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SM-ideal of A Non Associative Seminear –Ring With BCK Algebra

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Abstract: In this paper , we define a new type of ideal on a non associative seminear-ring with BCK algebra (NASNR-BCK algebra) which is called SM- ideal where we say that (X , . , * , 0) is a NASNR-BCK algebra if X is a non-empty set with two binary operations '*' and '.' defined on it and 0 is constant satisfying the following conditions :

a) (X , .) is a semigroup .

b) (X, *, 0) is a BCK algebra.

c) (x . y) * z = (x *z) . (y * z) , for all x, y, z ∈ X which is called thedistributive law

d) 0 . x = x . 0 = x , for all x ∈ X.

We prove some results then we define the notion of fuzzy SM-ideal on NASNR-BCK algebra and prove some results with examples.We explain the relationships among it and other type of ideals on NASNR-BCK algebra.

Keywords : Seminear-ring NASNR-BCK algebra , Ideal of type one ,Ideal of type two, Fuzzy set

1. INTRODUCTION

A BCK-algebra is a class of logical algebra introduced by Y.Imai and K.Iseki [23]. Many research on it in deferent branch mathematics introduced . "In 1967,Van Hoorn and Van Root selaar introduced the concept of seminear-rings and discussed a general theory of seminear-rings",[22]. Vasantha Kandasamy[19] ,in 2002 discuss a non-associative seminear-ring where she studies many concepts such as ideals .In 2015 A.T. Abdul Wehab [1] introduced a special kind of non- associative seminear-ring calling it Non Assocative Seminear Ring with BCK- algebra (NASNR-BCK algebra) where she introduced three types of ideals calling them the first ideal of type one ,the second ideal of type two and the third ideal of type then proved some of properties and gives examples on them.L.A.Zadeh [13] in 1965 introduced the notion of fuzzy sets where many research interested in this notion and many of papers appear in different mathematical branches .In this paper we present a new type of ideal on NASNR-BCK algebra in ordinary and fuzzy sense we called it SM- ideal where we study some properties and give some examples also explain the relation among it and first ideal of type one and second ideal of type two on NASNR-BCK algebra in ordinary and fuzzy sense.

2. BASIC CONCEPTS

In this section we view some concepts we needed in this paper .

Definition(2.1)[23]: Let Y be a non-empty set with binary operations ∗ , and 0 is a constant ,algebraic system (Y, ∗ , 0) is called a BCK algebra if it satisfies the following conditions:

1)((e *r) *(e *z)) *(z *r) = 0,

2) (e *(e *r)) *r = 0,

3) e*e = 0,

4) if e* r = 0 and r *e = 0 then e = r

5) 0 *e =0, ∀ e ,h ,z ∈Y

Remark(2.2)[4]: Let T be a BCK algebra then:
a)A partial ordering " ≼ " on T can be defined by r ≼ e if and only if r*e = 0.
b) T has the following properties:
1) e*0 = e.
2) if e*y=0 and y*r=0 imply e*r=0
4) (e*y)*r=(e*r)*(y*r)

5) (e*y)*e=0

7) if (e*y)*r=0 implies (e*r)*y=0, for all e, y, r ∈ T.

Definition (2.3) [1]:A BCK-algebra T is called commutative if n∗(n∗m) = m∗(m∗n) for any m, n ∈T .

Definition(2.4)[9]: A BCK-algebra (T,*,0) is said to be Bounded BCK-algebra if it is satisfying the identity: x* (y*x)=x, ∀ x, y ∈T .

Proposition(2.5)[19]: Let (T ,* , .) be a BCK-algebra ,then the following conditions are equivalent to each other:

1) T is negative implicative.

2) x * y= x * (y * x). ∀ x,y∈T

Definition(2.6)[22]: Let (N,+, .) be a non empty set with two binary operations "+" and ". " satisfying the following conditions:

1) (N, +) is a semigroup.

2) (N, .) is a groupoid.

3) (x+y) .z=x.z + y.z for all x,y,z∈N; (N,+, .) is called the right seminear- ring which is non-associative. If we replace 3 by a . (b + c) = a . b + a .c for all a,b,c∈ N; then (N,+, .) is a non-associative left seminear-ring .

Definition (2.7)[1]: Let (T, . , *, 0) be a non-empty set with two binary operations '*' and '.' and 0 is constant satisfying the following conditions :

a) (T ,.) is a semigroup .

b) (T, *, 0) is a BCK algebra.

c) (a.b) * c = (a *c) . (b * c) , for all a, b, c∈T which is called the distributive law ,

d) 0 .a = a. 0 = a , for all a∈ T .

Then ; (T , . , * , 0) is called A Non Associative Seminear-Ring With BCK algebra, we refer to by NASNR-BCKA algebra .

If only (T , .) is a commutative then we say T is commutative with respect to operation of semigroup in similar way if only (T , *) is commutative then we say T is commutative with respect to operation of BCK

Definition (2.8)[1]:Let (T , ., ∗ ,0) is a NASNR-BCK algebra a non-empty subset P of T is said to be a Non Associative Sub Seminear-Ring With BCK Algebra if (P, ., ∗,0) is a NASNR-BCK algebra , we denoted by sub NASNR-BCK algebra.

Definition(2.9)[1]: Let (T , . , * , 0) be a Non Associative Seminear-Ring With BCK Algebra then we say that T is (Commutative NASNR-BCK algebra) .If

(T , .) is commutative semigroup and (T , *) is commutative BCK algebra .

Definition(2.10) [1]: Let Y and Y' be NASNR-BCK algebra and $\mathcal{F}$: Y → Y' is mapping, then:

1) $\mathcal{F}$ is called a homomorphism if $\mathcal{F}$(x •y) = $\mathcal{F}$ (x) • $\mathcal{F}$ (y) and $\mathcal{F}$ (x ∗ y) = $\mathcal{F}$ (x) ∗ $\mathcal{F}$ (y) for all x, y ∈Y.

2) $\mathcal{F}$ is called a monomorphism if $\mathcal{F}$ is a one-to-one homomorphism.

3) $\mathcal{F}$ is called an epimorphism if $\mathcal{F}$ is an onto homomorphism.

4) $\mathcal{F}$ is called an isomorphismif $\mathcal{F}$ is a bijective homomorphism.

5) The set Ker $\mathcal{F}$ ={x∈X : $\mathcal{F}$(x)=0 } is called the kernel of $\mathcal{F}$.

Lemma (2.11) [1] : Let $\mathcal{F}$: Y → Y' be a NASNR-BCK algebra homomorphism . Then $\mathcal{F}$ (0) = 0.

Definition(2.12)[1]:Let Y is a NASNR-BCK algebra . A non- empty subset J of Y is called an ideal of type one in Y if satisfies the following condition:

1) (J , .) is a normal subsemigroup of (Y , .).

2)(n *(n_1.i)) . (n * n_1)) ∈J for each i∈J , n, n_1 ∈Y.

3) J * X ⊆ J.

Definition (2.13)[1]: Let Y be a NASNR-BCK algebra . A non-empty subset J of Y is called an ideal of type two in Y if satisfies the following conditions:

1)if a . b ∈J or b .a ∈J and a ∈J then b ∈J ∀ a ,b ∈ Y.

2) J * Y⊆ J.

Definition(2.14) [24]: Let Y be a non-empty set a fuzzy subset δ of Y is a function δ: Y→ [0, 1].

Definition(2.15) [24]: Let δ and ϑ be a fuzzy sets on Y . Define the fuzzy set ρ∩ϑ as follows: (ρ∩ϑ) (x)=min{ ρ (x) , ϑ (x)} for all x ∈Y .

Definition(2.16) [24]: Let ρ and ϑ be a fuzzy sets on Y . Define the fuzzy set

ρ∪ϑ as follows: (ρ∪ϑ)(x) = max{ ρ (x) , ϑ (x)} for all x ∈Y.

Definition(2.17) [7]: Let ρ and ϑ be the fuzzy subsets in a set Y the Cartesian product ρ× ϑ: Y×Y→ [0, 1] is defined by (ρ×ϑ)(x, y) = min{ ρ(x), ϑ (y)} for all x, y∈ Y.

Definition (2.18) [7]: Let Y be a non-empty set and let ϑ be the fuzzy subset of Y for a fixed 0 ≤ t ≤ 1,Then the set ρ_t = {x∈ X : ρ (x) ≥ t }is called an upper level set of ρ .

Definition(2.19)[17]: Let ϑ be a fuzzy subset of a semigroup S. α, β ∈ (0, 1] such that α < β. We define the fuzzy subset ϑ_α^β of S as follows, ϑ_α^β (x) = (ϑ(x) ∧β) ∨α , for all x ∈ S.

Definition(2.20) [7]: Let $\mathcal{F}$: X→Y be a mapping of NASNR-BCK algebra and ϑ be a fuzzy subset of Y . The map $\rho^{\mathcal{F}}$ is the pre-image of ρ under $\mathcal{F}$ if $\rho^{\mathcal{F}}$ = ρ ($\mathcal{F}$ (x)) , ∀ x ∈X.

Definition(2.21)[5]: If ξ is the family of all fuzzy sets in BCK algebra X .For $x \in Y$ and $\tau \in (0,1]$,$x_\tau \in \xi$ is a fuzzy point iff

$$x_\tau(y)=\begin{cases} \tau & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

We denote by $\widetilde{Y}=\{x_\tau : x\in Y, \tau \in (0,1]\}$ the set of all the fuzzy points on Y and we define a binary operation * on $\widetilde{Y}$ as follows: $x_\tau * h_\nu =(x*y)_{\min(\tau,\nu)}$.

It is easy to verify that $\forall x_\tau, x_\mu ,h_\nu ,z_\alpha \in \widetilde{Y}$, the conditions hold:

BCK-1: $((x_\tau*h_\nu) * (x_\tau * z_\alpha)) * (z_\alpha * h_\nu)=0_{\min(\tau,\nu,\alpha)}$

BCK-2: $[x_\tau* (x_\tau * h_\nu)] * h_\nu=0_{\min(\tau,\nu)}$

BCK-3: $x_\tau * x_\mu =0_{\min(\tau,\mu)}$

BCK- 4:$0_\nu * x_\tau=0_{\min(\nu,\tau)}$.

We recall that if $\mathbb{S}$ is a fuzzy subset of a BCK-algebra Y; then we have the following :

1) $\widetilde{\mathbb{S}}=\{x_\tau \in \widetilde{Y} : \mathbb{S}(x) \geq \tau ,\tau \in (0,1]\}$.

2) $\forall\tau \in (0, 1]$, $\widetilde{Y}_\tau =\{x_\tau: x \in Y\}$ and $\widetilde{\mathbb{S}}_\tau =\{x_\tau \in \widetilde{Y}_\tau : \mathbb{S}(x) \geq \tau \}$.

We have also $\widetilde{\mathbb{S}}_\tau\subseteq\widetilde{\mathbb{S}}$,$\widetilde{\mathbb{S}}\subseteq\widetilde{Y}$,$\widetilde{\mathbb{S}}_\tau\subseteq\widetilde{\mathbb{S}}$,$\widetilde{\mathbb{S}}_\tau\subseteq\widetilde{Y}_\tau$ and one can easily check that $(\widetilde{Y}_\tau ,* , 0_\tau)$ is a BCK-algebra.

Remark(2.22)[8]: Let $\tilde{S}$ be the set of all fuzzy points in semigroups X. Then

$x_\lambda.y_\tau = (x .y)_{\min(\lambda,\tau)} \in \tilde{S}$ for $x_\lambda,y_\tau \in \tilde{S}$.

Remark(2.23)[14]: Let ω be fuzzy sub NASNR-BCK algebra of Y .Then

$\omega(0) \geq \omega(x) \ \forall x \in Y$.

Definition(2.24)[14]: Let X be a NASNR- BCK algebra and let μ be a fuzzy subset of X, we say μ is a fuzzy sub NASNR- BCK algebra of X if:

1) $\mu(x*y) \geq \mu(x) \wedge \mu(y)$

2) $\mu(x.y) \geq \mu(x)\wedge\mu(y)$, $\forall x,y \in X$.

Definition(2.25)[15]:A fuzzy subset ζ of a NASNR-BCK algebra of X is called a fuzzy ideal of type one of X if it is satisfies the following conditions:-

1) $\zeta(x * y) \geq \zeta(x)$

2) $\zeta(x . y) \geq \zeta(x) \wedge \zeta(y)$

3) $\zeta(x . y) = \zeta(y . x)$

4) $\zeta((x * (y . i)) . (x * y)) \geq \zeta(i) \quad \forall x,y,i\in X$

Definition(2.26)[15]:A fuzzy set ζ of a NASNR-BCK algebra X is called a fuzzy ideal of type two of X if it is satisfies the following conditions :

1) $\zeta(x * y) \geq \zeta(x)$

2) $\zeta(y) \geq \{ \zeta(x.y) \vee\zeta(y.x)\}\wedge\zeta(x)$, $\forall x,y\in X$.

If X is a commutative with respect to (.) ,then(2) in definition above

become $\zeta(y) \geq \zeta(x.y) \wedge \zeta(x) \quad \forall x,y\in X$

3. SM- ideal of NASNR-BCK algebra

In this section ,we define the notion SM- ideal on NASNR-BCK algebra ,some results with examples are introduced .We explain the relation among SM-ideal and some other type ideals ideal on NASNR-BCK algebra.

Definition(3.1): Let X be a NASNR-BCK algebra .A nonempty subset J of X is called SM-ideal in X if satisfies the following conditions:

1) $x.y \in J \quad \forall x,y \in J$

2) $x* (y .i) \in J \quad \forall x ,(y \neq 0) \in X , i \in J$

3) $J* X \subseteq J$

Remark(3.2): In this paper X denotes the NASNR-BCK algebra unless otherwisespecified.

Example(3.3): Let X ={ 0 ,1 ,2} be defined by the following tables:

*	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

.	0	1	2
0	0	1	2
1	1	0	2
2	2	2	2

[Appendix in 1].

Let I ={0 ,2} ⊆ X then by usual calculation ,we can prove that is SM- ideal .

Proposition(3.4): Let X be NASNR- BCK algebra and I be SM-ideal .Then 0 ∈I.

Proof: Clear

Example(3.5): Let X={0,1,2 ,3} be defined by the following tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	0	0	3
3	3	0	0	0

.	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	1	1	1
3	3	1	2	3

,[Appendix in 1] .

Let A_1 = {0,1,3} and A_2= {0 ,2} , A_1 is SM-ideal, A_2 is not SM-ideal but
$A_1 \cap A_2$ ={0} is SM-ideal .

Preposition(3.6): Let {τ_j : j∈Λ} be a family of SM-ideal of X. Then $\bigcap_{j\in\Lambda}(\tau_j)$ is SM-ideal of X.

Proof:Let {τ_j : j ∈ Λ} be a family of SM-ideal of X, then

1)x, y∈ $\bigcap_{j\in\Lambda}(\tau_j)$, since τj is SM-ideal, then x .y∈ τ_j for each j , so x . y ∈ $\bigcap_{j\in\Lambda}(\tau_j)$

2)Let x,(y ≠0)∈ X , i∈ $\bigcap_{j\in\Lambda}(\tau_j)$,then i∈ τ_j, since τj is SM-ideal, then x*(y. i)∈ τ_j
for each j. Then x*(y . i)∈ $\bigcap_{j\in\Lambda}(\tau_j)$

3)Let x ∈ $\bigcap_{j\in\Lambda}(\tau_j)$ and y ∈ X, since τ_j is SM-ideal, then x*y ∈ τ_j for each j so x*y ∈ $\bigcap_{j\in\Lambda}(\tau_j)$.
Then $\bigcap_{j\in\Lambda}(\tau_j)$ is SM-ideal.

Preposition(3.7): Let {τ_i : i∈Λ}be a chain of SM-ideal of X .Then $\cup_{i\in\Lambda}(\tau_i)$ is SM-ideal

Proof:Let {τ_i : i∈Λ} be a chain of SM-ideal of X ,then

1)x, y∈ $\cup_{i\in\Lambda}$(τi) then ∃ τ_j ,τ_k∈{τ_i}i∈Λ , such that x∈ τ_j and y ∈ τ_k
then τ_j⊆ τk or τk ⊆ τ_j [since {τ_i}i∈Λ is a chain]
so either x∈ τ_j and y ∈ τ_j or x∈ τ_k and y ∈ τ_k then either x.y∈ τ_j or x .y∈ τ_k where τ_j and
τk are SM-ideal then x . y ∈ $\cup_{i\in\Lambda}(\tau_i)$

2)Let x,(y ≠0)∈ X, j∈ $\cup_{i\in\Lambda}(\tau_i)$ so x*(y . j)∈ τ_i for some τ_i . Then x*(y . j)∈ $\cup_{i\in\Lambda}(\tau_i)$

3)Let x ∈ $\cup_{i\in\Lambda}(\tau_i)$ and y ∈ X so ∃ τk∈ {τ_i}i∈Λ such that x∈ τ_k, then x*y ∈ τ_k
where τk is SM-ideal then x*y ∈ $\cup_{i\in\Lambda}(\tau_i)$. Then $\cup_{i\in\Lambda}(\tau_i)$ is SM-ideal.

Preposition(3.8): Let L:X→Y be a NASNR-BCK algebra isomorphism. If 𝔹 is SM-ideal of Y, then L^{-1}(𝔹)={x ∈ X : L(x) ∈ 𝔹} is SM-ideal of X.

Proof: Let L: X→Y be a NASNR- BCK algebra isomorphism if 𝔹 is SM-ideal of Y, then

1)Let x ,y ∈ L^{-1}(𝔹) then L(x) ,L(y) ∈ 𝔹 since 𝔹 is SM-ideal so x .y ∈ L^{-1}(𝔹)

2)let x ,(y≠0)∈ X , i∈ L^{-1}(𝔹), then L(i) ∈ 𝔹,since 𝔹 is SM- ideal ,then L(x) * (L(y). L(i)) ∈ 𝔹
so x * (y .i) ∈ L^{-1}(𝔹)

3)let x ∈ L^{-1}(𝔹) and y∈X then L(x) * L(y)= L(x*y) ∈ 𝔹 since 𝔹 is SM-ideal,
so x*y ∈ L^{-1}(𝔹) ∀ x ∈ L^{-1}(𝔹) and y∈ X so L^{-1}(𝔹) is SM-ideal .

Preposition(3.9): Let L: X→Y be NASNR- BCK algebra epimorphism .If 𝔸 is SM-ideal of X ,then L(𝔸) is SM -ideal of X.

Proof:Let L: X→Y be NASNR- BCK algebra epimorphism and 𝔸 be SM -ideal.

1)Let a, b∈ L(𝔸) then ∃ x, y ∈ 𝔸 such that L(x) = a and L(y) = b
but x . y∈ 𝔸 so L(x . y) ∈L(𝔸) so L(x). L(y) ∈ L(𝔸) then a . b∈ L(𝔸)

2)Let a , (b≠ 0)∈ Y, i∈ L(𝔸) such that L(x) = a, L(y) = b, L(c) =i then (x * (y . c))∈ 𝔸
where x ,(y ≠0) ∈X and c ∈ 𝔸 then L(x*(y . c)) ∈ L(𝔸) so a*(b . i) ∈L(𝔸)

3) in easy way we can prove L(𝔸) * y ⊆ L(𝔸), Then L(𝔸) is SM-ideal

Example(3.10): Let X={0,1,2}as in example(3.3).Let L : X → Y such that L(0) = L(1) =0 , L(2) =1 then ker L ={0,1} is not SM-ideal ,since 2*(1.1) =2 ∉ ker L.

Preposition(3.11): Let L : X → Y be a NASNR- BCK algebra homomorphism and let x*y =0 ∀(y≠ 0).Then ker L is SM-ideal of X.

Proof: Let L : X → Y be a NASNR- BCK algebra homomorphism ,then

1)Let a, b ∈ ker L, then L(a . b) = L(a) . L(b)=0 so a .b ∈ker L

2)Let x,(y ≠0) ∈ X and I ∈ ker L, then

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L(x*(y . i)) = L(x) * (L(y). L(i)) = L(x) * ((L(y).0)= L(x) * L(y) = L(0) =0
then x*(y.i) ∈ kerL
3)Let x ∈ ker L and y ∈ X then L(x*y) = L(x) * L(y)= 0 * L (y)= 0.Then x * y∈ ker L therefore ker L * X ⊆ ker L. Hence ker L is SM-ideal.
Proposition(3.12): Let X be NASNR- BCK algebra and ∅ ≠I ⊂ X such that there exist z ∈ X, i∈ I where z. i = 0 then I is not SM-ideal.
Proof: Let ∅ ≠I ⊂ X , z∈ X, i∈ I and z. i = 0 .Let I be SM-ideal .
x*(z.i) = x * 0=x such that x∈ X there are two cases:
case1: if x ∈ I then x*(z.i) ∈ I
case2: if x ∈ X/I then x ∉ I
Then we get contradiction with assumption hence I is not SM-ideal.
Preposition(3.13): Let X be NASNR- BCK algebra and I, J be SM-ideal of X .Then I×J is SM- ideal of X×X .
Proof: Clear
Proposition(3.14): If I is SM-ideal .Then I is a sub NASNR- BCK algebra.
Proof: Clear
Remark(3.15): The converse of the above proposition is not true in general as shown in the following example: Let X = {0,1,2,3} be defined by the following tables:

*****	**0**	**1**	**2**	**3**
0	**0**	**0**	**0**	**0**
1	**1**	**0**	**2**	**0**
2	**2**	**0**	**0**	**0**
3	**3**	**0**	**2**	**0**

.	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	1	2	1
3	3	1	1	1

[Appendix in 1].

I={0 ,1} is a sub NASNR- BCK algebra but not SM-ideal ,since 1∈I , 2∈X.
1*2 = 2 ∉ I ,then I *X ⊈ I .
Proposition(3.16): Let X be commutative NASNA-BCK algebra with (*) . If I be SM-ideal and (y * x) *(x * y)=y ∀x ≠ y and x * y ≠ 0 .Then I is an ideal of type two.
Proof: Let x . y∈ I or y. x∈I and x∈ I
If x .y∈I and x ∈ I , x ≠ y and y * x ≠ 0 ,then
(x . y) * x = (x * x) . (y * x) = y * x ∈ I by 3 of definition SM-ideal
but x * y ∈ I by 3 of definition SM-ideal then (y * x) * (x * y) =y∈ I by 3 of definition SM-ideal. Now, if x=y then clear y∈ I since x . x∈ I and x∈ I
If y * x =0 then y * (y * x) = y * 0 =y then x * (x * y)= y ∈ I since I is SM-ideal.
So I is an ideal of type two.
Example(3.17):Let X defined by the following tables:

*****	**0**	**1**	**2**
0	**0**	**0**	**0**
1	**1**	**0**	**1**
2	**2**	**0**	**0**

.	**0**	**1**	**2**
0	**0**	**1**	**2**
1	**1**	**1**	**1**
2	**2**	**1**	**0**

,[1].

Let I= {0,2} then I is an ideal of type two but not SM-ideal since if i=0, x=1, y=2, then 1*(2.0) = 1*2=1 ∉ I

4. A Fuzzy SM-Ideal of NASNR-BCK Algebra

In this section ,we define the notion of fuzzy SM-ideal on NASNR-BCK algebra and prove some results with examples .We explain the relation between a fuzzy SM-ideal and fuzzy (sub NASNR-BCK algebra ,ideal of type one and ideal of type two).
Definition(4.1): Let δ be a fuzzy set on a NASNR- BCK algebra X.δ is called a fuzzy SM-ideal of NASNR- BCK algebra if :
1)δ(x*y) ≥ δ(x) ∀x ,y∈ X

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2)$\delta(x . y) \geq \delta(x) \wedge \delta(y) \quad \forall x ,y \in X$
3)$\delta(x*(y.i)) \geq \delta(i) \quad \forall x,(y \neq 0)\ i \in X$
Remark(4.2): It is clear that if $\delta(x*y) \geq \delta(x)$ then $\delta(x*y) \geq \delta(x) \wedge \delta(y) \ \forall x ,y \in X$
Example(4.3): Let X={0,1,2,3} defined by the following tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	1	0	0

.	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

[1].

we define ϵ by $\epsilon(x) = 0.4$ if $x= 0$ and $\epsilon(x) = 0.3$ if $x= 1,2,3$
Then by usual calculation ,we can prove ϵ is a fuzzy SM- ideal of NASNR- BCK algebra .
Example(4.4): Let X={0 ,1,2,3} defined by the following tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	0	0	3
3	3	0	0	0

.	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	2	2	2
3	3	2	2	3

,Appendex in 1].

Let ϵ defined by $\epsilon(x) = 0.5$ if $x=0$ and $\epsilon(x)=0.4$ if $x= 1,2,3$ then ϵ is not fuzzy SM-ideal of a NASNR-BCK algebra since $\epsilon(1 * (3 . 0)) = \epsilon(3) =0.4 \ngeq \epsilon(0) = 0.5$.
Proposition(4.5): Let ∂ be a fuzzy SM-ideal of a commutative NASNR-BCK algebra with respect to * if $x \leq y$ .Then $\partial(x) \geq \partial(y)$.
Proof:Clear
Remark(4.6): Let η be a fuzzy SM-ideal of X .Then $\eta(0) \geq \eta(x)\ \forall x \in X$.
Proposition(4.7): Let $\mathbb{S}$ be fuzzy SM-ideal of NASNR-BCK algebra of Y then $\widetilde{\mathbb{S}}_\lambda$ is SM-ideal of NASNR-BCK algebra of $\widetilde{Y}_\lambda$.
Proof: Let $\mathbb{S}$ be a fuzzy SM-ideal of of X and let $\lambda \in (0 ,1]$,then
1) let $x\lambda \in \widetilde{\mathbb{S}}_\lambda$,then $\mathbb{S}(x) \geq \lambda$ and $y\lambda \in \widetilde{Y}_\lambda$ since $\mathbb{S}(x*y) \geq \mathbb{S}(x) \geq \lambda$ thus $(x* y)_\lambda \in \widetilde{\mathbb{S}}_\lambda$ then $x_\lambda * y_\lambda \in \widetilde{\mathbb{S}}_\lambda$.
2) since $\mathbb{S}(x.y) \geq \mathbb{S}(x) \wedge \mathbb{S}(y) \geq \lambda$ then $x_\lambda . y_\lambda \in \widetilde{\mathbb{S}}_\lambda$
3) let $i\lambda \in \widetilde{\mathbb{S}}_\lambda$ and $x_\lambda, y_\lambda \in \widetilde{Y}_\lambda$ such that $\mathbb{S}(i) \geq \lambda$ .Since $\mathbb{S}(x*(y.i)) \geq \mathbb{S}(i) \geq \lambda$
Then $(x_\lambda *(y_\lambda .i_\lambda)) \in \mathbb{S}_\lambda$. Hence $\widetilde{\mathbb{S}}_\lambda$ is SM- ideal of NASNR-BCK algebra of $\widetilde{Y}_\lambda$.
Example(4.8): By X in example (4.3) , we define $\eta1$ and $\eta2$ by $\eta1(x) =0.5$ if $x=0$ and $\eta1(x) =0.3$ if $x=1,2,3$, $\eta2 (x) = 0.4$ if $x=1$, $\eta2 (x) = 0.6$ if $x=0$ and $\eta2 (x) = 0.3$ if $x=2,3$ .Note that $\eta1$ is a fuzzy SM- ideal and $\eta2$ is not fuzzy SM-ideal but $(\eta1 \cap \eta2)(x) = 0$ if $x=0$ and $(\eta1 \cap \eta2)(x)=0.3$ if $x=1,2,3$,is a fuzzy SM-ideal
Preposition(4.9): Let $\{\mu_j: j \in \Lambda\}$ be a family of fuzzy SM-ideal of NASNR-BCK algebra of X .Then $\bigcap_{j\in\Lambda}(\mu_j)$ is a fuzzy SM-ideal.
Proof: Let μj be a fuzzy SM-ideal, in easily way we can have
$(\bigcap_{j\in\Lambda}(\mu_j))\ (x*y) \geq \bigcap_{i\in\Lambda}(\mu_j)(x)$ and $\bigcap_{j\in\Lambda}(\mu_j))(x.y) \geq \bigcap_{j\in\Lambda}(\mu_j)(x) \wedge \bigcap_{j\in\Lambda}(\mu_j)(y)$ for each
$x,y \in X$.Now, $\bigcap_{j\in\Lambda}(\mu_j)(x*(y.i)) = \inf \{\mu j(x*(y.i)): j \in \Lambda\}$ where x, $(y \neq 0) \in X$
$\geq \inf\{ \mu_j (i): j \in \Lambda\}$ [by 3 of definition (4.1)]
$= \bigcap_{j\in\Lambda}(\mu_j)(i)$.Then $\bigcap_{j\in\Lambda}(\mu_j)$ is a fuzzy SM-ideal.
Proposition(4.10): Let $\{\mu_j: j \in \Lambda\}$be a chain of fuzzy SM-ideal of NASNR-BCK algebra . Then$\bigcup_{j\in\Lambda} \mu_j$ is a fuzzy SM-ideal .
Proof:Let μj is a fuzzy SM-ideal, then in easily way we can have
$(\bigcup_{j\in\Lambda} \mu_j))\ (x*y) \geq \bigcup_{j\in\Lambda} \mu_j\ (x)$ and $\bigcup_{j\in\Lambda} \mu_j(x.y) \geq \bigcup_{j\in\Lambda} \mu_j\ (x) \wedge \bigcup_{j\in\Lambda} \mu_j(y)$ for each
$x,y \in X$. Now,$\bigcup_{j\in\Lambda} \mu_j(x*(y.i)) = \sup \{\mu j(x*(y.i)):j \in \Lambda\}$ where x, $(y \neq 0) \in X$
$\geq \sup\{\mu j\ (i) :j \in \Lambda\}$ [by 3 of definition (3.1)]

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$= \bigcup_{j\in\Lambda} \mu_j$ (i) . Then $\bigcup_{j\in\Lambda} \mu_j$ is a fuzzy SM-ideal.

Proposition(4.11): Let μ_1 and μ_2 are two a fuzzy SM- ideal of NASNR-BCK algebra of X.Then $\mu_1\times\mu_2$ is a fuzzy SM- ideal X ×X.

Proof:Let μ_1 , μ_2 be two fuzzy SM- ideal of NASNR-BCK algebra and x=(x_1,x_2) and y=(y_1,y_2) ∈X×X.We can prove 1 and 2 by a similar way of [17]

3) let (x_1,x_2) , y_1≠0 , y_2≠ 0 , (i1,i2)∈X ×X ,then

$((\mu_1\times\mu_2)((x_1,x_2)*((y_1,y_2).(i_1,i_2))) = \mu_1(x_1*(y_1.i_1))\wedge \mu_2(x_2*(y_2.i_2))$

$\geq \mu_1(i_1) \wedge \mu_2(i_2) = (\mu_1\times\mu_2)(i_1,i_2)$

, where (y_1, y_2) ≠0.Then $\mu_1\times\mu_2$ a fuzzy SM-ideal of X×X.

Proposition(4.12): Let ρ be a fuzzy SM-ideal of NASNR- BCK algebra of X. Then ρ+ is a fuzzy SM-ideal.

Preposition(4.13): Let L: X ⟶ Y be isomorphism if κ is a fuzzy SM-ideal of NASNR- BCK algebra of Y. Then κ^L is a fuzzy SM-ideal of X.

Proof:Let L: X ⟶ Y be isomorphism and let κ be a fuzzy SM-ideal and let x, y ,i∈X, then

in a similar way by 1and 2 of [15] we can prove $\kappa^L(x*y) \geq \kappa^L(x)$ and $\kappa^L(x . y) \geq \kappa^L(x) \wedge \kappa^L(y)$ for each x,y ∈X.Now,

$\kappa^L(x*(y.i)) = \kappa(L(x *(y . i)))$

$\geq \kappa(L(i))$ ∀ i∈I [by 3 of definition(3.1)]

$= \kappa^L(i)$ where y ≠0 ∈X.Then κ^L is a fuzzy SM-ideal.

Preposition(4.14): Let L: X ⟶ Y be an epimorphism if κ^L is a fuzzy SM-ideal of NASNR- BCK algebra of X. Then κ is a fuzzy SM-ideal of Y.

Proof: Let κ^L be a fuzzy SM-ideal of X and let L be an epimorphism and let x, y, i∈X, then

∃ a, b, c ∈X such that L(a) = x , L(b) = y and L(c) = i, then in a similar way by 1and 2 of [17].

κ (x*y) ≥ κ(x) and κ(x . y) ≥ κ (x) ∧ κ(y) for each x,y ∈X.Now,

κ (x*(y.i)) = κ (L (a)*(L (b). L (c))) where y ≠0 ∈ X

$= \kappa(L(a)*(b.c)) = \kappa^L(a*(b.c)) \geq \kappa^L(c)$ [since κ^L is a fuzzy SM-ideal]

= κ (L (c))= κ(i).Then κ is a fuzzy SM-ideal of Y.

Proposition(4.15): Let ω be a fuzzy SM-ideal of NASNR- BCK algebra of X .Then

H={x∈X: ω(x) = ω(0)} is a SM-ideal.

position(4.16):Let ϑ be a fuzzy SM-ideal of NASNR-BCK algebra X and α,β∈[0,1].Then ϑ_α^β is a fuzzy SM-ideal of X ∀α<β.

Proof:Let ϑ be a fuzzy SM-ideal of NASNR-BCK algebra of X and x,y,i∈Xsuch that α,β∈(0,1] , then in a similar way by 1 and 2 of [15] $\vartheta_\alpha^\beta(x*y) \geq \vartheta_\alpha^\beta(x)$ and $\vartheta_\alpha^\beta(x.y) \geq \vartheta_\alpha^\beta(x) \wedge \vartheta_\alpha^\beta(y)$ for each x,y ∈X.Now,

$\vartheta_\alpha^\beta((x*(y.i)) = (\vartheta(x*(y.i))\wedge\beta)\vee\alpha) \geq (\vartheta(i)\wedge\beta)\vee\alpha = \vartheta_\alpha^\beta(i)$ [by 3 of definition (3.1)]

5. Relationship Among Fuzzy SM-ideal And Other Types Of Ideals on NASNR-BCK Algebra

In this section ,some propositions and examples prove to explain relationships among some types of ideals of NASNR- BCK algebra.

Proposition(5.1): Let ω be a fuzzy SM-ideal of NASNR- BCK algebra. Then ω is a fuzzy sub NASNR- BCK algebra.

Remark(5.2): The converse of above proposition is not true in general. As we shown in the following example: Let X = {0,1,2,3} as in example (4.4).Letω defined by ω(x) = 0.5 if x= 0 and ω(1)= ω(2)= ω(3)= 0.4,ω is a fuzzy sub NASNR- BCK algebra, but not a fuzzy SM-ideal. let x=1 , y= 3, i=0 ,then ω(1*(3.0)) =ω(3) = 0.4 ≱ ω(0) = 0.5.

Proposition(5.3): Let ω be a fuzzy set of NASNR- BCK algebra and let X be commutative with respect to (.) and x≤ y ∀ x,(y≠ 0) ∈X then ω is a fuzzy ideal of type one iff ω is a fuzzy SM-ideal.

Proof: Let ω be a fuzzy SM-ideal and let x, y ∈X, then in easily way we can prove

1) ω (x*y) ≥ ω(x) , 2) ω(x.y) ≥ ω(x)∧ ω(y) for each x,y ∈X.Now,

3) ω(x.y) = ω(y. x) [since X be commutative with respect to .]

4) ω ((x*(y.i)).(x*y)) ≥ ω((x * (y.i)).0) = ω(x* (y.i)) where (y ≠0) ∈ X

≥ ω(i) [by 3 of definition (3.2)]

Conversely, let ω be a fuzzy ideal of type one ,then in easily way we can prove

1) ω (x*y) ≥ ω(x) and 2) ω (x.y) ≥ ω(x)∧ ω(y) for each x,y ∈X.

3) ω((x*(y.i)).(x * y)) ≥ ω(i) ∀x, y,i∈X

Let y≠0 so ω((x*(y.i)).(x * y)) ≥ ω(i) ∀x, (y≠0), i∈X .Since x * y=0 ∀ y≠0

so ω((x * (y.i)). (x*y))= ω((x*(y.i)).0)≥ ω((x *(y .i))≥ ω(i) .Then ω is a fuzzy SM-ideal.

Example(5.4): Let X = {0,1,2,3} as in example (4.4) .Let ω defined by ω(x) =0.6 if x= 0 and ω(1) = ω(2) =ω(3) = 0.5 ,ω is a fuzzy ideal of type two, but ω not a fuzzy SM-ideal ,since
let x=1 , y= 3, i=0 ,then $\omega(1*(3.0)) = \omega(3) = 0.5 \ngeq \omega(0) = 0.6$.

Proposition(5.5): Let ω be a fuzzy SM-ideal of NASNR- BCK algebra and let X be commutative with respect to (.) and let $x^2 = 0 \ \forall\ x \in X$ .Then ω is a fuzzy ideal of type two.

Proof: Let ω be a fuzzy SM-ideal and let x, y ∈X, then
1) $\omega(x*y) \geq \omega(x)$ [by 1 of definition (3.1)]
2) $\omega(y) = \omega(y.0) = \omega(y.x^2)= \omega(y.x.x)= \omega((y.x).x) \geq \omega(y.x) \wedge \omega(x)$ [by 2 of definition (4.1)]
The converse is not true in general by the following example.

Example(5.6): Let X as example(4.3) ,we define ω defined by ω(0) =0.6 and ω(1) = ω(2) = (3) =0.5, ω fuzzy ideal of type two, but ω not a fuzzy SM-ideal let x=1 , y= 3, i=0 ,
then $\omega(1*(3.0)) = \omega(3) = 0.5 \ngeq \omega(0) = 0.6$.

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