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The Dynamic Production/Inventory Lot sizing Problem with Stochastic Demand

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Abstract: *The paper addresses the multi-period single-item production-inventory lot sizing problem where demand is stochastic and non-stationary. Over the entire planning horizon, the inventory replenishment periods are uniformly fixed and considering inventory positions of the product, we formulate a finite state Markov decision process model where states of a Markov chain represent possible states of demand for the product. The objective is to determine for each period of the production –inventory problem an optimal lot sizing policy so that the long run production-inventory costs are minimized for the given state of demand. The decision of whether or not to produce additional inventory units is made using dynamic programming over a finite period planning horizon. The approach demonstrates the existence of an optimal state dependent production lot size (PLS) and produces an optimal lot sizing policy as well as the corresponding total production and inventory costs.*

Keywords— Inventory; lot-size; production; stochastic demand

1. INTRODUCTION

The dynamic production lot size (PLS) model has extensive applications in production-inventory problems of manufacturing industries. The model is primarily used to decide upon when and how much to produce for inventory under fluctuating demand. A critical foundation of the stochastic version for the dynamic lot-size problem by Vargas [1] determines the optimal solution to the deterministic case using the well-known Wagner-Whitin algorithm. However, the characteristics of cost structures for dynamic lot size model with stochastic demand as illustrated by Bhaskaran and Sethi [2] show the structure of the setup cost not having a convex structure that is usually assumed. Tarim and Kingsman [3] build on the existing literature by considering service constraints under the static-dynamic uncertainty strategy where replenishment periods are fixed at the beginning of the planning horizon.
Critical analysis of the problem from the social and behavioral science perspectives by Purohit , Choudhary and Shankar [4] induced authors to investigate the effects of emission and system related parameters on inventory lot sizing and supply chain performance under dynamic stochastic demand using integer programming. Although earlier scholars examined the dynamic lot sizing problem with deterministic demands but stochastic lead times as noted by Nevison and Burstem [5], the solutions were lumpy in the sense that each order satisfied a set not necessarily consecutive, of the demands. The earlier version of this problem by Mubiru[6] considers the economic production lot size model with stochastic demand ; where stationary assumption of demand over the planning horizon is considered. The demand of item is described by a two-state markov chain and using dynamic programming, the optimal production lot sizing policy and economic production quantity are determined. In similar contexts, modeling lot size with time dependent demand based on stochastic programming by Rojas and Leiva [7] considered a case study of drug supply in Chile where a lot sizing methodology was proposed for an inventory system that faces time dependent random demands. The stochastic lot sizing literature also benefits from Bijari and Shirnesshan [8] where authors maximize the probability of meeting target profit for a single item and single period.

Additional heuristics complement the solution approaches by Senyogdt E [9] where simultaneous considerations of both demand and price uncertainties are examined. When all the costs are constant over time, this represents the classical dynamic lot sizing problem whose optimal solution can be obtained by the Wagner-Whitin algorithm. The economic production lot size with stochastic demand and shortage with partial backlogging rate under imperfect quality items presented by Kumar and Chauham [10] considered constant deterioration and linear holding costs where shortages are permitted in inventory. The parameter effects upon the optimal solutions were numerically examined.

On a comparative note however. the dynamic/inventory lot sizing model we propose offer interesting results for discussion especially for the sake of characterizing random demand in a dynamic production/inventory setting when shortages are allowed or not allowed.

2. Model Description

We consider a production-inventory system whose demand during each time period over a fixed planning horizon is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of

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any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to produce additional stock units (a decision denoted by Z=1) or not to produce additional units (a decision denoted by Z=0) during each time period over the planning horizon, where Z is a binary decision variable.

Optimality is defined such that the lowest expected total production and inventory costs are accumulated at the end of a total of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-period (N=2) planning horizon is considered.

2.1 Notation

Sets

i,j	Set of states of demand
Z	Set of lot sizing policies

Parameters

D	Demand matrix
Q	Demand transition matrix
O	On-hand inventory matrix
V	Production-Inventory cost matrix
e	Expected costs
a	Accumulated costs
c_p	Unit production cost
c_h	Unit holding cost
c_s	Unit shortage cost
P	Production lot size
Q^Z_{ij}	Demand transition probability

Others

n,N	Stages
C	Customer matrix
F	Favorable demand
U	Unfavorable demand

i,j Є {F,U} Z Є{0.1}

2.2 Finite-Period Dynamic Programming Formulation

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal PLS and lot sizing policy may be expressed as a finite period dynamic programming model. Let $g_n(i)$ denote the optimal expected total production and inventory costs accumulated during periods *n+1*,…………***N*** given that the state of the system at the beginning of period *n* is i Є{F,U}.The recursive equation relating g_n and g_{n+1} is

$$g_n(i) = min_Z[Q^Z_{iF}(n)V^Z_{iF}(n) + g_{n+1}(F), Q^Z_{iU}(n)V^Z_{iU}(n) + g_{n+1}(U)]$$

i Є{F,U}, Z Є{0,1} n=1,2,……….N (1)

together with the conditions

$$g_{N+1}(F) = g_{N+1}(U) = 0$$

This recursive relationship may be justified by noting that the cumulative total production - inventory costs $V^Z_{ij}(n)$ + $g_{n+1}(j)$ resulting from reaching state j Є {F,U} at the start of period *n+1* from state i Є {F,U} at the start of period *n* occurs with probability $Q^Z_{ij}(n)$.

Clearly,

$$e^Z(n) = [Q^Z(n)][V^Z(n)]^T \quad Z \in \{0,1\} \tag{2}$$

where "T" denotes matrix transposition, and hence the dynamic programming recursive equations

$$\begin{aligned} g_N(i,n) &= min_Z[e^Z_i(n)] \\ &+ min_Z[Q^Z_{iF}(n)g_{N+1}(F,n)] \\ &+ min_Z[Q^Z_{iU}(n)g_{N+1}(U,n)] \end{aligned} \tag{3}$$

$$g_N(i,n) = min_Z[e^Z_i(n)] \tag{4}$$

result.

2.2.1 Computing $Q^Z(n)$ and $V^Z(n)$

The demand transition probability from state i Є {F,U} to state j Є {F,U},given lot sizing policy Z Є {0,1} may be taken as the number of customers observed with demand initially in state i and later with demand changing to state j , divided by the sum of customers over all states. That is,

$$Q^Z_{ij}(n) = C^Z_{ij}(n)/[C^Z_{iF}(n) + C^Z_{iU}(n)] \tag{5}$$

i Є {F,U} , Z Є {0,1}

When demand outweighs on-hand inventory, the cost matrix $V^Z(n)$ may be computed by means of the relation

$$V^Z(n) = (c_p + c_h + c_s)[D^Z(n) - O^Z(n)]$$

$$V^Z_{ij}(n) = \begin{cases} (c_p + c_h + c_s)[D^Z_{ij}(n) - O^Z_{ij}(n)] & if \ \ D^Z_{ij}(n) > O^Z_{ij}(n) \\ c_h[O^Z_{ij}(n) - D^Z_{ij}(n)] & if \ \ D^Z_{ij}(n) \le O^Z_{ij}(n) \end{cases} \tag{6}$$

for all i,j Є {F,U} and Z Є {0,1}

A justification for expression (6) is that $D^Z_{ij}(n) - O^Z_{ij}(n)$ units must be produced in order to meet excess demand. Otherwise production is cancelled when demand is less than or equal to on-hand inventory.

The PLS when demand is initially in state i Є {F,U} ,given lot sizing policy Z Є {0,1} is

$$p^Z_i(n) = [D^Z_{iF}(n) - O^Z_{iF}(n)] + [D^Z_{iU}(n) - O^Z_{iU}(n)]$$

i Є {F,U} , Z Є {0,1}

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The following conditions must, however, hold:

1. $P^Z_i(n) > 0$ when $D^Z_{ij}(n) > O^Z_{ij}(n)$, and $p^Z_i(n) = 0$ when $D^Z_{ij}(n) \leq O^Z_{ij}(n)$
2. Z=1 when $c_p > 0$, and Z=0 when $c_p = 0$
3. $c_s > 0$ when shortages are allowed, and $c_s = 0$ when shortages are not allowed

3. Computing an Optimal Lot sizing Policy and PLS

The optimal PLS and lot sizing policy are found in this section, for each period separately

3.1 Optimization strategy during period 1

When demand is favorable (i.e. in state F), the optimal lot sizing policy during period 1 is

$$Z = \begin{cases} 1 & if \;\; e_F^1(1) < e_F^0(1) \\ 0 & if \;\; e_F^1(1) \geq e_F^0(1) \end{cases}$$

The associated total production and inventory costs and PLS are then

$$g_1(F) = \begin{cases} e_F^1(1) & if \;\; Z = 1 \\ e_F^0(1) & if \;\; Z = 0 \end{cases}$$

and

$$p_F^Z(1) = \begin{cases} [D_{FF}^1(1) - O_{FF}^1(1)] + [D_{FU}^1(1) - O_{FU}^1(1)] & if \;\; Z = 1 \\ 0 & if \;\; Z = 0 \end{cases}$$

respectively. Similarly, when demand is unfavorable (i.e. in state U), the optimal lot sizing policy during period 1 is

$$Z = \begin{cases} 1 & if \;\; e_U^1(1) < e_U^0(1) \\ 0 & if \;\; e_U^1(1) \geq e_U^0(1) \end{cases}$$

while the associated total production and inventory costs and PLS are

and

$$p_U^Z(1) = \begin{cases} [D_{UF}^1(1) - O_{UF}^1(1)] + [D_{UU}^1(1) - O_{UU}^1(1)] & if \;\; Z = 1 \\ 0 & if \;\; Z = 0 \end{cases}$$

3.2 Optimization strategy during period 2

Using dynamic programming recursive equation (3) ,and recalling that $a^Z_i(1)$ denotes the already accumulated production and inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^Z(2) = e_i^Z(1) + Q_{iF}^Z(2)min[e_F^1(1), e_F^0(1)] + Q_{iU}^Z(2)min[e_U^1(1), e_U^0(1)]$$
$$= e_i^Z(1) + Q_{iF}^Z(2)g_2(F) + Q_{iU}^Z(2)g_2(U)$$

Therefore, when demand is favorable (ie. in state F), the optimal lot sizing policy during period 2 is

$$Z = \begin{cases} 1 & if \;\; a_F^1(2) < a_F^0(2) \\ 0 & if \;\; a_F^1(2) \geq a_F^0(2) \end{cases}$$

while the associated total production and inventory costs and PLS are

$$g_2(F) = \begin{cases} a_F^1(2) & if \;\; Z = 1 \\ a_F^0(2) & if \;\; Z = 0 \end{cases}$$

and

$$p_F^Z(2) = \begin{cases} [D_{FF}^1(2) - O_{FF}^1(2)] + [D_{FU}^1(2) - O_{FU}^1(2)] & if \;\; Z = 1 \\ 0 & if \;\; Z = 0 \end{cases}$$

respectively. Similarly, when demand is unfavorable (i.e. in state U), the optimal lot sizing policy during period 2 is

$$Z = \begin{cases} 1 & if \;\; a_U^1(2) < a_U^0(2) \\ 0 & if \;\; a_U^1(2) \geq a_U^0(2) \end{cases}$$

In this case, the associated total production and inventory costs and PLS are

$$g_2(U) = \begin{cases} a_U^1(2) & if \;\; Z = 1 \\ a_U^0(2) & if \;\; Z = 0 \end{cases}$$

and

$$P_U^Z(2) = \begin{cases} [D_{UF}^1(2) - O_{UF}^1(2)] + [D_{UU}^1(2) - O_{UU}^1(2)] & if \;\; Z = 1 \\ 0 \;\; if \;\; Z = 0 \end{cases}$$

respectively.

4. Case Study

In order to demonstrate use of the model in §3-4, a real case application from Plascon, a manufacturer of plastic utensils in Uganda is presented in this section. Plastic basins are among the products manufactured and the demand for basins fluctuates from month to month. The company wants to avoid over-producing when demand is low or under-producing when demand is high, and hence. seeks decision support in terms of an optimal lot sizing policy, the associated production-inventory costs and specifically a recommendation as to PLS of plastic basins over a two-week period.

4.1 Data collection

A sample of 50 customers was used. Past data revealed the following demand pattern and inventory levels of basins during the first and second weeks of the month when demand was favorable (state F) or unfavorable (state U).

Table 1: Customers versus state transitions

		Lot sizing policy 1		Lot sizing policy 0	
Week	States	F	U	F	U
1	F	34	16	24	26
	U	20	30	19	31

2	F	27	23	25	25
	U	22	28	9	41

Table 2: Demand (basins) versus state transitions

		Lot sizing policy 1		Lot sizing policy 0	
Week	States	F	U	F	U
1	F	75	10	80	25
	U	20	50	40	10
2	F	80	15	75	30
	U	45	60	90	45

Table 3: On hand inventory (basins) versus state transitions

		Lot sizing policy 1		Lot sizing policy 0	
Week	States	F	U	F	U
1	F	60	20	25	50
	U	40	25	30	65
2	F	75	30	45	100
	U	55	40	50	60

In either case, the unit production cost (c_p) is \$2.00,the unit holding cost per week(c_h) is \$0.50 and the unit shortage cost per week (c_s) is \$1.00

4.2 Computation of model parameters

Using (5) and (6)., the state transition matrices and production-inventory cost matrices for weeks 1 and 2 are

$$Q^1(1) = \begin{bmatrix} 0.68 & 0.32 \\ 0.67 & 0.33 \end{bmatrix} \qquad V^1(1) = \begin{bmatrix} 52.5 & 5 \\ 10 & 87.5 \end{bmatrix}$$

$$Q^1(2) = \begin{bmatrix} 0.54 & 0.46 \\ 0.79 & 0.21 \end{bmatrix} \qquad V^1(2) = \begin{bmatrix} 52.5 & 7.5 \\ 5 & 72 \end{bmatrix}$$

for the case where additional basins are produced during weeks 1 and 2, while these matrices are given by

$$Q^0(1) = \begin{bmatrix} 0.48 & 0.52 \\ 0.38 & 0.62 \end{bmatrix} \qquad V^0(1) = \begin{bmatrix} 192.5 & 12.5 \\ 38 & 17.5 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.50 & 0.50 \\ 0.18 & 0.82 \end{bmatrix} \qquad V^0(2) = \begin{bmatrix} 105 & 55 \\ 140 & 7.5 \end{bmatrix}$$

for the case where additional basins are not produced during weeks 1 and 2. When additional basins are produced (Z=1) during week 1, the matrices $Q^1(1)$ and $V^1(1)$ yield the costs

$$e_F^1(1) = (0.68)(52.5) + (0.32)(5) = 37.3$$
$$e_U^1(1) = (0.67)(10) + (0.33)(87.5) = 35.38$$

However, when additional basins are not produced (Z=0) during week 1, the matrices $Q^0(1)$ and $V^0(1)$ yield the costs

$$e_F^0(1) = (0.48)(192.5) + (0.52)(12.5) = 98.9$$
$$e_U^0(1) = (0.38)(35) + (0.62)(17.5) = 24.15$$

When additional units are produced (Z=1) during week 2, the matrices $Q^1(2)$ and $V^1(2)$ yield the costs

$$e_F^1(2) = (0.54)(52.5)) + (0.46)(7.5) = 29.96$$
$$e_U^1(2) = (0.79)(5)) + (0.21)(72) = 19.07$$

while the matrices $Q^0(2)$ and $V^0(2)$ yield the costs

$$e_F^0(2) = (0.50)(105)) + (0.50)(35) = 70.00$$
$$e_U^0(2) = (0.18)(140)) + (0.82)(7.5) = 13.35$$

4.3 The Optimal lot sizing policy and PLS

Since 37.3 < 98.9, it follows that Z=1 is an optimal lot sizing policy for week 1 with associated total production and inventory costs of \$37.3 and a PLS of 75 – 60 =15 units if demand is favorable. Since 24.15 < 35.58, if follows that Z=0 is an optimal lot sizing policy for week 1 with associated total production and inventory costs of \$24.15 and a PLS of 0 units if demand is unfavorable.

If demand is favorable, the accumulated production and inventory costs at the end of week1 are

$$a_F^1(1) = 37.3 + (0.54)(29.6) + (0.46)(13.35) = 59.43$$
$$a_F^0(1) = 98.9 + (0.50)(29.6) + (0.50)(13.35) = 120.38$$

Since 59.43< 120.38, it follows that Z=1 is an optimal lot sizing policy for week 2 with associated accumulated production and inventory costs of \$59.43 and a PLS of 80 – 75 = 5 units for the case of favorable demand. However, if demand is unfavorable, the associated accumulated production - inventory costs at the end of week 1 are

$$a_U^1(1) = 35.58 + (0.79)(29.6) + (0.21)(13.35) = 61.7$$
$$a_U^0(1) = 24.15 + (0.18)(29.6) + (0.82)(13.35) = 40.43$$

Since 40.43<61.7, it follows that Z=0 is an optimal lot sizing policy for week 2 with associated accumulated production and inventory costs of \$29.63 and a PLS of 0 units for the case of unfavorable demand. When shortages are not allowed, the values of Z, $g_n(i)$ and $P^Z{}_i(n)$ may be computed for i Є {F,U} in a similar fashion after substituting c_s=0 in the matrix function $V^Z(n) = [c_p+c_h+c_s][D^Z(n) – O^Z(n)]$

5. Conclusion

A dynamic production lot size model with stochastic demand was presented in this paper. The model determines an optimal lot sizing policy, production-inventory costs and the PLS of a given product with stochastic non-stationary demand. The decision of whether or not to produce additional stock units is modeled as a multi-period decision problem using dynamic programming over a finite period planning horizon. The working of the model was demonstrated by means of a case study. Therefore as a cost minimization strategy in production-inventory management, computational efforts of using markov decision processes show promising results. It is appropriate to conclude this paper by identifying the limitations of our model, which also indicate future research directions.

• Our model ignores production disruptions that influence lot sizing policies in manufacturing

• Production capacity limitations that influence lot size optimization for effective production planning have similarly not been considered.

• Extending this model to production/inventory lot sizing policies using continuous time markov chains (CTMC) is an important challenge

• Finally, better and more robust models are needed for production/lot sizing policies with highly uncertain demand conditions.

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