

Some Methods for Calculations Double Integrales Numerically

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Abstract: The main objective of this paper is to present a number of ways to find the values of double integrals with continuous integrands numerically, and compare us to each other to find out the best method, Newton Coates Rules (Simpson, Trapezoidal, and mid-point) and Suggested method, and use Romberg's acceleration or Aitken acceleration or both with methods.

Keywords: Newton Coates, Suggested method, Romberg acceleration, Aitken acceleration, double integrals.

1. Introduction

When the various sciences were complicated by the interferences between them and developed significantly and the scientists began to study the natural phenomena, whether chemical or physical, biological or engineering, and the integrations of various types played a prominent role in the interpretation of these phenomena and finding different solutions, whether analytical or numerical and integrations special importance between different mathematical sciences Such as differential and partial equations, and analytic analysis and the theory of influences. Therefore, many researchers have been able to find different numerical methods to calculate the single integrations Recall them [6], [7], [10]

In this paper we review the double integration of continuous integrations in the integration area using complex methods Romberg's acceleration and Aitken acceleration on the base of the Newton-Cotes Formulas

$$\text{Trapezoidal rule } T(k) = \frac{k}{2} [V(e) + V(r) + 2 \sum_{\alpha=1}^{\omega-1} V(\chi_{\alpha})]$$

$$\text{Simpson rule } J = \frac{k}{3} [Q(e) + 2 \sum_{\Gamma=1}^{(\frac{\omega}{2}-1)} Q(\chi_{2\Gamma}) + 4 \sum_{\Gamma=1}^{\frac{\omega}{2}} Q(\chi_{2\Gamma-1}) + Q(r)]$$

$$\text{Mid-point rule } M(k) = k \sum_{\Lambda=0}^{n-1} (f(x_{\Lambda} + \frac{k}{2}))$$

The correction limits for the above rules have been calculated by Fax [5] which respectively

$$E_T(k) = A_{T_1} k^2 + A_{T_2} k^4 + A_{T_3} k^6 + \dots \quad (1)$$

$$E_S(k) = \varpi_{S_1} k^4 + \varpi_{S_2} k^6 + \varpi_{S_3} k^8 + \dots \quad (2)$$

$$E_M(k) = \beta_{M_1} k^2 + \beta_{M_2} k^4 + \beta_{M_3} k^6 + \dots \quad (3)$$

where $A_{T_v}, \varpi_{S_v}, \beta_{M_v}, \alpha = 1, 2, 3, \dots$ are constants, These correction formula, which we use when applying the Romberg Acceleration [2], We do not need it to Aitken's delta – Squared Process (Aitken acceleration) [3], and Suggested method (Su) [9]

$$su(k) = V(e) + V(r) + 2 V(e + (n - \frac{1}{2})k) + 2 \sum_{\lambda=1}^{n-1} (V(\chi_{\lambda}) + V(e + (\lambda - \frac{1}{2})k))$$

$$\text{and correction terms } E(h) = \mu_{Su_1} k^2 + \mu_{Su_2} k^4 + \mu_{Su_3} k^6 + \dots \quad (4)$$

where $\mu_{Su_v}, v = 1, 2, 3, \dots$ are constants

If Integration J is defined as

$$J = \int_{\tau}^z \int_s^r T(u, v) dudv \quad (5)$$

the interval $[s, r]$ will divide for a number of intervals (n), and divide interval $[\tau, z]$ on a number of intervals (m), we take ($n = m$), and $\kappa = \frac{r-z}{m}, \bar{\kappa} = \frac{r-s}{n}$ and $\bar{\kappa} = \kappa$.

We review these methods

1- RM (RM) method

It is a composite method from the base of mid- point with Romberg's acceleration on both dimensions (inner dimension u and extrinsic dimension v). [8] , [4]

2- RM (AM) method

It is a composite method from the base of mid- point with Romberg's acceleration on the extrinsic dimension v , and the of mid- point with Aitken acceleration of it on the internal dimension u. [8]

3- AM (RM) method

It is a composite method from the base of mid- point with Aitken acceleration on the outer dimension v , and the mid- point with Romberg's acceleration on the internal dimension u.[7]

4- (AM)AM Method

It is a composite method from the base of the mid- point with Aitken acceleration on both dimensions. [7]

5- (TM)TM Method

It is a composite method from Trapezoidal rule with Aitken acceleration on both dimensions.[9]

6- RS (RS) method

This method is made up of the Simpson base with Romberg's acceleration on both dimensions. [4]

7- RS(AS) method

This method is made up of the Simpson base with Romberg's acceleration on the extrinsic dimension v and the Simpson base with Aitken acceleration on the internal dimension u.[7]

8 - AS (RS) Method

This method is composed of the Simpson base with Aitken acceleration on the outer dimension v and the Simpson base with Romberg's acceleration on the internal dimension u.[7]

9- AS(AS) method

It represents a complex method from the Simpson base with Aitken acceleration on both dimensions.[7]

10- RT(RSu)

we will apply the suggested method (on inner dimension u) with Romberg's acceleration and on outer dimension v by Trapezoidal rule with Romberg's acceleration[1]

11- RS(RSu) method

we will apply the suggested method (on inner dimension u) with Romberg's acceleration and on outer dimension v by Simpson rule with Romberg's acceleration.[1]

12- RM(RSu). method

we will apply the suggested method (on inner dimension u) with Romberg's acceleration and on outer dimension v by mid-point rule with Romberg's acceleration.[1]

After reviewing the roads, we will clarify one calculation of the methods, and let the rest of the roads be calculated in the same mode

To find the approximate value eq (5) we write:

$$J = \int_{\tau}^z Q(v)dv \quad \dots (6)$$

Where : $Q(v) = \int_s^r T(u,v)du \quad \dots(7)$

The approximate value of integration (6) (on the outer dimension v), for example the base point rule is

$$J = \kappa \sum_{\alpha=1}^m Q(v_{\alpha}) + E(\kappa) \quad \dots(8)$$

$$\beta = 1, 2, 3, \dots, m, v_{\alpha} = e + \frac{2\alpha - 1}{2}$$

$E(k)$ is the correction boundary Romberg's acceleration is applied to the outer dimension v) To calculate values $Q(v_{\alpha})$ roughly,

values v_{Γ} are compensated in (7).

$$Q(v_{\alpha}) = \int_s^r T(u, v_{\alpha})du \quad \dots(9)$$

When the base Simpson rule is also applied to the integration (9) (internal dimension u), the formula becomes the form

$$Q(v_{\alpha}) = \frac{\bar{\kappa}}{3} [T(s, v_{\alpha}) + 2 \sum_{\beta=1}^{(\frac{n}{2}-1)} Q(u_{2\beta}, v_{\alpha}) + 4 \sum_{\beta=1}^{\frac{n}{2}} Q(u_{2\beta-1}, v_{\alpha}) + Q(t, v_{\alpha})] + E(\bar{\kappa}) \quad (10)$$

$$\lambda = 1, 2, \dots, n, u_{(2\lambda-1)} = s + (2\lambda - 1)\kappa, \lambda = 1, 2, \dots, n - 1, u_{2\lambda} = s + 2\lambda\kappa$$

$E(\bar{\kappa})$ The correction limits are applied if the Romberg's acceleration is applied to the u internal dimension.

You will improve values $Q(v_{\alpha})$ by Romberg's acceleration or Aitken acceleration

The resulting values are only approximate values for integration (7) after applying Romberg's acceleration or Aitken acceleration it. After each compensation in formula (8) we can apply the Romberg's acceleration or Aitken acceleration on these values after finding a formula in the case of the application of Romberg's acceleration, Integration (6) in a way that Romberg's acceleration or Aitken acceleration, it with the base of the center point and this leads to knowing the value of integration (5) roughly

The formula of the error in the case of the application of Romberg's acceleration on the internal dimension

As for partial periods when applying (Trapezoidal rule ,mid-point , suggested method)all previous methods, the following values are used:

$$m = 1, 2, 4, 8, \dots, n = 1, 2, 4, 8, \dots$$

It is worth mentioning that the Simpson rule uses even numbers so do not start with one.

And to find the value of integration (6), Requires finding $Q(v_{\alpha})$ by use Romberg's acceleration , Aitken acceleration , (according to the method used) , with the mid-point rule on integration (9). After placing $m = 1$ In the formula (8) It must be calculated $Q(v_1)$ Of the formula (6) When $n = 1$ When using Simpson rule on internal integration then $n = 2$ thus the value is obtained where the absolute error is the EPS (on the inner dimension).

It was assumed that this value was obtained when $n = 16$ In this case the approximate value of the integration is shown in the tables as the resulting value when $n = 16, m = 1$, If they are $m = 1$ It must be calculated $Q(v_1)$ and $Q(v_2)$ With the formula (6), By applying the formula (8) When $n = 1$ then $n = 2$ and so on. Assuming that the error in value Less than or equal to EPS When $n = 32$ The absolute error in value $Q(v_2)$ Is less than or equal to EPS When $n = 64$, Then the installation in the table is the approximate value of the integration defined by the formula (5) As the result obtained when $n = 64, m = 2$, Install the largest value for n , And so when they are $m > 2$. [4]

Remark (1) :For many method we will work on a one table that includes all the methods will take the last value of the method and value after the application of the Romberg acceleration in the same partial period

3-EXAMPLE :

$$1- J = \int_0^1 \int_0^1 y e^{2.5x+0.6y} dx dy$$

Its analytical value is 3.36906774253669, Correct for fourteen decimal places, from the table(1), Note that the methods. Note that the methods (RM (RM), RS (RS), RT (RSu), RS (RSu)), But the method (RT (RSu), RS (RSu)) was because the number of partial intervals is less ($n=m=32$), And a method (RM (RSu)) that was correct for thirteen decimal places.

Table (1)

m	n	mothed	value
32	64	RM (RM)	3.36906774253669
16	64	RM (AM)	3.36906774107739
32	32	AM (RM)	3.3690677427688
64	32	ATAT	3.36906774101761
512	64	AM(AM)	3.36906773129494
64	32	RS (RS)	3.36906774618069
16	32	RS(AS)	3.36906774618069
16	16	AS (RS)	3.36906774185700
32	32	AS (AS)	3.36906772992585
32	32	RT (RSu)	3.36906774253669
32	32	RS (RSu)	3.36906774253669
16	32	RM (RSu)	3.36906774253669

$$2- I = \int_2^3 \int_1^2 \sqrt{u} \ln(v+u) dv du$$

Its analytical value 2.18658641209591 rounded to fourteen decimal places, from the table(2) , Note that the methods. Note that the methods (RM (RM), RS (RS), RM (RSu), RT (RSu), RS (RSu)), Gave the same analytical value ,But the method (RS (RSu)) was because the number of partial.(n-m=16) This is considered to be the best Of the rest of the methods.

Table (2)

m	n	mothed	value
16	32	RM (RM)	3.18658641209591
16	16	RM (AM)	2.18658641211953
16	16	AM (RM)	2.18658641176431
16	16	AM(AM)	2.18658641068667
64	16	ATAT	2.18658641491270
32	32	RS (RS)	2.18658641209591
8	16	RS(AS)	2.18658641235447
8	16	AS (RS)	2.18658641135228
8	16	AS (AS)	2.18658641156033
32	16	RT (RSu)	2.18658641209591
16	16	RS (RSu)	2.18658641209591
32	16	Rm (RSu)	2.18658641209591

4- Conclusion :

All numerical methods use the same pattern in the calculation. For example, we conclude that the best method to calculate double integrations in terms of precision and partial intervals is the method (RS(RSu), RT (RSu), RM (RM)).

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