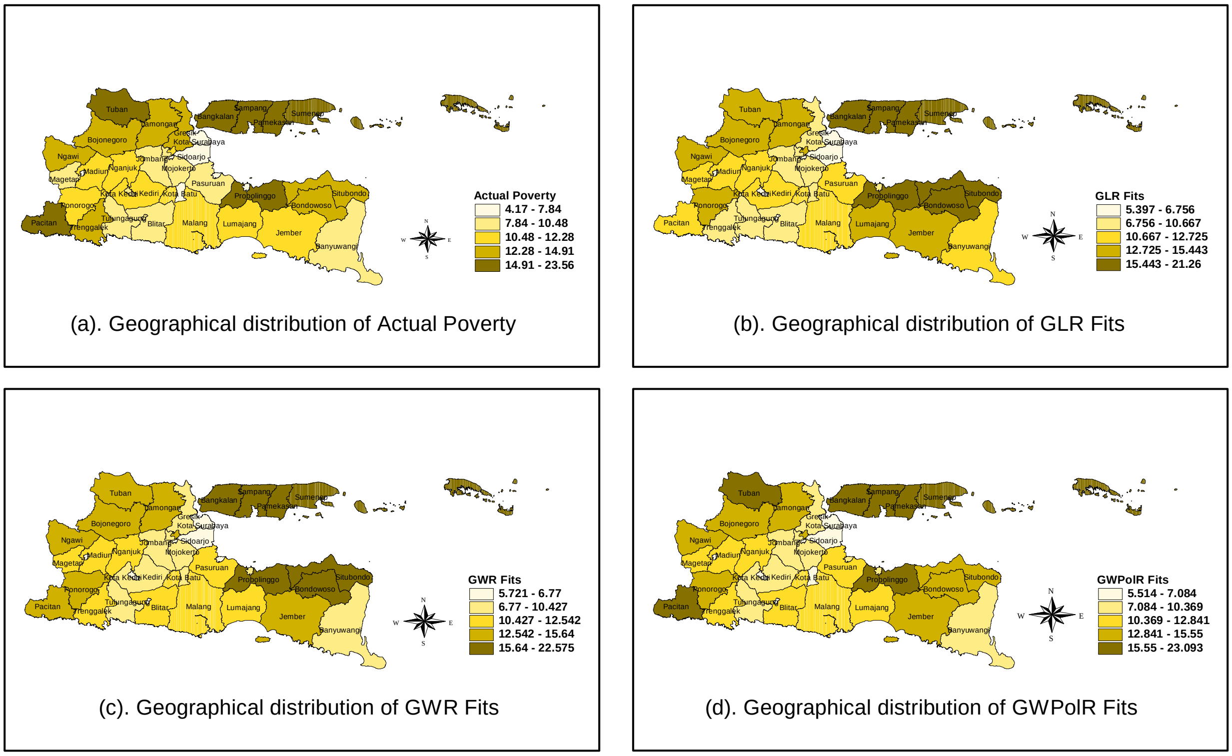


Fig. 2. *Residual of GLR, GWR, and GWPolR models for the poverty data over 38 districts in East Java, Indonesia*

Finally, we plot the percentage of poverty and its estimates on a map using quantile classification method to compare visually the model results. It is shown by Fig. 3. Based on Fig. 3, the goegraphical distribution of GWPolR fits is more similar to the actual poverty than that of GWR or GLR model. It shows that the GWPolR estimate is the closest to actual poverty.

Furthermore, based on the GWPolR estimate, we describe the estimated parameters on a map using quantile classification method to make the complex relationship easier to be understood. For the first degree explanatory variables, the geographical distribution of each estimated parameter is displayed in Fig. 4. The colour is graduated from dark to light. Areas with dark shade represent areas where that particular variable exhibit strong influence on poverty rate. While light shade represent areas where that specific variable exhibit weak or low influence on poverty rate.



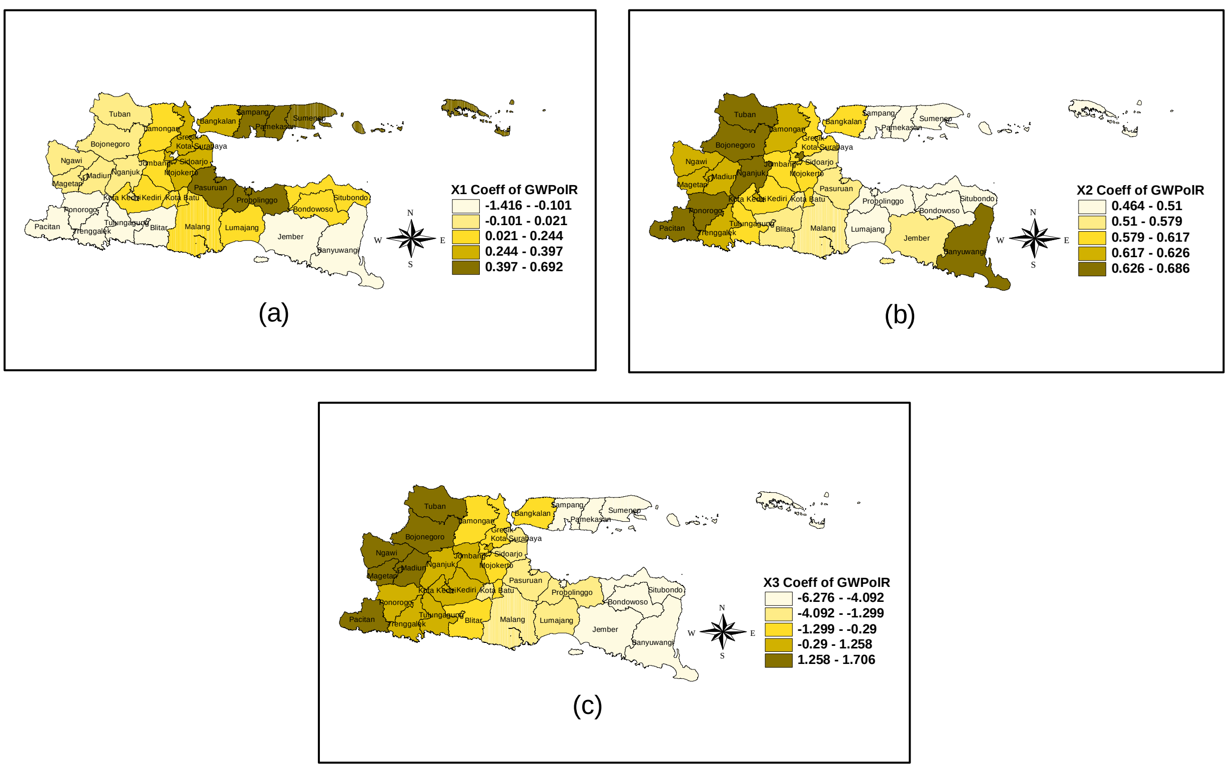
(a). Geographical distribution of Actual Poverty

(b). Geographical distribution of GLR Fits

(c). Geographical distribution of GWR Fits

(d). Geographical distribution of GWPolR Fits

Fig. 3. *Geographical distribution of poverty and its estimates*



(a)

(b)

(c)

Fig. 4. *Geographical distribution of the GWPolR coefficients for (a) x_1, (b) x_2, and (c) x_3*

Based on Fig. 4, X_1 gives a strong influence to the poverty in the region of Madura island and surrounding. While, X_2 and X_3 give a strong influence in western of East Java. Further east, the influence weakened.

For the second degree explanatory variables, the geographical distribution of each estimated parameter is displayed in Fig. 5. The strong influence of the X_1^2 variable is located in the southern of east Java, and the further north the influence decreases. Meanwhile, the strong influence of the X_3^2 variable is located in the eastern of East Java, and the further west the influence decreases.

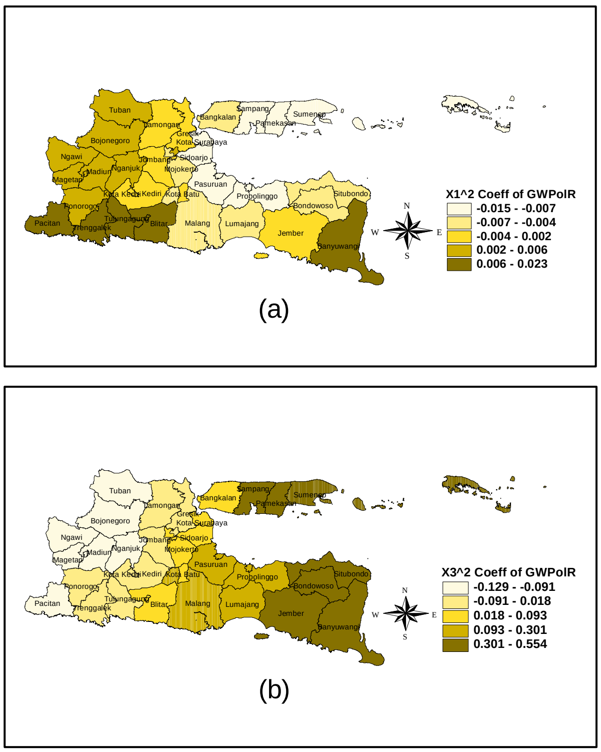


Fig. 5. *Geographical distribution of the GWPolR coefficients for (a) x_1^2, (b) x_3^2*

4. CONCLUSIONS

The parameter estimate of GWPolR depends on the weighting function selected, bandwidth, and polynomial degree of each explanatory variable. For a certain weighting function, the optimum bandwidth and the optimum polynomial degrees can be found by using a simple algorithm based on a certain criterion of the model optimization.

For the poverty data studied here, the GLR model was not feasible to use due to the existence of nonlinearity relationships between the poverty and some explanatory variables. With this condition, the GWPolR model produced the best performance in describing the poverty data. It gave a significant improvement and also explained more complete on how each explanatory variable was related to the poverty.

The spatial variation of the GWPolR parameter estimates exhibits some regional features. Hence, policy recommendation can be proposed specifically in each region. For example, to accelerate the reduction of poverty rate in some districts which have higher poverty rate, especially in Sampang, Pamekasan, Sumenep, and Probolinggo, poverty alleviation programs should focus more on education to reduce the percentage of people educated less than elementary school.

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