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Solving Linear Systems of First Order of Ordinary Differential Equations Using AL-Tememe Transform

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Abstract: *In these paper, we presented the solution of first-order differential equations for n equations. This method shortened a major step in solving this type using matrices, as we proved some hypotheses that we need to solve this type because this method gives very quick results in finding solutions.*

Keywords: AL-Tememe Transform, variable coefficients, partial fraction,

1. Introduction:

We will use Al-Tememe Transform ($\mathcal{T}$.T) to solve systems of n-linear first order system ordinary differential equations with variable coefficients. And the method summarized by taking ($\mathcal{T}$.T) to both sides of the equations then we take ($\mathcal{T}^{-1}$. T) to both sides of the equations and by using partial fraction decomposition we find the values of values constants.

Definition 2: [2]

Al-Tememe transformation for the function $f(x);\ x > 1$ is defined by the following integral:

$$\mathcal{T}\,[f(x)] = \int_1^\infty x^{-p} f(x)\,dx = F(p)$$

such that this integral is convergent , p is positive constant

ID	Function,**f**(**x**)	$F(p) = \int_1^\infty x^{-p} f(x)\,dx = \mathcal{T}\,[f(x)]$	Regional of convergence
1	k ; k =constant	$\frac{k}{p-1}$	$\mathbf{p > 1}$
2	$x^n, n \in R$	$\frac{1}{p-(n+1)}$	$\mathbf{p > n+1}$
3	lnx	$\frac{1}{(p-1)^2}$	$\mathbf{p > 1}$
4	$x^n lnx, n \in R$	$\frac{1}{[p-(n+1)]^2}$	$\mathbf{p > n+1}$
5	$sin(alnx)$	$\frac{a}{(p-1)^2 + a^2}$	$\mathbf{p > 1}$ a =constant
6	$cos(alnx)$	$\frac{p-1}{(p-1)^2 + a^2}$	$\mathbf{p > 1}$ a =constant
7	$sinh(alnx)$		$\lvert\mathbf{p-1}\rvert > \boldsymbol{a}$

		$\frac{a}{(p-1)^2-a^2}$	a =constant
8	$cosh(alnx)$	$\frac{p-1}{(p-1)^2-a^2}$	$\lvert p-1\rvert > \boldsymbol{a}$ a =constant

are given in table(1) **[1]**

Table(1)

From the Al-Tememe definition and the above table, we get:

Theorem 1:

If $\mathcal{T}\, f(x) = F(p)$ and a is constant, then $\mathcal{T}\, f\,(x^{-a}) = F(p+a)$.see **[2]**

Definition 3: [2]

Let $f(x)$ be a function where $(x > 1)$ and $\mathcal{T}\, f(x) = F(p)$, $f(x)$ is said to be an inverse for the Al-Tememe transformation and written as

$\mathcal{T}^{-1}\, F(p) = f(x)$, where $\mathcal{T}^{-1}$ returns the transformation to the original function.

Property 2: [2]

If $\mathcal{T}^{-1}\, F_1(p) = f_1(x)$, $\mathcal{T}^{-1}\, F_2(p) = f_2(x)$,…, $\mathcal{T}^{-1}\, F_n(p) = f_n(x)$ and $a_1, a_2, \ldots a_n$ are constants, then

$$\mathcal{T}^{-1}[a_1F_1(p) + a_2F_2(p) + \cdots + a_nF_n(p)] = a_1f_1(x) + a_2f_2(x) + \cdots + a_n\, f_n(x)$$

Theorem 2: [2]

If the function $f(x)$ is defined for $x > 1$ and its derivatives $f^{(1)}(x), f^{(2)}(x), \ldots, f^{(n)}(x)$ are exist then:

$$\mathcal{T}\big[x^n f^{(n)}(x)\big] = -f^{(n-1)}(1) - (p-n)f^{(n-2)}(1) - \cdots$$

$$-(p-n)\,(p-(n-1))\,\ldots(p-2)\,f(1) + (p-n)!\,F(p)$$

Definition 4: [3]

A function $f(x)$ is piecewise continuous on an interval $[a,b]$ if the interval can be partitioned by a finite number of points

$a = x_0 < x_1 < \cdots < x_n = b$ such that:

1. $f(x)$is continuous on each subinterval (x_i, x_{i+1}), for
$i = 0, 1, 2, \ldots, n-1$

2. The function f has jump discontinuity at x_i , thus

$$\left|\lim_{x\to x_i^+} f(x)\right| < \infty \;, i = 0, 1, 2, \ldots, n-1;$$

$$\left|\lim_{x\to x_i^-} f(x)\right| < \infty \;, i = 0, 1, 2, \ldots, n$$

Definition (5): [4]

The equation

$$a_0x^n\frac{d^ny}{dx^n} + a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}x\frac{dy}{dx} + a_ny = f(x)$$

where $a_0, a_2, \ldots, a_n$ are constants and $f(x)$ is a function of x , is called ***Euler's Equation.***

2.**A Procedure For Solving Linear Systems Of First Order Of Ordinary Differential Equations Using AL-Tememe Transform.**

Let $f_1(x), f_2(x), \ldots, f_n(x)$ be members of functions of Ω .where Ω is the class of all piecewise continuous functions with exponential order at infinity. Consider the vector-valued function .

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ . \\ . \\ . \\ f_n(x) \end{bmatrix} \qquad \dots(1)$$

AL-Tememe Transformation of $f(x)$ is

$$\mathcal{T}[f(x)] = \int_1^\infty x^{-p} f(x)\, dx$$

$$= \begin{bmatrix} \int_1^\infty x^{-p} f_1(x)\, dx \\ \int_1^\infty x^{-p} f_2(x) dx \\ . \\ . \\ . \\ \int_1^\infty x^{-p} f_n(x)\, dx \end{bmatrix} = \begin{bmatrix} \mathcal{T}[f_1(x)] \\ \mathcal{T}[f_2(x)] \\ . \\ . \\ . \\ \mathcal{T}[f_n(x)] \end{bmatrix} = \begin{bmatrix} F_1(p) \\ F_2(p) \\ . \\ . \\ . \\ F_n(p) \end{bmatrix} \qquad \dots(2)$$

In a same way, we define Al-Tememe transform of an $m \times n$ matrix to be the $m \times n$ matrix consisting of Al-Tememe transform of the component functions. If Al-Tememe transform of each component exists then we say $F(p)$ is Al-Tememe transform.

Example(1): To find the $\mathcal{T}$.T for vector-valued function to:

$$f(x) = \begin{bmatrix} x^3 \\ (lnx)^2 \\ cosh(3lnx) \end{bmatrix}$$

We see , $\mathcal{T}[f(x)] = \begin{bmatrix} \mathcal{T}[x^3] \\ \mathcal{T}[(lnx)^2] \\ \mathcal{T}[cosh(3lnx)] \end{bmatrix} = \begin{bmatrix} \frac{1}{p-4} \\ \frac{1}{(p-1)^3} \\ \frac{(p-1)}{(p-1)^2-9} \end{bmatrix}$

The inverse of Al-Tememe transform ($\mathcal{T}^{-1}$. T) to both sides of eq. (2) is:

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ . \\ . \\ . \\ f_n(x) \end{bmatrix} = \begin{bmatrix} \mathcal{T}^{-1}[F_1(p)] \\ \mathcal{T}^{-1}[F_1(p)] \\ . \\ . \\ . \\ \mathcal{T}^{-1}[F_1(p)] \end{bmatrix} = \mathcal{T}^{-1}[F(p)] \qquad \dots(3)$$

For example: $$\begin{bmatrix} \mathcal{T}^{-1}\left[\frac{1}{(p-3)^2}\right] \\ \mathcal{T}^{-1}\left[\frac{1}{(p-1)^2+25}\right] \\ \mathcal{T}^{-1}\left[\frac{1}{p+4}\right] \end{bmatrix} = \begin{bmatrix} x^2 lnx \\ \frac{1}{5} sin(5lnx) \\ x^{-5} \end{bmatrix}$$

Theorem 3: If A is constant $m \times m$ matrix and B an $m \times r$ matrix-valued function then :

$$\mathcal{T}[A\, B(x)] = A\, \mathcal{T}[B(x)]$$

Proof:

$$Let\ A = (a_{ij}) \quad and \quad B(x) = b_{ij}(x)$$

Then $\quad A\,B(x) = \sum_{k=1}^{m} a_{ik} b_{kr}$

Hence $\quad \mathcal{T}[A\,B(x)] = \mathcal{T}[\sum_{k=1}^{m} a_{ik} b_{kr}]$

$$= \sum_{k=1}^{m} a_{ik}\, \mathcal{T}[b_{kr}]$$

$$= A\, \mathcal{T}[B(x)]$$

Theorem4:

(a) Suppose that $y(x)$ is continuous for $x > 1$ and let the elements

of derivative vector xy' be member of Ω . Then

$$\mathcal{T}[xy'] = (p-1)\mathcal{T}(y) - y(1)$$

(b) Let xy' be continuous for $x > 1$ and let the entries x^2y'' be member of Ω , then :

$$\mathcal{T}[x^2y''] = (p-2)(p-1)\mathcal{T}(y) - (p-2)y(1) - y'(1)$$

(c) Let $x^{n-1}y^{(n-1)}$ be continuous for $x > 1$ and let the entries $x^n y^{(n)}$ be member of Ω , then :

$$\mathcal{T}[x^n y^n] = -y^{(n-1)}(1) - (p-n)y^{(n-2)}(1) - \cdots$$
$$-(p-n)\,(p-\ (n-1))\ \ldots(\,p-2)\,y(1)\ +(p-n)!\,F(p)$$

Proof:

(a) $$\mathcal{T}[xy'] = \begin{bmatrix} \mathcal{T}[xy_1'] \\ \mathcal{T}[xy_2'] \\ . \\ . \\ . \\ \mathcal{T}[xy_n'] \end{bmatrix} = \begin{bmatrix} (p-1)\mathcal{T}(y_1) - y_1(1) \\ (p-1)\mathcal{T}(y_2) - y_2(1) \\ . \\ . \\ . \\ (p-1)\mathcal{T}(y_n) - y_n(1) \end{bmatrix}$$

$(p-1)\mathcal{T}(y) - y(1) =$

(b) $\mathcal{T}[x^2 y''] = \begin{bmatrix} \mathcal{T}[x^2 y_1''] \\ \mathcal{T}[x^2 y_2''] \\ \cdot \\ \cdot \\ \cdot \\ \mathcal{T}[x^2 y_n''] \end{bmatrix}$

$$= \begin{bmatrix} (p-2)(p-1)\mathcal{T}(y_1) - (p-2)y(1) - y_1'(1) \\ (p-2)(p-1)\mathcal{T}(y_2) - (p-2)y(1) - y_2'(1) \\ \cdot \\ \cdot \\ \cdot \\ (p-2)(p-1)\mathcal{T}(y_n) - (p-2)y(1) - y_n'(1) \end{bmatrix}$$

$(p-2)(p-1)\mathcal{T}(y) - (p-2)y(1) - y'(1) \qquad =$

(c) $\mathcal{T}[x^n y^n] = \begin{bmatrix} \mathcal{T}[x^n y_1^n] \\ \mathcal{T}[x^n y_2^n] \\ \cdot \\ \cdot \\ \cdot \\ \mathcal{T}[x^n y_n^n] \end{bmatrix}$

$$= \begin{bmatrix} -y_1^{(n-1)}(1) - (p-n)y_1^{(n-2)}(1) - \cdots \\ -(p-n)\,(p-\,(n-1))\,\ldots(\,p-2)\,y_1(1)\; + (p-n)!\,F_1(p) \\ -y_2^{(n-1)}(1) - (p-n)y_2^{(n-2)}(1) - \cdots \\ -(p-n)\,(p-\,(n-1))\,\ldots(\,p-2)\,y_2(1)\; + (p-n)!\,F_2(p) \\ \cdot \\ \cdot \\ \cdot \\ -y_n^{(n-1)}(1) - (p-n)y_n^{(n-2)}(1) - \cdots \\ -(p-n)\,(p-\,(n-1))\,\ldots(\,p-2)\,y_n(1)\; + (p-n)!\,F_n(p) \end{bmatrix}$$

$$= -y^{(n-1)}(1) - (p-n)y^{(n-2)}(1) - \cdots$$
$$-(p-n)\,(p-\,(n-1))\,\ldots(\,p-2)\,y(1)\; + (p-n)!\,F(p)$$

Now,

Suppose we have a first order system of differential equations that can be written in the form .

$$xy_1' = a_{11}y_1 + a_{12}y_2 + \cdots + a_{1n}y_n + f_1(x)$$
$$xy_2' = a_{21}y_1 + a_{22}y_2 + \cdots + a_{2n}y_n + f_2(x) \qquad \ldots \quad (4)$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$xy_n' = a_{n1}y_1 + a_{n2}y_2 + \cdots + a_{nn}y_n + f_n(x)$$

We can write the linear system (4) in matrix form as

$$\begin{bmatrix} xy_1{}' \\ xy_2{}' \\ \vdots \\ xy_n{}' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} + \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

or $\quad xy' = \beta y + f(x) \qquad \ldots (5)$

If the system is homogeneous its matrix form is then,

$$xy' = \beta y \quad , \qquad \ldots (6)$$

where $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, $\beta = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$, and $f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$

An initial condition of eq.(4) consists of finding a solution of eq.(5) that equal a given vector.

$k = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$ and $y(1) = \begin{bmatrix} y_1(1) \\ y_2(1) \\ \vdots \\ y_n(1) \end{bmatrix}$

Hence $y_1(1) = k_1 , y_2(1) = k_2 , \ldots , y_n(1) = k_n$ so $y(1) = k$

The above two theorem (3) and (4) can be used for solving the following initial condition valued .

$$xy' = \beta y + f(x) \quad , \quad y(1) = k \quad , \; x > 1$$

If we take T.$\mathcal{T}$ to above equation (4) and by using theorem (3) and (4) we can write :

$$(p-1)\mathcal{T}(y) - y(1) = \beta\, \mathcal{T}(y) + G(p)$$

$$[(p-1)I - \beta]\mathcal{T}(y) = G(p) + y(1)$$

Where I is the $n \times n$ identity matrix , put $Y = \mathcal{T}(y)$, $\mathcal{T}[f(x)] = G(p)$,

if $(p-1)$ is not an eigenvalue of β then the matrix $[(p-1)I - \beta]$ is invertible and in this cases we have :

$$Y = [(p-1)I - \beta]^{-1}\, [G(p) + y(1)]$$

$$Y = \left(\begin{bmatrix} \frac{q_1(p)}{h_1(p)} \\ \frac{q_2(p)}{h_2(p)} \\ \vdots \\ \frac{q_n(p)}{h_n(p)} \end{bmatrix} \right) \quad h_1(p) \neq 0, h_2(p) \neq 0 , \ldots , h_n(p) \neq 0 \qquad \ldots (7)$$

Hence,

$$y = \mathcal{T}^{-1}\left(\begin{bmatrix} \frac{q_1(p)}{h_1(p)} \\ \frac{q_2(p)}{h_2(p)} \\ \vdots \\ \frac{q_n(p)}{h_n(p)} \end{bmatrix}\right) \quad \ldots (8)$$

Then $$y = \begin{bmatrix} A_{11}Q_{11}(x) + A_{12}Q_{12}(x) + \cdots + A_{1n}Q_{1n}(x) \\ A_{21}Q_{21}(x) + A_{22}Q_{22}(x) + \cdots + A_{2n}Q_{2n}(x) \\ \vdots \quad\quad \vdots \quad\quad \vdots \\ A_{n1}Q_{n1}(x) + A_{n2}Q_{n2}(x) + \cdots + A_{nn}Q_{nn}(x) \end{bmatrix} \quad \ldots (9)$$

$$y = \begin{bmatrix} \sum_{i=1}^{n} A_{1i}Q_{1i}(x) \\ \sum_{i=1}^{n} A_{2i}Q_{2i}(x) \\ \vdots \\ \sum_{i=1}^{n} A_{ni}Q_{ni}(x) \end{bmatrix} \quad \ldots (10)$$

Where $Q_{1i}(x)$, $Q_{2i}(x)$, … , $Q_{ni}(x)$ are functions of x , and $A_{1i}, A_{2i}, \ldots, A_{ni}$ are constants , which are equal in number to the degree of $h_i(p)$, where $i = 1, 2, \ldots, n$ .To find the values of constants of $A_{1i}, A_{2i}, \ldots, A_{ni}$ we use the initial conditions $y(1)$ in system . But the conditions $y(1)$ are not enough to find out the above constants, thus we find $y'(1), y''(1), \ldots, y^{n-1}(1)$ by using system (4) we get nm equations which is formed linear system, this linear system can be solved to obtain the values of A_{ij} .

Example(2): To solve the following linear systems

$$xy_1' = 2y_1 + y_2 - 2y_3 + 1 \qquad y_1(1) = 0$$

$$xy_2' = -3y_3 - x \qquad y_2(1) = 0$$

$$xy_3' = -2y_2 + y_3 + x^{-2} \qquad y_3(1) = 0$$

We can write

$$xy' = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & -3 \\ 0 & -2 & 1 \end{bmatrix} y + \begin{bmatrix} 1 \\ -x \\ x^{-2} \end{bmatrix}$$

By theorem (3) and (4)

$$Y = [(p-1)I - A]^{-1} \begin{bmatrix} \frac{1}{p-1} \\ \frac{-1}{p-2} \\ \frac{1}{p+1} \end{bmatrix} \quad , \quad A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & -3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$Y=\left(\begin{bmatrix} p-1 & 0 & 0 \\ 0 & p-1 & 0 \\ 0 & 0 & p-1 \end{bmatrix}-\begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & -3 \\ 0 & -2 & 1 \end{bmatrix}\right)^{-1}\begin{bmatrix} \frac{1}{p-1} \\ \frac{-1}{p-2} \\ \frac{1}{p+1} \end{bmatrix}$$

$$Y=\left(\begin{bmatrix} p-3 & -1 & 2 \\ 0 & p-1 & 3 \\ 0 & 2 & p-2 \end{bmatrix}\right)^{-1}\begin{bmatrix} \frac{1}{p-1} \\ \frac{-1}{p-2} \\ \frac{1}{p+1} \end{bmatrix}$$

$$=\frac{1}{(p-3)\,(p-4)\,(p+1)}\begin{bmatrix} p^2-3p+4 & p+2 & -2p-1 \\ 0 & p^2-5p+6 & 9-3p \\ 0 & 6-2p & p^2-4p+3 \end{bmatrix}\begin{bmatrix} \frac{1}{p-1} \\ \frac{-1}{p-2} \\ \frac{1}{p+1} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{p^4-4p^3-3p^2+10p+8}{(p-1)(p-2)(p-3)\,(p-4)\,(p+1)^2} \\ \frac{-3p^3+p^2+14p-24}{(p-2)(p-3)\,(p-4)\,(p+1)^2} \\ \frac{p^3-4p^2+7p-12}{(p-2)(p-3)\,(p-4)\,(p+1)^2} \end{bmatrix}$$

$$Y_1=\frac{A_1}{p-1}+\frac{B_1}{p-2}+\frac{C_1}{p-3}+\frac{D_1}{p-4}+\frac{E_1}{p+1}+\frac{F_1}{(p+1)^2}$$

$$y_1=A_1+B_1x+C_1x^2+D_1x^3+E_1x^{-2}+F_1x^{-2}lnx$$

$$y_1(1)=0\ ,\ y_1{}'(1)=1\ ,\ y_1{}''(1)=-2\ ,\ y_1{}'''(1)=-3$$

$$y_1{}^{(4)}(1)=-15\ ,\ y_1{}^{(5)}(1)=96$$

$$A_1=\frac{-1}{2},B_1=-\frac{2}{3},C_1=\frac{35}{16},D_1=-\frac{24}{25},E_1=-\frac{73}{1200},F_1=\frac{1}{20}$$

$$y_1=\frac{-1}{2}-\frac{2}{3}x+\frac{35}{16}x^2-\frac{24}{25}x^3-\frac{73}{1200}x^{-2}+\frac{1}{20}x^{-2}lnx$$

$$Y_2=\frac{A_2}{p-2}+\frac{B_2}{p-3}+\frac{C_2}{p-4}+\frac{D_2}{p+1}+\frac{E_2}{(p+1)^2}$$

$y_2 = A_2x + B_2x^2 + C_2x^3 + D_2x^{-2} + E_2x^{-2}lnx$

$y_2(1) = 0$, $y_2'(1) = -1$, $y_2''(1) = -3$,

$y_2'''(1) = 6$, $y_2^{(4)}(1) = -54$

$A_2 = 0$, $B_2 = 0$, $C_2 = \frac{-8}{25}$, $D_2 = \frac{8}{25}$, $E_2 = \frac{3}{5}$

$$y_2 = \frac{-8}{25}x^3 + \frac{8}{25}x^{-2} + \frac{3}{5}x^{-2}lnx$$

$$Y_3 = \frac{A_3}{p-2} + \frac{B_3}{p-3} + \frac{C_3}{p-4} + \frac{D_3}{p+1} + \frac{E_3}{(p+1)^2}$$

$y_3 = A_3x + B_3x^2 + C_3x^3 + D_3x^{-2} + E_3x^{-2}lnx$

$y_3(1) = 0$, $y_3'(1) = 1$, $y_3''(1) = 0$, $y_3'''(1) = 12$,

$y_3^{(4)}(1) = -60$

$A_3 = \frac{-1}{3}$, $B_3 = 0$, $C_3 = \frac{8}{25}$, $D_3 = \frac{1}{75}$, $E_3 = \frac{2}{5}$

$$y_3 = \frac{-1}{3}x + \frac{8}{25}x^3 + \frac{1}{75}x^{-2} + \frac{2}{5}x^{-2}lnx$$

Example(3): To solve the following linear systems

$xy_1' = 3y_1 + 2$ $\qquad y_1(1) = 0$

$xy_2' = y_2 - 2y_3$ $\qquad y_2(1) = 0$

$xy_3' = 4y_1 - y_4 + x^3$ $\qquad y_3(1) = 0$

$xy_4' = -4y_3$ $\qquad y_1(1) = 0$

We can write

$$xy' = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 4 & 0 & 0 & -1 \\ 0 & 0 & -4 & 0 \end{bmatrix} y + \begin{bmatrix} 2 \\ 0 \\ x^3 \\ 0 \end{bmatrix}$$

By theorem (3) and (4)

$$Y = [(p-1)I - A]^{-1} \begin{bmatrix} \frac{2}{p-1} \\ 0 \\ \frac{1}{p-4} \\ 0 \end{bmatrix} \quad , \quad A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 4 & 0 & 0 & -1 \\ 0 & 0 & -4 & 0 \end{bmatrix}$$

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$$Y = \left(\begin{bmatrix} p-4 & 0 & 0 & 0 \\ 0 & p-2 & 2 & 0 \\ -4 & 0 & p-1 & 1 \\ 0 & 0 & 4 & p-1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \frac{2}{p-1} \\ 0 \\ \frac{1}{p-4} \\ 0 \end{bmatrix}$$

$$Y = \frac{1}{(p+1)(p-2)\,(p-3)\,(p-4)} \times$$

$$\begin{bmatrix} p^3-4p^2+p+6 & 0 & 0 & 0 \\ -(8p-8) & p^3-6p^2+5p+12 & -(2p^2-10p+8) & (2p-8) \\ 4p^2-12p+8 & 0 & p^3-7p^2+14p-8 & -(p^2-6p+8) \\ -(16p-32) & 0 & -(4p^2-24p+32) & (p^3-7p^2+14p-8) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{2(p^3-4p^2+p+6)}{(p+1)(p-1)(p-2)\,(p-3)\,(p-4)} \\ \dfrac{-2p^3-4p^2+62p-56}{(p+1)(p-1)(p-2)\,(p-3)\,\,(p-4)^2} \\ \dfrac{p^4-35p^2+90p-56}{(p+1)(p-1)(p-2)\,(p-3)\,\,(p-4)^2} \\ \dfrac{-4p^3-4p^2+136p-224}{(p+1)(p-1)(p-2)\,(p-3)\,\,(p-4)^2} \end{bmatrix}$$

$$Y_1 = \frac{A_1}{p+1} + \frac{B_1}{p-1} + \frac{C_1}{p-2} + \frac{D_1}{p-3} + \frac{E_1}{p-4}$$

$$y_1 = A_1x^{-2} + B_1 + C_1x + D_1x^2 + E_1x^3$$

$$A_1 + B_1 + C_1 + D_1 + E_1 = 0 \qquad \cdots(11)$$

$$-10A_1 - 8B_1 - 7C_1 - 6D_1 - 5E_1 = 2 \qquad \cdots(12)$$

$$35A_1 + 17B_1 + 7D_1 + 5E_1 = -8 \qquad \cdots(13)$$

$$-50A_1 + 2B_1 + 7C_1 + 6D_1 + 5E_1 = 2 \qquad \cdots(14)$$

$$24A_1 - 24B_1 - 12C_2 - 8D_1 - 6E_1 = 12 \qquad \cdots(15)$$

After we solve the system of equations $(11), (12), (13), (14)$ and (15) we get:

$$A_1 = 0\ ,\ \ B_1 = \frac{-2}{3}\ ,\ C_1 = 0\ ,\ \ D_1 = 0\ ,\ \ E_1 = \frac{2}{3}\ ,$$

$$y_1 = \frac{2}{3}x^3 - \frac{2}{3}$$

$$Y_2 = \frac{A_2}{p+1} + \frac{B_2}{p-1} + \frac{C_2}{p-2} + \frac{D_2}{p-3} + \frac{E_2}{p-4} + \frac{F_2}{(p-4)^2}$$

$$y_2 = A_2x^{-2} + B_2 + C_2x + D_2x^2 + E_2x^3 + F_2x^3 lnx$$

$A_2 + B_2 + C_2 + D_2 + E_2 = 0$ $\cdots(16)$

$-14A_2 - 12B_2 - 11C_2 - 10D_2 - 9E_2 + F_2 = 0$ $\cdots(17)$

$75A_2 + 49B_2 + 39C_2 + 31D_2 + 25E_2 - 5F_2 = -2$ $\cdots(18)$

$-190A_2 - 66B_2 - 37C_2 - 22D_2 - 15E_2 + 5F_2 = -4$ $\cdots(19)$

$224A_2 - 32B_2 - 40C_2 - 32D_2 - 26E_2 + 5F_2 = 62$ $\cdots(20)$

$-48A_2 + 48B_2 + 24C_2 + 16D_2 + 12E_2 - 3F_2 = -28$ $\cdots(21)$

After we solve the system of equations $(16), (17), (18), (19),$

(20) and (21) we get:

$A_2 = \frac{1}{5}$, $B_2 = 0$, $C_2 = -3$, $D_2 = 5$, $E_2 = \frac{-11}{5}$, $F_2 = 0$

$$y_2 = \frac{1}{5}x^{-2} - 3x + 5x^2 - \frac{11}{5}x^3$$

$$Y_3 = \frac{A_3}{p+1} + \frac{B_3}{p-1} + \frac{C_3}{p-2} + \frac{D_3}{p-3} + \frac{E_3}{p-4} + \frac{F_3}{(p-4)^2}$$

$$y_3 = A_3x^{-2} + B_3 + C_3x + D_3x^2 + E_3x^3 + F_3x^3 lnx$$

$A_3 + B_3 + C_3 + D_3 + E_3 = 0$ $\cdots(22)$

$-14A_3 - 12B_3 - 11C_3 - 10D_3 - 9E_3 + F_3 = 1$ $\cdots(23)$

$75A_3 + 49B_3 + 39C_3 + 31D_3 + 25E_3 - 5F_3 = 0$ $\cdots(24)$

$-190A_3 - 66B_3 - 37C_3 - 22D_3 - 15E_3 + 5F_3 = -35$ $\cdots(25)$

$224A_3 - 32B_3 - 40C_3 - 32D_3 - 26E_3 + 5F_3 = 90$ $\cdots(26)$

$-48A_3 + 48B_3 + 24C_3 + 16D_3 + 12E_3 - 3F_3 = -28$ $\cdots(27)$

After we solve the system of equations $(22), (23), (24), (25),$

(26), and (27) we get:

$A_3 = \frac{3}{10}$, $B_3 = 0$, $C_3 = 0$, $D_3 = \frac{-5}{2}$, $E_3 = \frac{11}{5}$, $F_3 = 0$

$$y_3 = \frac{3}{10}x^{-2} - \frac{5}{2}x^2 + \frac{11}{5}x^3$$

$$Y_4 = \frac{A_4}{p+1} + \frac{B_4}{p-1} + \frac{C_4}{p-2} + \frac{D_4}{p-3} + \frac{E_4}{p-4} + \frac{F_4}{(p-4)^2}$$

$$y_4 = A_4x^{-2} + B_4 + C_4x + D_4x^2 + E_4x^3 + F_4x^3 lnx$$

$$A_4 + B_4 + C_4 + D_4 + E_4 = 0 \quad \cdots (28)$$

$$-14A_4 - 12B_4 - 11C_4 - 10D_4 - 9E_4 + F_4 = 0 \quad \cdots (29)$$

$$75A_4 + 49B_4 + 39C_4 + 31D_4 + 25E_4 - 5F_4 = -4 \quad \cdots (30)$$

$$-190A_4 - 66B_4 - 37C_4 - 22D_4 - 15E_4 + 5F_4 = -4 \quad \cdots (31)$$

$$224A_4 - 32B_4 - 40C_4 - 32D_4 - 26E_4 + 5F_4 = 136 \quad \cdots (32)$$

$$-48A_4 + 48B_4 + 24C_4 + 16D_4 + 12E_4 - 3F_4 = -112 \quad \cdots (33)$$

After we solve the system of equations $(28), (29), (30), (31)$,
(32) and (33) we get:

$$A_4 = \frac{3}{5}, \quad B_4 = \frac{-8}{3}, \; C_4 = 0, \; D_4 = 5, \; E_4 = \frac{-44}{15}, \quad F_4 = 0$$

$$y_4 = \frac{3}{5}x^{-2} - \frac{8}{3} - \frac{44}{15}x^3 + 5x^2$$

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