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The Cyclic Decomposition of the Group $(Q_{2m}\times C_2)$ when $m=2^h$, $h \in Z^+$

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Abstract— *The main purpose of this paper, is determination of the cyclic decomposition of the abelian factor group AC(G) =* $\overline{R}$ *(G)/T(G) where G =* $Q_{2m}\times C_2$ *when* $m=2^h$ *,* $h \in Z^+$ *(the group of all Z-valued characters of G over the group of induced unit characters from all cyclic subgroups of G).*

We have found that the cyclic, decomposition $AC(Q_{2m}\times C_2)$ *depends on the elementary divisor of m as follows.*

if $m = 2^h$ *, h any positive integer, then:*

$$AC(Q_{2m}\times C_2) = \bigoplus_{i=1}^{2(h+1)} C_2$$

Keywords— ; Artin's characters; Decomposition of the Group G; Decomposition of the Group Q_{2m}; Decomposition of the Group $Q_{2^h}\times C_2$.

1. INTRODUCTION

The problem of determining the cyclic decomposition of AC(G) seem to be untouched. We use the concepts of invariant matrix in linear algebra to find the cyclic decomposition of AC(G), G is considered to be the group $Q_{2^{h+1}}\times C_2$.
In 1968 T.Y Lam [2] defined AC(G) and he studied AC(G),when G is a cyclic group.
In 2000 H.R .Yassin [4] studied the cyclic decomposition of AC(G) when G is an elementary abelian group . In 2006 R.H. Abass [9] found $Ar(Q_{2m}\times C_2)$When m is an even Number .
In this paper ,we find the cyclic decomposition of the factor group $AC(Q_{2^{h+1}}\times C_2)$ where Q_{2m} is the Quaternion group of order 4m When $m=2^h$,$h \in Z^+$ and C_2 is the Cyclic group of order 2

2. The Factor Group AC(G)

This section is devoted to the study of the factor group AC(G) of a group G and the cyclic decomposition of the group AC (Q_{2m}) and Artin's characters table of the group $(Q_{2^{h+1}}\times C_2)$ when $m=2^h$,$h\in Z^+$.

2.The Factor Group AC(G):

This section is devoted to the study of the factor group AC(G) of a group G and the cyclic decomposition of the group AC $(Q_{2m}\times C_2)$ when $m=2^h$, $h \in Z^+$

2.1 Definition:[1]Let T(G) be the subgroup of $\overline{R}$(G) generated by Artin's characters.T (G) is normal subgroup of $\overline{R}$(G) and denotes the factor abelian group $\overline{R}$ (G)/T(G) by AC(G) which is called **Artin cokernel of** G.

2.2 Definition:[5]Let M be a matrix with entries in a principal domain R. **A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

2.3 Definition:[5]A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M. This is denoted by D_k (M).

2.4 Lemma:[5]Let M, P and W b e matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

2.5Theorem:[5]Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1- P and W are invertible.
2- P M W = D.

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3- D is diagonal matrix.

4- if we denote D_{ii} by d_i then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d_j = 0$ and $j \le m$ implies $d_j \ne 0$ and $1 \le j \le m$ implies $d_j \mid d_{j+1}$.

2.6 Definition:[5]Let M be matrix with entries in a principal domain R, be equivalent to a matrix D = diag $\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j \mid d_{j+1}$ for $1 \le j < m$**.** We call D **the invariant factor matrix of** M and $d_1, d_2, \dots, d_m$ the invariant factors of M

2.7 Theorem:[5] Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal

$<d_1>, <d_2>, \dots, <d_m>$, $d_j \mid d_{j+1}$ for $j = 1, 2, \dots, K-1$.

2.8 Proposition:[1]AC(G) is a finitely generated Z-module.

2.9Theorem (Artin's):[6]Every rational valued character of G can be written as a linear combination of Artin characters with coefficient rational numbers.

2.10 Proposition:[9]The general from of the Artin's characters table of the group $(Q_{2^{h+1}} \times C_2)$ when $m=2^h, h\in Z^+$ is give as follows:

$Ar(Q_{2^{h+1}} \times C_2)$=

Γ- classes of $(Q_{2^{h+1}})\times\{I\}$							Γ- classes of $(Q_{2^{h+1}})\times\{z\}$					
Γ-classes	[1,I]	$[x^{2^h}, I]$	…	[x,I]	[y,I]	[xy,I]	[1,z]	$[x^{2^h}, z]$	…	[x,z]	[y,z]	[xy,z]
$\lvert CL_\alpha \rvert$	1	1	…	2	2^h	2^h	1	1	…	2	2^h	2^h
$\lvert C_{Q_{2^{h+1}}\times C_2}(CL_\alpha)\rvert$	2^{h+3}	2^{h+3}	…	2^{h+2}	8	8	2^{h+3}	2^{h+3}	...	2^{h+2}	8	8
$\Phi_{(1,1)}$	**$2Ar(Q_{2^{h+1}})$**						**0**					
$\Phi_{(2,1)}$												
⋮												
$\Phi_{(l,1)}$												
$\Phi_{(l+1,1)}$												
$\Phi_{(l+2,1)}$												
$\Phi_{(1,2)}$	**$Ar(Q_{2^{h+1}})$**						**$Ar(Q_{2^{h+1}})$**					
$\Phi_{(2,2)}$												
⋮												
$\Phi_{(l,2)}$												
$\Phi_{(l+1,2)}$												
$\Phi_{(l+2,2)}$												

Table (1)

3. The Matrix M(G) :[3]

Let l be the number of all distinct Γ- classes then Ar(G) and $\overset{*}{\equiv}(G)$ are of rank l according to the Artin's theorem there exists an invertible matrix

$M^{-1}(G)$ with entries in Q such that :

$$\overset{*}{\equiv}(G) = M^{-1}(G)\,.Ar\,(G)$$

and this implies,

$$M\,(G) = Ar(G)\,.\,(\overset{*}{\equiv}(G))^{-1}$$

M (G) is the matrix expressing the T(G) basis in terms of the $\overline{\boldsymbol{R}}$ (G) basis.
By Theorem (2.5) there exist two matrix P(G) and W(G) with determinant ± 1 such that:

$$P(G).\ M(G).W(G) = \operatorname{diag}\{ d_1, d_2, \dots, d_l \} = D(G)$$

where $d_i = \pm D_i(G)/D_{i-1}(G)$ This process yields a new basis for T(G) and $\overline{\boldsymbol{R}}$ (G),$\{v_1, v_2, \dots, v_l\}$ and $\{u_1, u_2, \dots, u_l\}$ respectively, with the property $v_j = d_j u_j$.

Hence, by Theorem (2.7) and Proposition (2.8) the z-module AC (G) is the direct sum of cyclic sub modules with annihilating ideals

$<d_1>, <d_2>, \dots, <d_l>$

3.1Theorem:[4]AC(G) = $\bigoplus_{i=1}^{l} C_{d_i}$ where $d_i = \pm D_i(G) / D_{i-1}(G)$ where l is the number of all distinct Γ-classes.

3.2 Corollary:[1] $|AC(G)|$ =| det (M(G))| .

3.3 Lemma:[1]If A and B are two matrices of degree m and t respectively, then:
$\det(A\otimes B) = (\det(A))^t . (\det(B))^m$.

3.4 Lemma:[1]Let A and B be two non-singular matrices of rank l and m respectively, over a principal domain R and let:
$P_1 A W_1 = D(A) = \operatorname{diag}\{d_1(A), d_2(A), \dots, d_l(A)\}$
and
$P_2 A W_2 = D(B) = \operatorname{diag}\{d_1(B), d_2(B), \dots, d_m(B)\}$
The invariant factor matrices of A and B then:
$(P_1 \otimes P_2)(A \otimes B)(W_1 \otimes W_2) = D(A) \otimes D(B)$
and from this the invariant factor matrices of A ⊗ B can be obtained.

3.5 Proposition:[4]Let H_1 and H_2 be $p_1$and p_2 - groups respectively where p_1 and p_2 are distinct primes and if M_1 is the matrix from all cyclic subgroups of $\overline{\boldsymbol{R}}$ (H_1) basis and M_2 is the matrix which expresses the T(H_2) basis terms of $\overline{R}$ (H_2) basis then the matrix which expresses the T ($H_1 \times H_2$) basis of $\overline{\boldsymbol{R}}$ ($H_1 \times H_2$) basis is $M_1 \otimes M_2$.

3.6 Proposition:[7]If p is prime number and s is positive integer, then:

$$M(C_{p^s}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

which is of the order (s+1)×(s+1)

3.7 Proposition:[7]The general form of the matrices P (C_{p^s}) and W (C_{p^s}) is:

$$P(C_{p^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

which is (s+1)×(s+1) square matrix.

$W(C_{p^s}) = I_{s+1}$, where I_{s+1} is (s+1)×(s+1) identity matrix

and $D(C_{p^s}) = \text{diag}\,\underbrace{\{1,1,\ldots\ldots,1\}}_{s+1}$.

3.8 Remarks: [6]If $m=2^h$,h is any positive integer ,then we can write $M(C_m)$ as the following :

$$M(C_m)=\begin{bmatrix} & & & & 1 & 1 \\ & & & & 1 & 1 \\ & & R_1(C_m) & & \vdots & \vdots \\ & & & & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

which is (h+1)×(h+1) square matrix ,$R_1(C_m)$ is the matrix obtained by omitting the last two rows $\{0,0,\cdots,1,1\}$ and $\{0,0,\cdots,0,0,1\}$ and the last two columns $\{1,1,\cdots,1,0\}$ and $\{1,1,\cdots,1,1\}$ from the matrix $M(C_{2^h})$ in the Proposition (3.6).

3.9 Proposition:[8]If $m=2^h$,h any positive integers ,then the matrix $M(Q_{2m})$ of the quaternion group Q_{2m} is :

$$M(Q_{2m}) = \left[\begin{array}{cccccc|cccc} & 2R_1(C_{2m}) & & & & & 1 & 1 & 1 & 1 \\ & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

which is (h+4)×(h+4) square matrix ,$R(C_{2m})$ is similar to the matrix in the remarks (3.8).

3.10 Example: By the proposition (3.9) we can find matrix $M(Q_{128})$ as follow

$M(Q_{128})=M(Q_{2^7})=$

$$R_1(C_{2^7}) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

which 6×6 square matrix .
Then

$$M(Q_{2^7})=\begin{bmatrix} & & & & & & & 1 & 1 & 1 & 1\\ & 2R_1(C_{2^7}) & & & & & & 1 & 1 & 1 & 1\\ & & & & & & & 1 & 1 & 1 & 1\\ & & & & & & & 1 & 1 & 1 & 1\\ & & & & & & & 1 & 1 & 1 & 1\\ & & & & & & & 1 & 1 & 1 & 1\\ & & & & & & & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1\\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1\\ 0 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1\\ 0 & 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1\\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

which is 10×10 square matrix .

3.11 Proposition:[8]If $m=2^h$,h any positive integer then the matrices $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms:

$$P(Q_{2m})=\begin{bmatrix} & & & & & 0 & 0\\ & & & & & 0 & 0\\ & & P(C_{2m}) & & & \vdots & \vdots\\ & & & & & \vdots & \vdots\\ & & & & & 0 & 0\\ & & & & & -1 & 1\\ & & & & & 0 & -1\\ 0 & 0 & \cdots & \cdots & 0 & 1 & -1\\ 0 & 0 & \cdots & \cdots & 0 & 0 & 1\end{bmatrix}$$

And

$$W(Q_{2m})=\begin{bmatrix} & & & & & & 0 & 0 & 0\\ & & & & & & 0 & 0 & 0\\ & & I_{h+1} & & & & \vdots & \vdots & \vdots\\ & & & & & & \vdots & \vdots & \vdots\\ & & & & & & 0 & 0 & 0\\ 0 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0\\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0\\ 0 & -1 & -1 & \cdots & -1 & -1 & 0 & 0 & 1\end{bmatrix}$$

where I_{h+1} is the identity matrix .They are (h+4)×(h+4) square matrix .

3.12 Example: To find $P(Q_{128})$ and $W(Q_{128})$, by the proposition (3.11) and to find $AC(Q_{128})$

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$$P(Q_{128})=P(Q_{2^7})=\left[\begin{array}{cccccccc|cc}1&-1&0&0&0&0&0&0&0&0\\0&1&-1&0&0&0&0&0&0&0\\0&0&1&-1&0&0&0&0&0&0\\0&0&0&1&-1&0&0&0&0&0\\0&0&0&0&1&-1&0&0&0&0\\0&0&0&0&0&1&-1&0&0&0\\0&0&0&0&0&0&1&-1&-1&1\\0&0&0&0&0&0&0&1&0&-1\\\hline 0&0&0&0&0&0&0&0&1&-1\\0&0&0&0&0&0&0&0&0&1\end{array}\right]$$

which is 10×10 square matrix
And

$$W(Q_{128})=W(Q_{2^7})=\left[\begin{array}{ccccccc|ccc}1&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&1&0&0&0\\\hline 0&1&1&1&1&1&1&1&0&0\\0&0&0&0&0&0&1&0&1&0\\0&-1&-1&-1&-1&-1&-1&0&0&1\end{array}\right]$$

which is 10×10 quare matrix
$D(Q_{128})=P(Q_{128}).M(Q_{128}).W(Q_{128})=diag\{2,2,2,2,2,2,2,1,1,1\}$
3.13 Theorem: [8]If $m=2^h$,h any positive integers then the cyclic decomposition of $AC(Q_{2m})$ is:

$$AC(Q_{2m})=\mathop{\oplus}_{i=1}^{h+1}C_2$$

4 The Main Results

In this section we give the general form of the cyclic decomposition of the factor group $AC(Q_{2m}\times C_2)$ when $m=2^h$, $h \in Z^+$.

4.1Proposition: If $m=2^h$,h any positive integer , then the matrix $M(Q_{2m}\times C_2)$ of the group $(Q_{2m}\times C_2)$ is :

$$M(Q_{2m}\times C_2)=\left[\begin{array}{c|c}M(Q_{2m})&M(Q_{2m})\\\hline 0&M(Q_{2m})\end{array}\right]$$

Which is $2(h+4)\times 2(h+4)$ square matrix ,$M(Q_{2m})$ is similar to the matrix in Proposition (3.9).

Proof : By Proposition (2.10) we obtain the Artin's characters Table $Ar(Q_{2m}\times C_2)$ of the group $(Q_{2m}\times C_2)$ when $m=2^h, h\in Z^+$ and from the theorem (1.6) we get the rational valued characters $(\overset{*}{\equiv}(Q_{2m}\times C_2))$ table of the group $(Q_{2m}\times C_2)$ when $m=2^h, h\in Z^+$.

Thus , by definition of M(G) we can find the matrix $M(Q_{2m}\times C_2)$ when $m=2^h, h\in Z^+$.

$M(Q_{2m}\times C_2)=Ar(Q_{2m}\times C_2)\cdot(\overset{*}{\equiv}(Q_{2m}\times C_2))^{-1}$.

$$
=\left[\begin{array}{cccccccccccccccccccc}
2 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
0 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 2 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 0 & 2 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
 & & \ddots & & & & & & & & 0 & 0 & 2 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
\vdots & \vdots & & & & & & & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & & & \ddots & & & & \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & & & & & & & \vdots & \vdots & 0 & 0 & 0 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
 & & & & & & & & & & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\
 & & & & & & & \ddots & & & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1
\end{array}\right]
$$

$$
=\left[\begin{array}{cccccccccccccccccccc}
 & & & & & & 1 & 1 & 1 & 1 & & & & & & & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 1 & 1 & 1 & & & & & & & 1 & 1 & 1 & 1 \\
 & & 2.R_1(C_{2m}) & & & & 1 & 1 & 1 & 1 & & & 2.R_1(C_{2m}) & & & & 1 & 1 & 1 & 1 \\
 & & & & & & \vdots & \vdots & \vdots & \vdots & & & & & & & \vdots & \vdots & \vdots & \vdots \\
 & & & & & & \vdots & \vdots & \vdots & \vdots & & & & & & & \vdots & \vdots & \vdots & \vdots \\
 & & & & & & 1 & 1 & 1 & 1 & & & & & & & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & & & & & & & 1 & 1 & 1 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & & & & & & & 1 & 1 & 1 & 1 \\
 & & \ddots & & & & & & & & & & 2.R_1(C_{2m}) & & & & 1 & 1 & 1 & 1 \\
\vdots & \vdots & & & & & & & \vdots & \vdots & & & & & & & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & & & \ddots & & & & \vdots & \vdots & & & & & & & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & & & & & & & \vdots & \vdots & & & & & & & 1 & 1 & 1 & 1 \\
 & & & & & & & & & & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\
 & & & & & & & \ddots & & & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & \cdots & & \cdots & & \cdots & & 0 & 0 & 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1
\end{array}\right]
$$

$$
=\left[\begin{array}{c|c} M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) \end{array}\right] = M(Q_{2m}\times C_2)
$$

4.2Example : Consider the group $(Q_{128}\times C_2)$, we can find the matrix $M(Q_{128}\times C_2)$ by using two ways :

First : By the definition of M(G)

$$M(Q_{128}\times C_2)=M(Q_{2^7}\times C_2)=Ar(Q_{2^7}\times C_2)\cdot(\overset{*}{\equiv}(Q_{2^7}\times C_2))^{-1}$$

$$=\begin{bmatrix}
512&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
256&256&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
128&128&128&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
64&64&64&64&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
32&32&32&32&32&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
16&16&16&16&16&16&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
8&8&8&8&8&8&8&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&4&4&4&4&4&4&4&0&0&0&0&0&0&0&0&0&0&0&0\\
128&128&0&0&0&0&0&0&4&0&0&0&0&0&0&0&0&0&0&0\\
128&128&0&0&0&0&0&0&0&4&0&0&0&0&0&0&0&0&0&0\\
256&0&0&0&0&0&0&0&0&0&256&0&0&0&0&0&0&0&0&0\\
128&128&0&0&0&0&0&0&0&0&128&128&0&0&0&0&0&0&0&0\\
64&64&64&0&0&0&0&0&0&0&64&64&64&0&0&0&0&0&0&0\\
32&32&32&32&0&0&0&0&0&0&32&32&32&32&0&0&0&0&0&0\\
16&16&16&16&16&0&0&0&0&0&16&16&16&16&16&0&0&0&0&0\\
8&8&8&8&8&8&0&0&0&0&8&8&8&8&8&8&0&0&0&0\\
4&4&4&4&4&4&4&0&0&0&4&4&4&4&4&4&4&0&0&0\\
2&2&2&2&2&2&2&2&0&0&2&2&2&2&2&2&2&2&0&0\\
64&64&0&0&0&0&0&0&2&0&64&64&0&0&0&0&0&0&2&0\\
64&64&0&0&0&0&0&0&0&2&64&64&0&0&0&0&0&0&0&2
\end{bmatrix}\bullet$$

$$\begin{bmatrix}
1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512&1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512\\
-1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512&-1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512\\
0&-1/128&1/128&1/128&1/128&1/128&1/256&1/256&1/256&1/256&0&-1/128&1/128&1/128&1/128&1/128&1/256&1/256&1/256&1/256\\
0&0&-1/64&1/64&1/64&1/64&1/128&1/128&1/128&1/128&0&0&-1/64&1/64&1/64&1/64&1/128&1/128&1/128&1/128\\
0&0&0&-1/32&1/32&1/32&1/64&1/64&1/64&1/64&0&0&0&-1/32&1/32&1/32&1/64&1/64&1/64&1/64\\
0&0&0&0&-1/16&1/16&1/32&1/32&1/32&1/32&0&0&0&0&-1/16&1/16&1/32&1/32&1/32&1/32\\
0&0&0&0&0&-1/8&1/16&1/16&1/16&1/16&0&0&0&0&0&-1/8&1/16&1/16&1/16&1/16\\
0&0&0&0&0&0&-1/8&1/8&-1/8&1/8&0&0&0&0&0&0&-1/8&1/8&-1/8&1/8\\
0&0&0&0&0&0&-1/8&-1/8&1/8&1/8&0&0&0&0&0&0&-1/8&-1/8&1/8&1/8\\
0&0&0&0&0&0&1/8&-1/8&-1/8&1/8&0&0&0&0&0&0&1/8&-1/8&-1/8&1/8\\
-1/256&-1/256&-1/256&-1/256&-1/256&-1/256&-1/512&-1/512&-1/512&-1/512&1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512\\
1/256&-1/256&-1/256&-1/256&-1/256&-1/256&-1/512&-1/512&-1/512&-1/512&-1/256&1/256&1/256&1/256&1/256&1/256&1/512&1/512&1/512&1/512\\
0&1/128&-1/128&-1/128&-1/128&-1/128&-1/256&-1/256&-1/256&-1/256&0&-1/128&1/128&1/128&1/128&1/128&1/256&1/256&1/256&1/256\\
0&0&1/64&-1/64&-1/64&-1/64&-1/128&-1/128&-1/128&-1/128&0&0&-1/64&1/64&1/64&1/64&1/128&1/128&1/128&1/128\\
0&0&0&1/32&-1/32&-1/32&-1/64&-1/64&-1/64&-1/64&0&0&0&-1/32&1/32&1/32&1/64&1/64&1/64&1/64\\
0&0&0&0&1/16&-1/16&-1/32&-1/32&-1/32&-1/32&0&0&0&0&-1/16&1/16&1/32&1/32&1/32&1/32\\
0&0&0&0&0&1/8&-1/16&-1/16&-1/16&-1/16&0&0&0&0&0&-1/8&1/16&1/16&1/16&1/16\\
0&0&0&0&0&0&1/8&-1/8&1/8&-1/8&0&0&0&0&0&0&-1/8&1/8&-1/8&1/8\\
0&0&0&0&0&0&1/8&1/8&-1/8&-1/8&0&0&0&0&0&0&-1/8&-1/8&1/8&1/8\\
0&0&0&0&0&0&-1/8&1/8&1/8&-1/8&0&0&0&0&0&0&1/8&-1/8&-1/8&1/8
\end{bmatrix}$$

$$
=\begin{bmatrix}
2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&1&1&1&1\\
0&2&2&2&2&2&1&1&1&1&0&2&2&2&2&2&1&1&1&1\\
0&0&2&2&2&2&1&1&1&1&0&0&2&2&2&2&1&1&1&1\\
0&0&0&2&2&2&1&1&1&1&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&2&2&1&1&1&1&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&1\\
0&1&1&1&1&1&0&0&1&1&0&1&1&1&1&1&0&0&1&1\\
0&1&1&1&1&1&1&0&0&1&0&1&1&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1
\end{bmatrix}
$$

Second : By the proposition (3.9) ,from example (3.10) then

$$
M(Q_{128})=\begin{bmatrix}
2&2&2&2&2&2&1&1&1&1\\
0&2&2&2&2&2&1&1&1&1\\
0&0&2&2&2&2&1&1&1&1\\
0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&1&0&1\\
0&1&1&1&1&1&0&0&1&1\\
0&1&1&1&1&1&1&0&0&1
\end{bmatrix}
$$

And by the proposition (4.1) then $M(Q_{128}\times C_2)$ equal to:

$$
M(Q_{128}\times C_2)=\left[\begin{array}{c|c} M(Q_{128}) & M(Q_{128}) \\ \hline 0 & M(Q_{128}) \end{array}\right]=
$$

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$$\left[\begin{array}{cccccccccc|cccccccccc}
2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&1&1&1&1\\
0&2&2&2&2&2&1&1&1&1&0&2&2&2&2&2&1&1&1&1\\
0&0&2&2&2&2&1&1&1&1&0&0&2&2&2&2&1&1&1&1\\
0&0&0&2&2&2&1&1&1&1&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&2&2&1&1&1&1&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&1\\
0&1&1&1&1&1&0&0&1&1&0&1&1&1&1&1&0&0&1&1\\
0&1&1&1&1&1&1&0&0&1&0&1&1&1&1&1&1&0&0&1\\
\hline
0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1
\end{array}\right]$$

4.3Proposition: If $m=2^h$, h any positive integers then the matrices $P(Q_{2m}\times C_2)$ and $W(Q_{2m}\times C_2)$ are taking the forms :

$$P(Q_{2m}\times C_2)=\left[\begin{array}{c|c} P(Q_{2m}) & -P(Q_{2m}) \\ \hline 0 & P(Q_{2m}) \end{array}\right]$$

which is 2(h+4)×2(h+4) square matrix .

And

$$W(Q_{2m}\times C_2)=\left[\begin{array}{c|c} W(Q_{2m}) & 0 \\ \hline 0 & W(Q_{2m}) \end{array}\right]$$

which is 2(h+4)×2(h+4) square matrix .

Proof : By using the proposition (4.1) taking the matrix $M(Q_{2m}\times C_2)$ and the above forms $P(Q_{2m}\times C_2)$ and $W(Q_{2m}\times C_2)$ then we have : $P(Q_{2m}\times C_2)$. $M(Q_{2m}\times C_2)$.$W(Q_{2m}\times C_2)$ = diag $\{\underbrace{2,2,2,2,\cdots,2}_{2(h+1)},1,1,1,1,1,1\}$

$$=D(Q_{2m}\times C_2)$$

which is 2(h+4)×2(h+4) square matrix .

4.4 Example: To find the matrices $P(Q_{128}\times C_2)$ and $W(Q_{128}\times C_2)$ by the proposition (4.3) from Example (3.12) to find $P(Q_{128})$ and $W(Q_{128})$:

$$P(Q_{128}\times C_2)=\left[\begin{array}{c|c} P(Q_{128}) & -P(Q_{128}) \\ \hline 0 & P(Q_{128}) \end{array}\right]=$$

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$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 00 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

And

$$W(Q_{128}\times C_2)=\left[\begin{array}{c|c} W(Q_{128}) & 0 \\ \hline 0 & W(Q_{128}) \end{array}\right]=$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

4.5 Example: To find $D(Q_{128}\times C_2)$ and the cyclic decomposition of the factor group
We find the matrices $P(Q_{128}\times C_2)$ and $W(Q_{128}\times C_2)$ as in example (4.4) and $M(Q_{128}\times C_2)$ as in example (4.2),then :
$P(Q_{128}\times C_2).M(Q_{128}\times C_2).W(Q_{128}\times C_2)=$
diag{2,2,2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1}=$D(Q_{128}\times C_2)$
Then by Theorem (3.1) we have

$$AC(D(Q_{128}\times C_2)= \bigoplus_{i=1}^{14} C_2$$

The following theorem gives the cyclic decomposition of the factor group $AC(D(Q_{2m}\times C_2))$ when $m=2^h$, $h \in Z^+$.

4.6 Theorem: If $m= 2^h$, h any positive integer then the cyclic decomposition of $AC(Q_{2m}\times C_2)$ is :

$$AC(D(Q_{2m}\times C_2))= \bigoplus_{i=1}^{2(h+1)} C_2$$

Proof : By using the proposition (4.1),we can find matrix $M(Q_{2m}\times C_2)$ and by the proposition (4.3),we find $P(Q_{2m}\times C_2)$ and $W(Q_{2m}\times C_2)$:

$P(Q_{2m}\times C_2)$. $M(Q_{2m}\times C_2)$. $W(Q_{2m}\times C_2)$=

$$\mathrm{diag}\{2,2,2,2,2,2,\ldots,2,2,2,1,1,1,1,1,1\}$$

Then ,by the theorem (3.1) we have :

$$AC(D(Q_{2m}\times C_2))= \bigoplus_{i=1}^{2(h+1)} C_2$$

4.7 Example : Consider the groups $(Q_{32768}\times C_2)$,$(Q_{268435456}\times C_2)$,then :

1. $AC(Q_{32768}\times C_2) = AC(Q_{2^{15}}\times C_2) = \bigoplus_{i=1}^{30} C_2$
2. $AC(Q_{268435456}\times C_2) = AC(Q_{2^{28}}\times C_2) = \bigoplus_{i=1}^{56} C_2$

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