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The Effect of Strain Hardening and Cross Section Shape on Post Yield Behavior of Steel Box Girder Bridge

Muttaqin Hasan, Mochammad Afifuddin, Muhammad Jamil
Dept. of Civil Engineering
Syiah Kuala University
Banda Aceh, Indonesia
E-mail: muttaqin@unsyiah.ac.id

Yuslinda
Dept. of Structural Engineering
Civil Engineering Consultant
Banda Aceh, Indonesia

Abstract: *The purpose of this study is to analyze the post yield behavior of 4 different types of box girder cross sections. In order to see the effect of strain hardening, bilinear stress-strain curve and stress-strain curve with strain hardening were used. The studied parameters were moment-curvature relationships, plastic moments, inelastic region lengths, and shape factors. The analyzed bridge has the span length of 40 meters and width of 10 meters. All cross sections were designed to have the same yield moment. For analysis with bilinear model, the higher plastic moment was reached at multiple box section while the lower plastic moment was reached at single box section. For analysis with strain hardening model, the higher plastic moment was reached at twin box section while the lower plastic moment was reached at single box section. The higher increase of plastic moment due to strain hardening was reached at single box section, which was 33.5% increasing, while the lower increase of plastic moment was reached at multiple box section, which was 25.8% increasing. The average increase of plastic moment due to strain hardening for all analyzed cross-sections was 29.1%. The higher increase of inelastic region length due to strain hardening was reached at single box section, which was 4 times increasing. The moment-curvature relationships for all cross sections with and without strain hardening were compared.*

Keywords— strain hardening, box girder, plastic moment, yield moment, moment-curvature relationship

1. INTRODUCTION

There have been many studies about behavior of box girder bridges [e.g. 1-4]. Box girder bridges have several cross-sectional shapes, i.e. single box, twin boxes, multiple boxes, and cellular box. The cross-sections affect post yield behavior of girder although the cross sections were designed to have the same yield moments.

In elastic range, the stress-strain relationship is linear. When the yield stress is reached, the strain increases without increasing stress. At a certain level of strain, the stress again increases as increasing the strain until it reaches the maximum tensile stress. This range is called strain hardening [5,6]. In design and analysis of steel structures, the strain hardening is always ignored. The steel is idealized as an elastic-perfectly plastic material. Bilinear stress-strain relationship as shown in Fig. 2(a) is always assumed [5,7].

In this paper, elastoplastic analysis of steel box girder with four different cross-sectional shapes as mentioned above is presented. In order to see the effect of strain hardening on post yield behavior of those box girders, bilinear stress-strain curve and stress-strain curve with strain hardening were used in the analysis. In analysis, only flexural deformation was considered.

2. ANALITICAL METHOD

Bridge cross sections were designed based on Indonesian Standard for Design of Bridge Structures (RSNI T-03-2005) [7] and Indonesian Standard for Bridge Loads (RSNI T-02-2005) [8]. The analyzed bridge has the span length of 40 meters and width of 10 meters. All cross sections were designed to have the same yield moment (by choosing the cross sections which have the same elastic section modulus). The steel used for the girder is BJ 55 steel based on RSNI T-03-2005 [3]. The properties of BJ 55 steel are as follows: yield stress (f_y) = 410 MPa, ultimate stress (f_{su}) = 550 MPa, modulus of elasticity (E) = 200 GPa, yield strain (ε_y) = 0.00205, ultimate strain (ε_{su}) = 0.13, Poisson ratio (ν) = 0.3, strain at the beginning of strain hardening (ε_{sh}) = 0.02, and strain hardening modulus (E_{sh}) = 10 GPa. The designed cross section of box girders are shown in Figs. 1-4.

Two stress-strain models were used in section analysis as shown in Fig. 5. For bilinear model in Fig. 5 (a), the stress-strain relationship is written as in Eqs. (1) and (2) [5-7].

$$f_s = E_s\, \varepsilon_s \quad ; \varepsilon_s < \varepsilon_y \qquad (1)$$

$$f_s = f_y \quad ; \varepsilon_s \geq \varepsilon_y \qquad (2)$$

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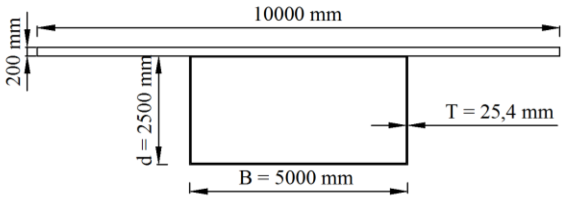

Fig. 1. Sigle box girder

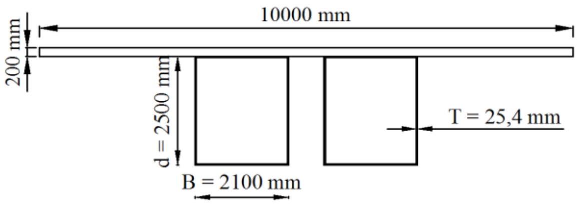

Fig. 2. Twin box girder

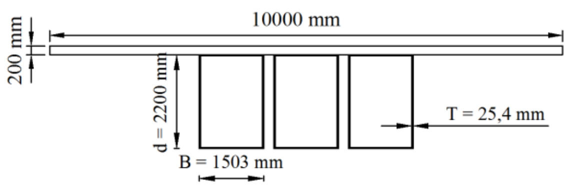

Fig. 3. Multiple box girder

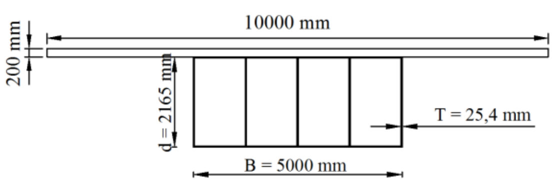

Fig. 4. Cellular box girder

For, stress-strain model with strain hardening, the model proposed by Thomson and Park (1978) [9] as shown in Fig. 5(b) and Eq. (3) to Eq. (7) was adopted as follows.

Region AB: $\varepsilon_s \leq \varepsilon_y$; $f_s = E_s\ \varepsilon_s$ (3)
Region BC: $\varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh}$; $f_s = f_y$ (4)
Region CD: $\varepsilon_s \geq \varepsilon_{sh}$;

$$f_s = f_y \left\{ \frac{m(\varepsilon_s-\varepsilon_{sh})+2}{60(\varepsilon_s-\varepsilon_{sh})+2} + \frac{(\varepsilon_s-\varepsilon_{sh})(60-m)}{2(30q+1)^2} \right\} \tag{5}$$

where:

$$m = \frac{\left(\frac{f_{su}}{f_y}\right)(30q+1)^2 - 60q - 1}{15q^2} \tag{6}$$

and:

$$q = \varepsilon_{su} - \varepsilon_{sh} \tag{7}$$

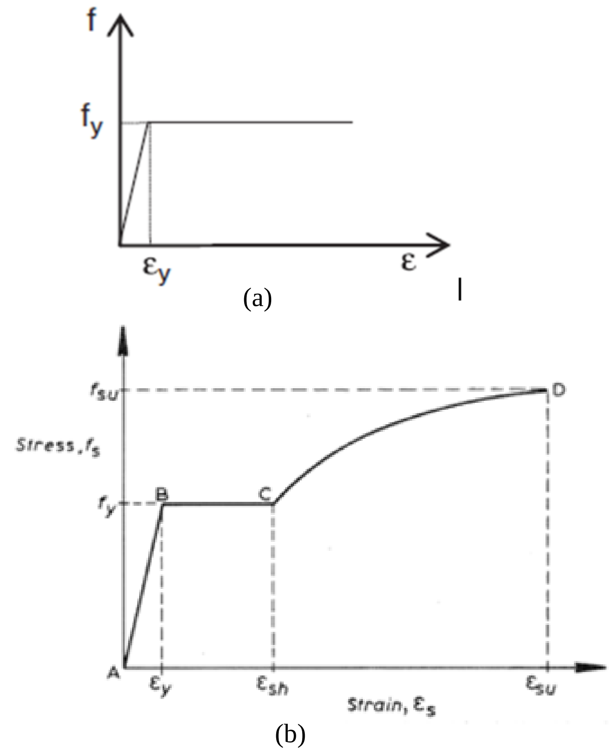

Fig. 5. Stress-strain model: (a) bilinear model; (b) model with strain hardening

To obtain plastic momen and momen-curvature relationship, the elastoplastis section analysis was conducted with the following step:

1) Devide the top and bottom flange of the section to 10 strips, respectively.
2) Devide the web of the section to 180 strips.
3) Assign the value of strain at the outer fiber of the section as yield strain.
4) Calculate the value of β for the partial plastification of the cross section. The value of β is a half of the ratio between the height of cross section that has not plastically yet to overall height of cross section.
5) Calculate strain at every strip.
6) Calculate stress at every strip. For bilinier model, Eqs. (1) and (2) were used for stress calculation. For model with strain hardening, Eqs. (3) to (7) were used for stress calculation.
7) Calculate the area of every strip.
8) Calculate axial force at every strip.
9) Calculate the bending moment of the strip about the neutral axis
10) The sum of the moments from all strips is the moment capacity of the section at this condition.
11) Calculate the curvature of the section (κ) using the following equation [5]:

$$\kappa = \frac{f_y}{E\beta d} \tag{8}$$

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12) Increase the strain at the outer fiber of the section.
13) Repeat step 3 to step 11 until all the section already plastically. The moment when all the section already plastically is plastic moment.
14) Draw the moment-curvature curve.

3. RESULTS AND DISCUSSIONS

3.1 ANALITICAL RESULTS

The plastic momen, the shape factor (ratio between plastic moment and yield moment) and the length of inelastic region for all box girder cross sections using bilinear model and model with strain hardening are summarized in Table 1. From this table, it can be seen that in analysis with bilinear model, the highest plastic moment occured at moltiple boxes, followed by twin boxes, cellular box and single box. Because of yield moments of all sections were the same, then the multiple boxes section had highest shape factor and single box section had smallest shape factor. From the design criteria, the multiple boxes section had smallest ($M_u/\phi M_n$) ratio and the single box section had the highest ($M_u/\phi M_n$) ratio. From this fact, it can be concluded that the multiple boxes section was very safe compared to the other sections.

Table 1 Plastic moment, shape factor and length of inelastic region

Cross section	Bilinear model			Strain hardening model			M_{ps}/M_p	s_s/s	x_{ps}/x_p
	M_p (kNm)	s	x_p (m)	M_{ps} (kNm)	s_s	x_{ps} (m)			
Single box	160069	1.08	3.026	213678	1.44	12.302	1.335	1.333	4.065
Twin boxes	170670	1.15	5.325	223166	1.51	13.482	1.308	1.313	2.532
Multiple boxes	174213	1.18	6.032	219214	1.48	13.005	1.258	1.254	2.156
Cellular box	169551	1.15	5.084	214346	1.45	12.381	1.264	1.261	2.435

Note: M_p = Bilinear model plastic moment; M_{ps} = Strain hardening model plastic moment; s = Bilinear model shape factor; s_s = Strain hardening model shape factor; x_p = Bilinear model inelastic region length; x_{ps} = Strain hardening model inelastic region length

In analysis with strain hardening model, the highest plastic moment occured at twin boxes, followed by multiple boxes, cellular box and single box. Therefore, the twin boxes section had highest shape factor and single box section had smallest shape factor. The highest increase in plastic moment due to strain hardening was happened at single box section, which was 33.5% and the smallest increase in plastic moment due to strain hardening was happened at multiple box section, which was 22.8%. The average increase in plastic moment due to strain hardening for all analyzed section in this study was 29.1%.

Because the length of inelastic range is a function of plastic moment and yield moment, and the the same yield moment was designed in this study, then the cross section with highest plastic moment will has longest inelastic region. In this study, as can be seen in Table 1, in analysis with bilinear model, the multiple boxes section had longest inelastic region. In analysis with strain hardening model, the twin boxes section had longest inelastic region. The maximum increase in the length of inelastic region due to strain hardening was happened at single box section. The length of inelastic region of single box with strain hardening model was four time of that with bilinear model. By those fact, it can be concluded that strain hardening not only increases the strength, but also increses the ductility of steel members.

3.2 Momen – Curvature Relationships

Fig. 6 shows the comparison of momen-curvature relationships for all box girder sections in analysis with bilinear and strain hardening models. The relationship was linear until yield moment was reached. The yield moment and corresponding curvature at the yield of outer fiber of section was 147.947 kNm dan 0,0016 /m. After yield moment was reached, the non linear relationships were observed until certain value of moment (says M_x) for all sections. The value of M_x and corresponding curvature for single box, twin boxes, multiple boxes and cellular box section were (0,0046 /m, 158681 kNm), (0,0059 /m, 169004 kNm), (0,0059 /m, 171714 kNm), dan (0,0068 /m, 168034 kNm), respectively. The value of β at those point was 0.14, 0.18, 0.16 and 0.14 for single box, twin boxes, multiple boxes and cellular box section, respectively. For bilinear model, the momen-curvature relationship again become linear for the increasing of curvature further.

For strain hardening model, at the curvature of 0.0137 /m which was corresponding to the strain in which strain hardening starting, the moment again increased until all the section get plastification. The moments at this curvature for single box, twin boxes, multiple boxes and cellular box section were 159930 kNm, 170392 kNm, 173798 kNm and 169204 kNm, respectively. The value of β at those point was 0.06 for single box dan twin boxes section; and 0.07 for multiple boxes and cellular box section. The increasing of moments after this point until plastic moment was reached is shown in Table 2. The highest increase was at single box section which was 33.60% and the lowest increase was at multiple boxes which was 26.1%. The comparison of momen-curvature relationship in analysis with bilinear and strain hardening model for all cross section are shown in Figs. 7-10.

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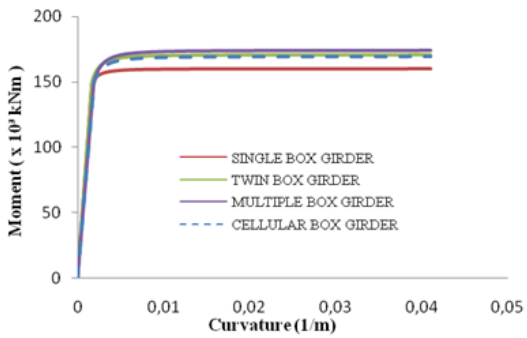
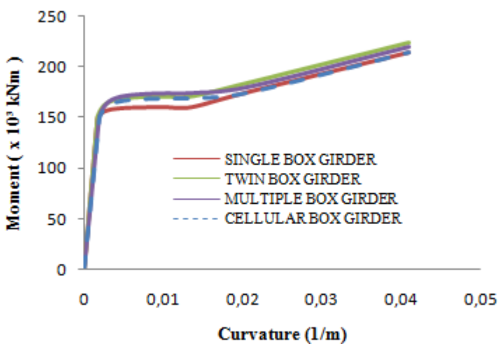


(a) Bilinear model



(b) Strain hardening model

Fig. 6. Moment-curvature relationships

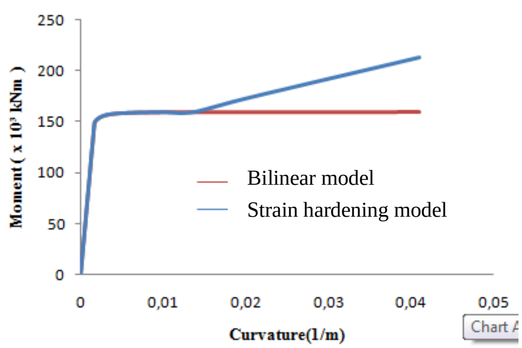


Fig.7. Comparison M-κ relationship for single box girder

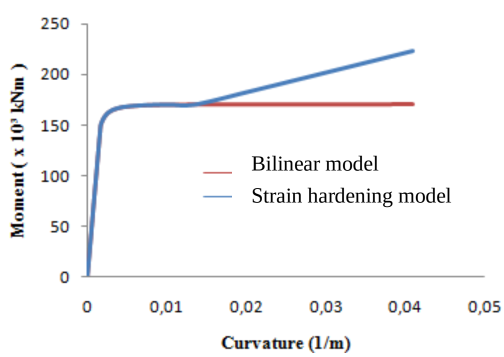


Fig. 8. Comparison M-κ relationship for twin box girder

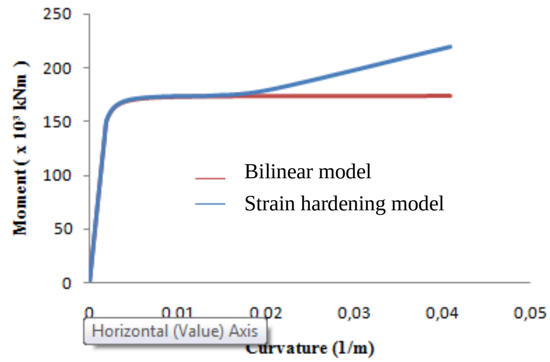


Fig. 9. Comparison M-κ relationship for multiple cross section

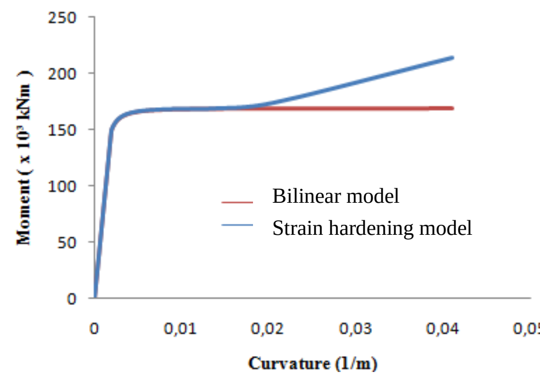


Fig. 10. Comparison M-κ relationship for cellular cross section

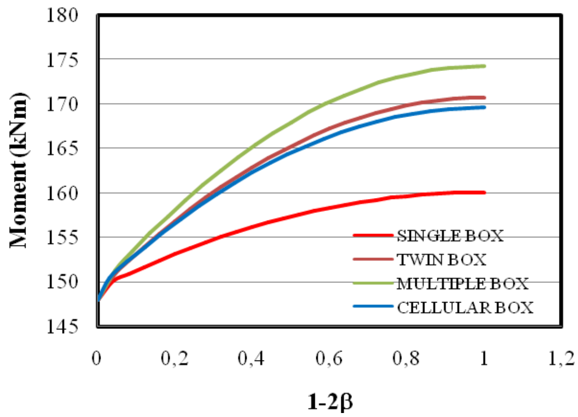
Table 2 The ratio between plastic moment and moment at curvature of 0.0137 /m

Section	M_z (kNm)	M_{ps} (kNm)	M_{ps}/M_z
Single box	159.930	213.678	1.336
Twin box	170.392	223.116	1.309
Multiple box	173.798	219.214	1.261
Cellular box	168.204	214.346	1.267

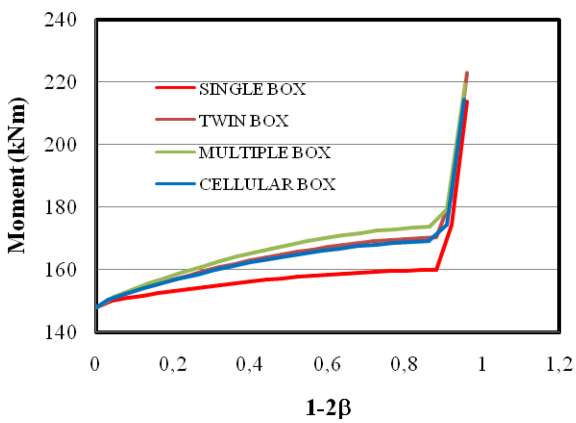
Note: M_z = moment at curvature of 0.0137 /m

3.3 Plasticification Process

In order to see the plastification process, the relationship between moment and (1-2β) was plotted in Fig. 11. The value (1-2β) is the ratio between the plastic portion of the section height to overall section height. Plastification started when the strain at the outer section fibers get yield. The depth of plastification increased by increasing the moment. As can be seen from Fig. 5, the relationship between the moment and the pastic portion ratio was non linear. For bilinear model, the curves were smooth until all the section gets plastic. For strain hardening model, the curves were linear until 88 % of section height gets plastic. For the further increase in plastic portion, the moment increased suddenly until it reaches plastic moment.



(a) Bilinear model



(b) Strain hardening model

Fig.11. Moment and (1-2β) relationships

4. Conclusions

Based on this study, the following conclusions can be drawn:

1. For analysis with bilinear model, the higher plastic moment was reached at multiple box section while the lower plastic moment was reached at single box section. For analysis with strain hardening model, the higher plastic moment was reached at twin box section while the lower plastic moment was reached at single box section. The higher increase of plastic moment due to strain hardening was reached at single box section, which was 33.5% increasing, while the lower increase of plastic moment was reached at multiple box section, which was 25.8% increasing. The average increase of plastic moment due to strain hardening for all analyzed cross-sections was 29.1%.
2. For analysis with bilinear model, the multiple boxes section had longest inelastic region. For analysis with strain hardening model, the twin boxes section had longest inelastic region. The maximum increase in the length of inelastic region due to strain hardening was happened at single box section. The length of inelastic region of single box with strain hardening model was four time of that with bilinear model.
3. Plastification started when the strain at the outer section fibers get yield. The depth of plastification increased by increasing the moment.

5. References

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