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A Method TTT for Calculating Triple Integrals of Numerically When the Integrand Is Continuous

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***Abstract**: The main objective of this research is to derive a base for the calculation of numerical values of triple integrals with continuous coefficients using the three-dimensional deviant base and how to find the correction limits (error formula) and to improve these results using the Romperk acceleration method [1], [4] Which we found, when the number of partial periods divided by the integration period on the internal dimension X is equal to the number of partial periods divided by the integration period on the middle dimension Y and equal to the number of partial periods divided by the integration period on the outer dimension Z.*

Keywords: Numerical integration ; triple integrals ; Romberg's acceleration ; continuous integrand

1.Introduction

Numerical analysis is characterized by the creation of diverse methods for finding approximate solutions to certain mathematical issues in an effective manner. The efficiency of these methods depends on both the accuracy and the ease with which they can be implemented. The numerical analysis is the numerical interface of the wide field of applied analysis. Tripartite integrations are of great importance in finding sizes and middle positions and the determination of the inertia of the volumes and finding the blocks with variable density, for example the size of the inside and above and below and the calculation of the average position of the size of the impact in and above the level and below the level, Such as a piece of thin wire or a thin sheet of metal. Frank Ayers [8], prompting a number of researchers to work in the field of triple integrals. Which led many researchers to work in the field of tripartite integrations, including Hans Jarr and Jacobsen [4] in 1973, Frank Ayers [3] in 1975, Muhammad [6] in 1984, Akkar [1] in 2010, Hassan[5]in2015 and Fadel [2] in2015.

In this paper we present a theorem with proof to derive a new base for calculating approximate values of triple integrations with constant inversions and their error formula. This rule is the result of the application of the Rumbark acceleration method to the values resulting from the use of the two point bases on the internal x and outer dimensions and the trapezoid base on the middle dimension (The number of partial periods divided by the internal dimension period, the number of partial periods divided by the middle dimension period and the number of partial periods divided by the external dimension period and we will mark this method with the symbol where the method of accelerating Rumbark and Several derivative we have got good results in terms of accuracy and speed of approaching the number of partial periods of relatively few and very short time.

1-1 The numerical TTT base and its correction limits:

Theorem:

Let the function $f(x,y,z)$ be continuous and derivable at each point of the region $[a,b]\times[c,d],[e,g]$, the approximate value of the triple integral $I=\int_e^g\int_c^d\int_a^b f(x,y,z)dxdydz$

of the triple integral Can be calculated from the following rule:

$$TTT=\frac{h^3}{8}[f(a,c,e)+f(a,c,g)+f(a,d,e)f(a,d,g)+f(b,c,e)$$
$$+f(b,c,g)+f(b,d,e)+f(b,d,g)+2\sum_{k=1}^{n-1}(f(a,c,z_k)+f(a,d,z_k)+$$
$$f(b,c,z_k)+f(b,d,z_k))+2\sum_{j=1}^{n-1}(f(a,y_j,e)+f(a,y_j,g)+f(b,y_j,e)$$
$$+f(b,y_j,g))+2\sum_{i=1}^{n-1}(f(x_i,c,e)+f(x_i,c,g)+f(x_i,d,e)+f(x_i,d,g)+$$
$$2\sum_{k=1}^{n-1}(f(x_i,c,z_k)+f(x_i,c,z_k)+f(x_i,d,z_k))+4\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}(f(x_i,y_j,e)+f(x_i,y_j,g)+2\sum_{k=1}^{n-1}f(x_i,y_j,z_k))]+$$

The formula of error is:

$$E_{TTT}(h) = I - TTT(h) = A_{TTT}h^2 + B_{TTT}h^4 + C_{TTT}h^6 + \cdots$$

Since $A_{TTT}, B_{TTT}, C_{TTT}, \ldots$ the constants depend on partial derivatives of the function f only and do not rely on h.

Proof:

Let's I say that integration is defined by $I = \int_e^g \int_c^d \int_a^b f(x,y,z)\, dxdydz$

It can be written in the format

$$I = \int_e^g \int_c^d \int_a^b f(x,y,z)\, dxdydz = TTT(h) + E_{TTT}(h) \qquad \cdots(3.1)$$

Where define at each point of the integration area $[a,b]\times[c,d]\times[e,g]$

whereas TTT Represents an approximate value for integration using the rule, $E_{TTT}(h)$ It is a series of correction terms that can be added to values $TTT(h)$. We will divide $[a,b]$ the period of integration on the internal dimension of the number (n) of partial periods and divide $[c,d]$ the period of integration on the middle dimension of the number (m) of partial periods and divide $[e,g]$ the period of integration on the external dimension of the number (n_1) of partial periods where $h = \frac{b-a}{n}$, $\bar{h} = \frac{d-c}{m}$

$\bar{\bar{h}} = \frac{g-e}{n_1}$ We will take $h = \bar{h} = \bar{\bar{h}}$ So we can use the Rompark acceleration

And that the limits of the correction chain to the base trapezoidal be: -

$$E_T(h) = -\frac{1}{12}h^2(f_n' - f_0') + \frac{1}{720}h^4(f_n^{(3)} - f_0^{(3)}) - \frac{1}{30240}h^6(f_n^{(5)} - f_0^{(5)}) + \cdots \qquad \ldots(1)$$

Using the mean value theorem in the differential of formula (1)

$$E_T(h) = \frac{-(x_n - x_0)}{12}h^2 f^{(2)}(\zeta_1) + \frac{(x_n - x_0)}{720}h^4 f^{(4)}(\zeta_2) + \ldots \qquad \ldots(2)$$

Where $i = 1,2,\ldots \;, \zeta_i \in (x_0, x_n)$

Frank Ayers [3]

By applying the trapezoidal rule to the inner dimension we obtain,

$$\int_a^b f(x,y,z)dx = \frac{h}{2}[f(a,y,z) + f(b,y,z) + 2\sum_{i=1}^{n-1} f(x_i,y,z)] +$$
$$\frac{b-a}{-12}h^2 \frac{\partial^2 f(\mu_1,y,z)}{\partial x^2} + \frac{b-a}{720}h^4 \frac{\partial^4 f(\mu_2,y,z)}{\partial x^4} + \cdots]$$

Whereas $x_i = a + ih, (i = 1,2,3\cdots,n-1)$ & $h = \frac{b-a}{n}$ &

$$y_j = c + jh, j = 1,2,3,\cdots$$

$$z_k = e + kh, k = 1,2,3,\cdots$$

Applying a base trapezoidal rule to the middle dimension of both ends of the formula and compensating the end of the period using the formula of error (1)and the theory of the mean value of differentiation(2) we get:

$$TT = \frac{h^2}{4}[f(a,c,z) + f(a,d,z) + f(b,d,z) + f(b,c,z) + 2\sum_{j=1}^{n-1}(f(a,y_j,z) + f(b,y_j,z)) +$$

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$$2\sum_{i=1}^{n-1}(f(x_i,c,z)+f(x_i,d,z)+2\sum_{j=1}^{n-1}f(x_i,y_j,z))]+\int_c^d[\frac{b-a}{-12}h^2\frac{\partial^2 f(\mu_1,y,z)}{\partial x^2}+\frac{b-a}{720}h^4\frac{\partial^4 f(\mu_2,y,z)}{\partial x^4}+\cdots]dy+$$

$$\frac{h}{2}[\frac{d-c}{-12}h^2\frac{\partial^2 f(a,\lambda_{11},z)}{\partial y^2}+\frac{d-c}{720}h^4\frac{\partial^4 f(a,\lambda_{12},z)}{\partial y^4}+\cdots+\frac{d-c}{-12}h^2\frac{\partial^2 f(b,\lambda_{21},z)}{\partial y^2}+\frac{d-c}{720}h^4\frac{\partial^4 f(b,\lambda_{22},z)}{\partial y^4}+\cdots$$

$$h\sum_{i=1}^{n-1}(\frac{d-c}{-12}h^2\frac{\partial^2 f(x_i,\lambda_{2i},z)}{\partial y^2}+\frac{d-c}{720}h^4\frac{\partial^4 f(x_i,\lambda_{2i+1},z)}{\partial y^4}+\cdots)]$$

By applying the trapezoidal rule to the outer dimension on both ends of the formula above we obtain:

$$1)\int_e^g\frac{h^2}{4}f(a,c,z)dz=\frac{h^3}{8}[f(a,c,e)+f(a,c,g)+2\sum_{k=1}^{n-1}f(a,c,z_k)]+\frac{h^2}{4}[\frac{g-e}{-12}h^4\frac{\partial^2 f(a,c,\beta_{11})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(a,c,\beta_{12})}{\partial z^4}+\cdots]$$

$$2)\int_e^g\frac{h^2}{4}f(a,d,z)dz=\frac{h^3}{8}[f(a,d,e)+f(a,d,g)+2\sum_{k=1}^{n-1}f(a,d,z_k)]+\frac{h^2}{4}[\frac{g-e}{-12}h^4\frac{\partial^2 f(a,d,\beta_{21})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(a,d,\beta_{22})}{\partial z^4}+\cdots]$$

$$3)\int_e^g\frac{h^2}{4}f(b,c,z)dz=\frac{h^3}{8}[f(b,c,e)+f(b,c,g)+2\sum_{k=1}^{n-1}f(b,c,z_k)]+\frac{h^2}{4}[\frac{g-e}{-12}h^4\frac{\partial^2 f(b,c,\beta_{31})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(b,c,\beta_{32})}{\partial z^4}+\cdots]$$

$$4)\int_e^g\frac{h^2}{4}f(b,d,z)dz=\frac{h^3}{8}[f(b,d,e)+f(b,d,g)+2\sum_{k=1}^{n-1}f(b,d,z_k)]+\frac{h^2}{4}[\frac{g-e}{-12}h^2\frac{\partial^2 f(a,d,\beta_{41})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(a,d,\beta_{42})}{\partial z^4}+\cdots]$$

$$5)\int_e^g\frac{h^2}{4}2\sum_{j=1}^{n-1}f(a,y_j,z)dz=\frac{h^3}{4}\sum_{j=1}^{n-1}[f(a,y_j,e)+f(a,y_j,g)+2\sum_{k=1}^{n-1}f(a,y_j,z_k)]+\frac{h^2}{2}\sum_{j=1}^{n-1}[\frac{g-e}{-12}h^2\frac{\partial^2 f(a,d,\beta_{51})}{\partial z^2}$$

$$+\frac{g-e}{720}h^4\frac{\partial^4 f(a,y_j,\beta_{52})}{\partial z^4}+\cdots]$$

$$6)\int_e^g\frac{h^2}{4}2\sum_{j=1}^{n-1}f(b,y_j,z)dz=\frac{h^3}{4}\sum_{j=1}^{n-1}[f(b,y_j,e)+f(b,y_j,g)+2\sum_{k=1}^{n-1}f(b,y_j,z_k)]+\frac{h^2}{2}\sum_{j=1}^{n-1}[\frac{g-e}{-12}h^2\frac{\partial^2 f(b,d,\beta_{61})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(b,y_j,\beta_{62})}{\partial z^4}+\cdots]$$

$$7)\int_e^g\frac{h^2}{4}2\sum_{j=1}^{n-1}f(x_i,c,z)dz=\frac{h^3}{4}\sum_{i=1}^{n-1}[f(x_i,c,e)+f(x_i,c,g)+2\sum_{k=1}^{n-1}f(x_i,c,z_k)]+\frac{h^2}{2}\sum_{i=1}^{n-1}[\frac{g-e}{-12}h^2\frac{\partial^2 f(x_i,c,\beta_{71})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(x_i,c,\beta_{72})}{\partial z^4}+\cdots]$$

$$8)\int_e^g \frac{h^2}{4} 2\sum_{i=1}^{n-1} f(x_i,d,z)dz = \frac{h^3}{4}\sum_{i=1}^{n-1}[f(x_i,d,e)+f(x_i,d,g)+2\sum_{k=1}^{n-1} f(x_i,d,z_k)] + \frac{h^2}{2}\sum_{i=1}^{n-1}[\frac{g-e}{-12}h^2\frac{\partial^2 f(x_i,d,\beta_{81})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(x_i,d,\beta_{82})}{\partial z^4}+\cdots]$$

$$9)\int_e^g \frac{h^2}{4} 2\sum_{i=1}^{n-1} 2\sum_{j=1}^{n-1} f(x_i,y_j,z)dz = \frac{h^3}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}[f(x_i,y_j,e)+f(x_i,y_j,g)+2\sum_{k=1}^{n-1} f(x_i,y_j,z_k)] + h^2\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}[\frac{g-e}{-12}h^2\frac{\partial^2 f(x_i,y_j,\beta_{91})}{\partial z^2}+$$

$$\frac{g-e}{720}h^4\frac{\partial^4 f(x_i,y_j,\beta_{92})}{\partial z^4}+\cdots]$$

By combining the formulas above we get the:

$$TTT = \frac{h^3}{8}[f(a,c,e)+f(a,c,g)+f(a,d,e)f(a,d,g)+f(b,c,e)$$

$$+f(b,c,g)+f(b,d,e)+f(b,d,g)+2\sum_{k=1}^{n-1}(f(a,c,z_k)+f(a,d,z_k)+f(b,c,z_k)+f(b,d,z_k))+$$

$$+2\sum_{j=1}^{n-1}(f(a,y_j,e)+f(a,y_j,g)+f(b,y_j,e)+f(b,y_j,g)+2\sum_{i=1}^{n-1}(f(x_i,c,e)+f(x_i,c,g)+f(x_i,d,e)+f(x_i,d,g)+$$

$$2\sum_{k=1}^{n-1}(f(x_i,c,z_k)+f(x_i,c,z_k)+f(x_i,d,z_k))+4\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}(f(x_i,y_j,e)+f(x_i,y_j,g)+2\sum_{k=1}^{n-1} f(x_i,y_j,z_k))]+$$

$$h^2[[\frac{(g-e)(d-c)(b-a)}{-12}\frac{\partial^2 f(\mu_1,\alpha_1,\theta_1)}{\partial x^2}+[\frac{(g-e)(d-c)}{-12}\frac{h}{2}(\frac{\partial^2 f(a,\lambda_{11},\theta_{21})}{\partial y^2}+\frac{\partial^2 f(b,\lambda_{21},\theta_{31})}{\partial y^2}+$$

$$+h\sum_{i=1}^{n-1}((\frac{\partial^2 f(x_i,\lambda_{2i},\theta_{41})}{\partial y^2}+(g-e)[\frac{h^2}{4}\frac{\partial^2 f(a,d,\beta_{21})}{\partial z^2}+$$ where

$$\frac{\partial^2 f(b,c,\beta_{31})}{\partial z^2}+\frac{\partial^2 f(b,d,\beta_{41})}{\partial z^2}+2\sum_{j-1}^{n-1}(\frac{\partial^2 f(a,y_j,\beta_{51})}{\partial z^2}+\frac{\partial^2 f(b,y_j,\beta_{61})}{\partial z^2})+2\sum_{i=1}^{n-1}(\frac{\partial^2 f(x_i,c,\beta_{81})}{\partial z^2}+\frac{\partial^2 f(x_i,d,\beta_{91})}{\partial z^2}+$$

$$4\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}(\frac{\partial^2 f(x_i,y_j,\beta_{91})}{\partial z^2})]]+h^4[\cdots$$

$\mu_1,\cdots,\alpha_1,\cdots,\theta_1,\cdots,\lambda_{11},\theta_{21},\cdots,\beta_{21},\cdots$ is aconstant.

2.Example:

	integration	ANALYTICAL VALUE FOR INTEGRATIONS
1	$\int_1^2\int_1^2\int_1^2 \log(x+y+z)dxdydz$	1.497802288575 the value is rounded to twelve decimal places
2	$\int_2^3\int_1^2\int_0^1 xe^{-(x+y+z)}dxdydz$	0.0052567434550 the value is rounded to thirteen decimal places

3	$\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z))dxdydz$	0.5160245509312 the value is rounded to thirteen decimal places
4	$\int_1^2\int_1^2\int_1^2 1/\sqrt{x^4+y^4+z^4}dxdydz$	Unknown analytical value

1-- Integration function $\int_1^2\int_1^2\int_1^2 \log(x+y+z)dxdydz$ Continuous and derivable for each $(x,y,z)\in[1,2]\times[1,2]\times[1,2]$ So the error formulas for the integration are mentioned when applying the TTT

$$E_{TTT}(h) = I - TTT(h) = A_{TTT}h^2 + B_{TTT}h^4 + C_{TTT}h^6 + \cdots$$

We conclude from Table (1) where $n=16$ the value of integration using the rule is correct For three decimal places and when the use of Rumbark's acceleration with the base to be mentioned is correct twelve decimal places and is also identical to the real value rounded to twelve decimal places.

2-Integration function $\int_2^3\int_1^2\int_0^1 xe^{-(x+y+z)}dxdydz$ Continuous and derivable for each $(x,y,z)\in[0,1]\times[1,2]\times[2,3]$ So the error formulas for the integration are mentioned when applying the TTT

$$E_{TTT}(h) = I - TTT(h) = A_{TTT}h^2 + B_{TTT}h^4 + C_{TTT}h^6 + \cdots$$

We conclude from Table (2) that $n=16$ the value of integration using the base is valid for five decimal places and when the use of Rumbark's acceleration with the above rule is correct for thirteen decimal places (partial period 2^{12}) and is identical to the real value rounded to thirteen decimal places. Akkar [1] obtained the same result when using the RMMM method when $m=n_1=n_2=16$.

3- Integration function $\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z))dxdydz$ Continuous and derivable $(x,y,z)\in[0,1]\times[0,1]\times[0,1]$ So the error formulas for the integration are mentioned when applying the TTT

$$E_{TTT}(h) = I - TTT(h) = A_{TTT}h^2 + B_{TTT}h^4 + C_{TTT}h^6 + \cdots$$

We conclude from Table (3) that $n=32$ the value of integration using the base is valid for two decimal places and when using Rumbark's acceleration with the base, the value is identical to the real value rounded to thirteen decimal places(partial period 2^{15}).

4- Integration function $\int_1^2\int_1^2\int_1^2 1/\sqrt{x^4+y^4+z^4}dxdydz$ Continuous and derivable $(x,y,z)\in[1,2]\times[1,2]\times[1,2]$ So the error formulas for the integration are mentioned when applying the TTT

$$E_{TTT}(h) = I - TTT(h) = A_{TTT}h^2 + B_{TTT}h^4 + C_{TTT}h^6 + \cdots$$

Note that the integration is unknown analytical value.

Note from Table (4) when $n=128$ and k=6,8,10,12,14 the value is constant (for fifteen decimal places) horizontally.

So we can say that the value is correct for at least fifteen decimal places when applying the method $RTTT$,we also note when $n=64$ and k = 8,10,12 that the value is constant (for fourteen decimal places) horizontally. (This means that the convergence is correctly towards the analytical value). From this we conclude that when using the above methods with the acceleration of Rumbark we obtained a fixed value of thirteen decimal places by taking the common value in all the tables so it can be said that the value of this integration is 0.2488674852276 close to thirteen Decimal order.

n	TTT	K=2	K=4	K=6	K=8
1	1.484696072320				
2	1.494577658768	1.497871520917			
4	1.496999384782	1.497806626786	1.497802300511		
8	1.497601766058	1.497802559817	1.497802288685	1.497802288498	
16	1.497752170661	1.497802305529	1.497802288577	1.497802288575	1.497802288575
table 1 $\int_1^2\int_1^2\int_1^2 \log(x+y+z)dxdydz$				the real value	1.497802288575

n	TTT	K=2	K=4	K=6	K=8
1	0.0042837856329				
2	0.0050493378533	0.0053045219267			
4	0.0052071387021	0.0052597389851	0.0052567534556		
8	0.0052444827529	0.0052569307699	0.0052567435555	0.0052567433984	
16	0.0052536870608	0.0052567551635	0.0052567434564	0.0052567434548	0.0052567434550
table 2 $\int_2^3\int_1^2\int_0^1 xe^{-(x+y+z)}dxdydz$					the real value 0.0052567434550

	TTT	K=2	K=4	K=6	K=8	K=10
1	0.2500000000000					
2	0.4397208691208	0.5029611588277				
4	0.4963347424286	0.5152060335312	0.5160223585115			
8	0.5110637426113	0.5159734093389	0.5160245677261	0.5160246027930		
16	0.5147819518942	0.5160213549885	0.5160245513651	0.5160245511055	0.5160245509028	
32	0.5157137513674	0.5160243511918	0.5160245509386	0.5160245509319	0.5160245509312	0.5160245509312
$\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z))dxdydz$ table 3					the real value	**0.5160245509312**

n	TTT	K=2	K=4	K=6	K=8	K=10	K=12	K=14
1	0.243878448192584							
2	0.246053305771514	0.246778258297824						
4	0.248109679261973	0.248795137092126	0.248929595678412					
8	0.248675209744218	0.248863719904967	0.248868292092489	0.248867319019697				
16	0.248819247474090	0.248867260050714	0.248867496060431	0.248867483425001	0.248867484069728			
32	0.248855415349103	0.248867471307440	0.248867485391222	0.248867485221869	0.248867485228916	0.248867485230049		
64	0.248864467107269	0.248867484359991	0.248867485230161	0.248867485227604	0.248867485227627	0.248867485227626	0.248867485227625	
128	0.248866730656891	0.248867485173431	0.248867485227661	0.248867485227621	0.248867485227621	0.248867485227621	0.248867485227621	0.248867485227621

table 4 $\int_1^2\int_1^2\int_1^2 1/\sqrt{x^4+y^4+z^4}\,dxdydz$

3.Discussion:

It is clear from the results of this research table that when calculating the approximate values of triple integrations with continuous inversions on the three dimensions, and when the number of partial periods divided by the period on the internal dimension is equal to the number of partial periods on the middle and equal to the number of partial periods on the external dimension, The external dimension has given this (TTT) rule true values (for several decimal places) compared to the real values of integrations and using a number of partial periods without using Rumbark's acceleration, for example in the first integration, Feather mattresses when decimal n=16, the second integration the value is valid for five decimal places when n=16, In the third integration, the value was correct for two decimal places when n=32, In the fourth integration, the value was valid for five decimal places when n = 128.
However, when using the Rumbark acceleration method with the above rule, it gave better results in terms of the velocity of approaching a few of the partial intervals compared with the values of the analytic integrations, which corresponded to the analytical value in the first integration (12 after interval) when n = 16 and in the second integral) Where n = 16 and in the third integration when n = 32 and in the fourth integration the result was accurate for fifteen orders after the interval and thus the RTTT method can be used to calculate the binary integrations with continuous calls

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