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Some Types of Numerical Methods For Account Of Double Integral

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Abstract: *the main aim of this paper we introduce number of methods to find the values of the double integrals numerically, and compare with each other to see the best way , which were method use Newton Coates Rules (Simpson, trapezoidal, and middle point) and Suggested method , all method were used equal dimensions and using Romberg's acceleration with methods.*

Keywords: Newton Coates, Suggested mothed ,Romberg acceleration, double integrals.

1. Introduction

Many Researchers,have worked in the field of single integrations because of their importance in calculating flat spaces, in finding the sizes of rotational objects, in finding middle positions, in the inertia of flat surfaces, in solid rotary bodies, and in the pressure of liquid, And remember them Fox [6] in 1967, Fox and Haze [7] in 1970 and Shanks [11] in 1972, who dealt with this subject in many ways.

finding value for duoble integrals is a much complex issue, than finding the value of a single integration ,because the complement here count on two variables ,In this paper researchers used several numerical methods using Newton Coates rules [5]

Trapezpoial rule $T(h)=\frac{h}{2}[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)$

Simpsion rule $S(h)=\frac{h}{3}[f(a)+2\sum_{v=1}^{(n/2-1)} f(\chi_{2v})+4\sum_{v=1}^{n/2} f(\chi_{2v-1})+f(b)]$

, Mid-point rule $M(h)=h\sum_{i=0}^{n-1}\left(f(x_i+\tfrac{h}{2})\right)$

and correction terms to this methods improve the value of integral and accelerate the numerical value of integral to the analytical or exact one and correction terms for each rule of Newton-Coates rules for continuous integrals for *T (h), S(h)* and *M (h)* respectively are Fox [6]

$$E_T(h)=A_{T_1}h^2+A_{T_2}h^4+A_{T_3}h^6+\cdots$$
$$E_S(h)=\mu_{S_1}h^4+\mu_{S_2}h^6+\mu_{S_3}h^8+\cdots$$
$$E_M(h)=\eta_{M_1}h^2+\eta_{M_2}h^4+\eta_{M_3}h^6+\cdots$$

where $A_{T_\omega},\mu_{S_\omega},\eta_{M_\omega},\omega=1,2,3,\ldots$ are constants These correction limits help us in applying Romberg's acceleration [4]

And use Suggested method (Su) [9]

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$$su(h)=f(a)+f(b)+2f(a+(n-\frac{1}{2})h)+2\sum_{i=1}^{n-1}(f(x_i)+f(a+(i-\frac{1}{2})h)$$

and correction terms $E(h)=A_{Su}h^2+B_{Su}h^4+C_{Su}h^6+\cdots$ where $A_{Su},B_{Su},C_{Su},\ldots$ are constants double Integration J can be written as follows

$$J=\int_r^e\int_s^t g(p,q)dpdq=QQ(k)+E(k)$$

Where $QQ(k)$ is an approximate value representing the integral using one of the methods, and that $E(k)$ is series of possible correction terms added to the values, will divide the integral interval on the internal dimension $[s \;\; , t]$ for a number of intervals (n) , and divide the integral interval on the external dimension [r, e] for a number of intervals (m), we take $(n=m)$, and $k=\frac{e-r}{m},\bar{k}=\frac{t-s}{n}$ and $\left(k=\bar{k}\right)$.

We review these methods

2. Main results

1-MM method

Applying the method of midpoint rule on both dimensions, Calculated by base

$$MM(k)=k^2\sum_{j=1}^{m}\sum_{i=1}^{n}g(p_i,q_j)$$

Where $p_i=s+\frac{2i-1}{2}k$, $i=1,2,\ldots,n$, $q_i=r+\frac{2\nu-1}{2}k,\nu=1,2,\ldots,m$

And correction *terms*

$$E(k)=J-MM(k)=A_1k^2+A_2k^4+A_3k^6+\ldots$$

where $A_i,,i=1,2,3,\ldots$ are constants [1]

2- SS method

Applying the method of Simpson rule on both dimensions , we choose sub-intervals $2n$ to equal sub-intervals $2m$ and $\left(k=\bar{k}\right)$ Calculated by base

$$SS=\frac{k^2}{9}\Bigg[\; g(s,r)+g(s,e)+g(t,r)+g(t,e)+4\sum_{i=1}^{n}\left(g\left(p_{(2i-1)},r\right)+g\left(p_{(2i-1)},e\right)\right)$$

$$+2\sum_{i=1}^{n-1}\left(g\left(p_{2i},r\right)+g\left(p_{2i},e\right)\right)+4\sum_{j=1}^{n}\Bigg(g(p,q_{(2j-1)})+g(t,q_{(2j-1)})+4\sum_{i=1}^{n}g(p_{(2i-1)},q_{(2j-1)})$$

$$+2\sum_{i=1}^{n-1}g(p_{2i},q_{(2j-1)})\Bigg)+2\sum_{j=1}^{n-1}\Bigg(g(s,q_{2j})+g(t,q_{2j})+4\sum_{i=1}^{n}g(p_{(2i-1)},q_{2j})+2\sum_{i=1}^{n-1}g(p_{2i},q_{2j})\Bigg)\Bigg]$$

$$\lambda=1,2,\ldots,n\ ,\ p_{(2\lambda-1)}=s+(2\lambda-1)k \quad,\ \lambda=1,2,\ldots,n-1\ ,\ p_{2\lambda}=s+2\lambda k$$

$$\omega=1,2,\ldots n\ ,\ q_{(2\omega-1)}=r+(2\omega-1)k \quad,\ \omega=1,2,\ldots,n-1\ ,\ q_{2\omega}=r+2\omega k$$

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And correction *terms*

$$E(k)=I-SS\left(k\right)=B_1h^4+B_2h^6+B_3h^8+\cdots$$

where $B_i ,, i=1,2,3,...$ are constants [10]

3- MS method

Base the mid-point on the outer dimension q , and the Simpson base on the inner dimension p We choose *n,m* even numbers It is calculated as follows

$$SM=\frac{k^2}{3}\sum_{j=1}^{2n}\left[g(s,q_j)+g(t,q_j)+4\sum_{i=1}^{n}g(p_{(2i-1)},q_j)+2\sum_{i=1}^{n-1}g(p_{2i},q_j)\right]$$

where $p_{(2\omega-1)}=s+(2\omega-1)k \quad ,\nu=1,2,...,2n \;\; p_{2i}=s+2\nu k \quad , \;\; ,\omega=1,2,...,n$

$$q_j=r+\frac{(2\alpha-1)}{2}k \; , \qquad \alpha=1,2,...,n-1$$

And correction *terms*

$$E(k)=J-MS\left(k\right)=C_1k^2+C_2k^4+C_3k^6+\cdots$$

, where $C_i ,, i=1,2,3,...$ are constants [10]

4- SM method

Simpson's rule on the outer dimension ,and the base mid- point base on the inner dimension. We choose $2n = 2m$ It is calculated as follows

$$SM=\frac{k^2}{3}\sum_{i=1}^{2n}\left[g(p_i,r)+g(p_i,e)+4\sum_{j=1}^{n}g(p_i,q_{(2j-1)})+2\sum_{j=1}^{n-1}g(p_i,q_{2j})\right]$$

$$\lambda=1,2,...,2n \;\; p_i=s+\frac{2\lambda-1}{2}k \quad , \qquad \omega=1,2,...,n \quad , q_{(2\omega-1)}=r+(2\omega-1)k$$

$$\omega=1,2,...,n-1 \qquad y_{2\omega}=r+2\omega k$$

And correction *terms*

$$E(k)=J-MS\left(k\right)=H_1k^2+H_2k^4+H_2k^6+\cdots$$

,where $H_i ,, i=1,2,3,...$ are constants [10]

5- TT method

Derivation of the base of the use of the trapezoidal base on both dimensions (internal *p* and external *q*) It is calculated as follows

$$TT=\frac{k^2}{4}\left[g(s,r)+g(s,e)+g(t,r)+g(t,e)+2\sum_{i=1}^{n-1}\left(g(p_i,r)+g(p_i,e)\right)\right.$$

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$$+2\sum_{\alpha=1}^{n-1}\left(g(s,q_{\alpha})+g(s,q_{j})+2\sum_{i=1}^{n-1}g(p_{i},q_{\alpha})\right)\Bigg]$$

$\alpha=1,2,\ldots,n\text{-}1$, $q_{\alpha}=r+\varpi k$ $\lambda=1,2,3,\ldots,n-1,$ $p_{i}=s+\lambda k$

And correction *terms*

$E(k)=J-TT(k)=L_1k^{2}+L_2k^{4}+L_3k^{6}+\ldots.$,where $L_i,,i=1,2,3,\ldots$ are constants [2]

6- TS method

The trapezoidal base applying the method of on the outer dimension q , and the Simpson rule on the inner dimension p , It is calculated as follows.

$$TS=\frac{k^{2}}{6}\Bigg[\quad g(s,r)+g(s,e)+g(t,r)+g(t,e)$$

$$+4\sum_{i=1}^{n/2}\left(f(p_{(2i-1)},r)+f(p_{(2i+1)},e)\right)+2\sum_{i=1}^{(n/2)-1}\left(g(p_{2i},r)+g(p_{2i},e)\right)$$

$$+2\sum_{j=1}^{n-1}\left(g(s,q_{j})+g(t,q_{j})+4\sum_{i=1}^{n/2}g(p_{(2i-1)},q_{j})+2\sum_{i=1}^{(n/2)-1}g(p_{2i},q_{j})\right)\Bigg]$$

$\alpha=1,2,\ldots,(n/2)\text{-}1$, $p_{2\alpha}=s+2\alpha k$ $\alpha=1,2,\ldots,n/2$, $p_{(2\alpha\text{-}1)}=s+(2\alpha-1)k$

$(\lambda=0,1,2,\cdots,n-1$, $q_{\lambda}=r+\lambda k)$, Only take even values n (for using Simpson's rule).

And correction *terms*

$E_{TS}(k)=J-TS(k)=M_1h^{2}+M_2h^{4}+M_3h^{6}+\ldots.$

where $M_i,,i=1,2,3,\ldots$ are constants .[3]

7- ST method

Applying the method of The trapezoidal base on the inner dimension p , and the Simpson base on the outer dimension q , It is calculated as follows

$$ST=\frac{k^{2}}{6}\Bigg[g(s,r)+g(s,e)+g(t,r)+g(t,e)+4\sum_{i=1}^{n-1}\left(g(p_{i},r)+g(p_{i},e)\right)$$

$$+4\sum_{j=1}^{n/2}\left(g(s,q_{(2j-1)})+g(t,q_{(2j-1)})+2\sum_{i=1}^{n-1}g(p_{i},q_{(2j-1)})\right)$$

$$+2\sum_{j=1}^{(n/2)-1}\left(g(s,p_{2j})+g(t,p_{2j})+2\sum_{i=1}^{n-1}g(p_{i},q_{2j})\right)\Bigg]$$

Only take even values n (for using Simpson's rule) , Where

$\lambda=1,2,\ldots,n/2$, $q_{(2\lambda\text{-}1)}=r+(2\lambda-1)k$, $\lambda=1,2,\ldots,(n/2)\text{-}1$, $q_{2\lambda}=r+2\lambda k$

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$(\beta = 0,1,2,\cdots,n-1\ ,\ p_\beta = s+\beta k)$, *And* correction *terms*

$$E_{ST}(k) = J - ST(k) = N_1 k^2 + N_2 k^4 + N_3 k^6 + ...$$

,where $N_i,, i = 1,2,3,...$ are constants .[3]

8-MT method

A base is derived from the use of the trapezoidal rule on the internal dimension p , and the base point base on the outer dimension q ,It is calculated as follows

$$MT = \frac{k^2}{2}\sum_{j=0}^{n-1}\left(g(s,q_j+\tfrac{k}{2}) + g(t,q_j+\tfrac{k}{2}) + 2\sum_{i=1}^{n-1} g(p_i,q_j+\tfrac{k}{2})\right)$$

Where $q_j = r+\lambda k$, . $\omega = 0,1,2,3,\cdots,n-1$ $p_{\text{i}} = a+\omega k$, λ=1,2,3,$\cdots$ n-1,

And correction *terms*

$$E_{MT}(k) = J - MT(k) = Q_1 k^2 + Q_2 k^4 + Q_3 k^6 + \cdots$$

,where $Q_i,, i = 1,2,3,...$ are constants . [2]

9 -TM method

In which a complex base is derived from the use of the trapezoidal base on the internal dimension *p*, and the center point base on the outer dimension q , It is calculated as follows .

$$TM = \frac{k^2}{2}\sum_{i=0}^{n-1}\left(g(p_i+\tfrac{k}{2},r) + g(p_i+\tfrac{k}{2},e) + 2\sum_{j=1}^{n-1} g(p_i+\tfrac{g}{2},q_j)\right)$$

Where $p_\lambda = a+\lambda k$ $\lambda = 0,1,2,3,\cdots n-1$, ( ) $q_j = r+\beta k$, $\beta = 1,2,\cdots,n-1$

And correction *terms*

$$E_{TM}(k) = J - TM(k) = R_1 k^2 + R_2 k^4 + R_3 k^6 + \cdots\ ,$$

where $R_i,, i = 1,2,3,...$ are constants [2]

10 -SuSu MOTHED

Applying Suggested method on both dimensions, It is calculated as follows:

$$\begin{aligned} susu = \frac{k^2}{16}[&g(s,r)+g(s,e)+g(t,r)+g(t,e)+2(g(s,r+(n-0.5)k) \\ &g(t,r+(n-0.5)k)+g(s+(n-0.5)k,r)+g(s+(n-0.5)k,e)+ \\ &g(s+(n-0.5)k,r+(c+0.5)k)+2\sum_{i=1}^{n-1}(g(s,q_i)+g(s,r+(i-0.5)k) \\ &+g(t,q_i)+g(t,r+(i-0.5)k+2g(s+(n-0.5)k,q_i)+g(p_i,r) \\ &2g(s+(n-0.5)k,r+(n+(i-0.5)k)+g(p_i,e)+2g(p_i,r+(n-0.5)k)+ \end{aligned}$$

$$g(s+(i-0.5)k,e)+2g(s+(i-0.5)k,r+(n-0.5)k))+4\sum_{j=1}^{n-1}(g(p_i,q_i)+$$
$$g(p_i,r+(j-0.5)k)+g(s+(i-0.5)k,p_i)+g(s+(i-0.5)k,r+(j-0.5)k)]$$

Where $p_i=a+\lambda k \quad \lambda=0,1,2,3,\cdots n-1$, ( ), $q_j=c+\beta k \quad ,\beta=1,2,\cdots,n-1$

And correction *terms*

$E_{SuSu}(k)=J-SuSu(k)=V_1k^2+V_2k^4+V_3k^6+\cdots,$

where $V_i,,i=1,2,3,\ldots$ are constants [8].

11 –SuT MOTHED

the use of the trapezoidal rule on the internal dimension p, and applying Suggested method on the outer dimension q ,It is calculated as follows

$$suT=\frac{k^2}{16}[g(s,r)+g(s,e)+g(t,r)+g(t,e)+2(g(s,r+(n-0.5)k))$$
$$2\sum_{i=1}^{n-1}(g(s,q_i)+g(s,r+(i-0.5)k)+g(t,q_i)+g(t,r+(i-0.5)k+g(p_i,r)$$
$$2g(s+(n-0.5)k,q_i)+4\sum_{j=1}^{n-1}\sum_{i=1}^{n-1}(g(p_i,q_i)+g(p_i,r+(j-0.5)k))]$$
$$p_\lambda=a+\lambda k \quad \lambda=0,1,2,3,\cdots n-1,(),y_\beta=c+\beta k\ ,\quad \beta=1,2,\cdots,n-1$$

And correction *terms*

$E_{SuT}(k)=J-SuT(k)=U_1k^2+U_2k^4+U_3k^6+\cdots,$

where $U_i,,i=1,2,3,\ldots$ are constants. [8]

12 –TSu MOTHED

A base is derived from the use of the Suggested method on the internal dimension p, and the trapezoidal rule base base on the outer dimension q, It is calculated as follows

$$TSu=\frac{k^2}{16}[g(s,r)+g(s,e)+g(t,r)+g(t,e)+2(g(s+(n-0.5)k,e))$$
$$2\sum_{i=1}^{n-1}(g(s,q_i)+g(s,r+(i-0.5)k)+g(t,q_i)+g(t,r+(i-0.5)k+g(p_i,r)$$
$$2g(s+(n-0.5)k,q_i)+4\sum_{j=1}^{n-1}\sum_{i=1}^{n-1}(g(p_i,q_i)+g(p_i,r+(j-0.5)k))]$$
$$p_\lambda=a+\lambda k \quad \lambda=0,1,2,3,\cdots n-1,(),q_\omega=c+\omega k\ ,\quad \omega=1,2,\cdots,n-1$$

And correction *terms*

$$E_{TSu}(k) = J - TSu(k) = Z_1 k^2 + Z_2 k^4 + Z_3 k^6 + \cdots \ ,$$

where $Z_i ,, i = 1,2,3,...$ are constants, [8]

Remark(1) : we will use the romberg acceleration with all the previous methods and we will mark it with the symbol before the mothed symbol R , for example RMM
Remark (2) :For many mothed we will work on a single table that includes all the methods will take the last value of the method and value after the application of the Romberg acceleration in the same partial period

3. EXAMPLE :

1- $I = \int_1^2 \int_0^1 x e^{-(x+y)} dxdy \quad ,(x,y) \in [0,1] \times [1,2]$

Its analytical value is 0.06144772819733 rounded to fourteen decimal places, from the table(1) , we find the results of all methods with the Romberg acceleration. The results were identical to the analytical value , And by comparing the partial periods we obtained, the method is the best RSuSu (n=m=16)is the best. The method RMS (n=m=64) is the slower compared to the methods of the rest whose results were equal in the same number of partial periods (n=m=32).

n=m	mothed	value	methods with the Romberg acceleration	value
32	MM	0.061454689204892	RMM	0.06144772819733
32	SS	0.06144772573367	RSS	0.06144772819733
64	MS	0.06144710294882	RMS	0.06144772819733
32	SM	0.06145719015870	RSM	0.06144772819733
32	TT	0.06143380341025	RTT	0.06144772819733
32	TS	0.06145272595546	RTS	0.06144772819733
32	ST	0.06142880472813	RST	0.06144772819733
32	MT	0.06142630492945	RMT	0.06144772819733
32	TM	0.06146219115063	RTM	0.06144772819733
16	SuSu	0.06143380341025	RSuSu	0.06144772819733
32	SuT	0.06143005416985	RSuT	0.06144772819733
32	TSu	0.06144799728044	RTSu	0.06144772819733

Table (1)

2- $\int_1^2 \int_1^2 \ln(x+y) dxdy$ Its analytical value is 0.06144772819733rounded to fourteen decimal places, We observe from the table (2) the results of all methods with Romberg acceleration, the results were identical to the analytical value, but the least partial periods (n=m=16)were for the method (RSuSu and RSuT and RTSu) (n=m=16), the method (SuSu)was correct for four decimal places while the method (SuT and TSu) was valid for three orders of the correct and thus the method RSuSu is the best and the method (RMS and RSM) (n=m=64) was to slow compared to the methods of the rest, Same number of partial periods(n=m=32).

n=m	mothed	value	methods with the Romberg acceleration	value
32	MM	1.08914823691532	RMM	1.08913865206603

32	SS	1.08913865110972	RSS	1.08913865206603
64	MS	1.08913985018059	RMS	1.08913865206603
64	SM	1.08913985018059	RSM	1.08913865206603
32	TT	1.08911948129138	RTT	1.08913865206603
32	TS	1.08912906649936	RTS	1.08913865206603
32	ST	1.08912906649936	RST	1.08913865206603
32	MT	1.08913385977589	RMT	1.08913865206603
32	TM	1.08913385977589	RTM	1.08913865206603
16	SuSu	1.08911948129137	RSuSu	1.08913865206603
16	SuT	1.08909072566744	RSuT	1.08913865206603
16	TSu	1.08909072566744	RTSu	1.08913865206603

Table (2)

4. Conclusion :

All numerical methods for computation of integration used the same method of derivation and by making all methods gave good results but the best is a method SuSu which gives the result less partial than the rest of the methods.

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