

Some New Results About Generalized Difference Triple Sequence Spaces Defined by a Double Orlicz-Functions

Ali Hussein Battor¹, Dalael Saad Abdulzahra²

Department of Mathematics, Faculty of Education for Girls, Iraq.

Abstract: In this study we introduced some new results about generalized difference triple sequence spaces by using a double Orlicz-functions and we will examine some new properties of these spaces.

Keywords: Triple sequence; Difference sequence; Double Orlicz functions.

1. Introduction

A triple sequence (complex or real) can be defined as a function $f: N \times N \times N \rightarrow R(C)$ where N, R and C denote the sets of natural numbers, real numbers and complex numbers respectively [2][9]. Some new results of triple sequences spaces would studied by Orlicz function using a function F where

$$F = (F_1(r), F_2(u)).$$

Let $(x, y) = (x_{r, u, j}, y_{r, u, j})$ be a triple infinite array of elements $(x_{r, u, j}, y_{r, u, j})$ and Ω^3 denotes the family of all triple sequences of real or complex numbers.

Let $3l_\infty, 3c_0$ be the linear spaces of bounded, null, and convergent sequences with complex terms, respectively, normed by :

$$\| (x, y) \| = \sup_{r, u, j} \{ |x_{r, u, j}|, |y_{r, u, j}| \}$$

$$\|x\| = \sup_{r, u, j} |x_{r, u, j}|$$

$$\|y\| = \sup_{r, u, j} |y_{r, u, j}|$$

where r, u and $j \in \mathbb{N}$ the set of positive integers.

In this study we define the triple sequence spaces $3c_0(\Delta_u^v, F_{r,u}, p, \varphi)$, $3c(\Delta_u^v, F_{r,u}, p, \varphi)$, and $3l_\infty(\Delta_u^v, F_{r,u}, p, \varphi)$.

2. Definitions and Preliminaries:

Definition.2.1[7]

The double Orlicz-functions is a function

$$F: [0, \infty) \times [0, \infty) \rightarrow [0, \infty) \times [0, \infty) \text{ such that } F(u, v) = (F_1(u), F_2(v))$$

$$F_1: [0, \infty) \rightarrow [0, \infty) \text{ and } F_2: [0, \infty) \rightarrow [0, \infty),$$

such that F_1, F_2 are Orlicz functions which are even, convex, continuous, non-decreasing and satisfies three conditions :

$$i) F_1(0) = 0, F_2(0) = 0 \rightarrow F(0,0) = (F_1(0), F_2(0))$$

$$ii) F_1(u) > 0, F_2(v) > 0 \rightarrow F(u, v) = (F_1(u), F_2(v)) > (0,0) \text{ for } u > 0,$$

$$v > 0, \text{ we mean that by } F(u, v) > (0,0) \text{ that } F_1(u) > 0, F_2(v) > 0$$

$$iii) F_1(u) \rightarrow \infty, F_2(v) \rightarrow \infty \text{ as } u, v \rightarrow \infty \text{ then}$$

$$F(u, v) = (F_1(u), F_2(v)) \rightarrow (\infty, \infty) \text{ as } (u, v) \rightarrow (\infty, \infty) \text{ we mean by}$$

$$F(u, v) \rightarrow (\infty, \infty), \text{ that } F_1(u) \rightarrow \infty, F_2(v) \rightarrow \infty$$

Definition.2.2[2][4]

A triple sequences $(x, y) = (x_{r,u}, y_{r,u})$ is called convergent to M in pringsheim's sense for every $\epsilon > 0$, there exists $N(\epsilon) \in \mathbb{N}$ such that $|x_{r,u} - M| < \epsilon$, whenever $r \geq N, u \geq N, j \geq N$ and we write

$$\lim_{r,u,j \rightarrow \infty} x_{r,u,j} = M$$

Definition.2.3[3][4]

A triple sequence $(x_{r,u})$ called Cauchy sequence if $\forall \epsilon > 0, \exists N(\epsilon) \in \mathbb{N}$ such that

$$|x_{r,u,j} - x_{m,b,l}| < \epsilon \text{ whenever } r, m \geq N, u \geq b \geq N, j \geq l \geq N.$$

Definition.2.4[3]

A triple sequence $(x_{r,u})$ called bounded if $\exists \mu > 0$ such that

$$|x_{r,u}| < \mu \text{ for all } r, u \text{ and } j \in \mathbb{N}.$$

Note.2.1[3] A triple sequence $(x_{r,u})$ is convergent in pringsheim's sense may not be bounded.

Definition.2.5[4]

A triple sequence $(x_{r,u})$ is called convergent regularly if it is convergent in pringsheim's sense, and satisfy the following limit :

$$\lim_{j \rightarrow \infty} x_{r,u,j} = \alpha_{r,u}$$

$$\lim_{u \rightarrow \infty} x_{r,u,j} = \alpha_{r,j}$$

$$\lim_{r \rightarrow \infty} x_{r,u,j} = \alpha_{u,j}$$

Let Ω^3 denoted the family of all triple sequence spaces of complex or real numbers. Then the class of triple sequence $3c_0, 3c, 3l_\infty$ clearly these classes are linear spaces, then $3c_0 \subset 3c \subset 3l_\infty$

Theorem.2.1[3]

The spaces $3c_0, 3c, 3l_\infty$ with the normed $\|x\| = \sup_{r,u} |x_{r,u}| < \infty$ and

$\|y\| = \sup_{r,u} |y_{r,u}| < \infty$ are complete normed linear spaces.

proof. Simple.

By Kizmaz [6] we can introduced difference of single sequence spaces as following:

$$\Omega(\Delta) = \{(x_r) \in \Omega : (\Delta x_r) \in \Omega\} \text{ for } \Omega = c, c_0, l_\infty \text{ where } \Delta x_r = x_r - x_{r+1} \text{ for all } r \in \mathbb{N}.$$

For the differences of a double sequences $(\Delta x, \Delta y)$ is defined by

$$(\Delta x, \Delta y) = (\Delta x_{r,u}, \Delta y_{r,u})_{r,u=1}^\infty.$$

Let $\Omega^2(\Delta_{ru})$ be denote the difference triple sequence space, and a triple sequence space is defined as :

$$\Delta_{r,u,j} = x_{r,u,j} - x_{r,u+1,j} - x_{r,u,j+1} + x_{r,u+1,j+1} - x_{r+1,u,j+1} - x_{r+1,u+1,j} + x_{r+1,u,j+1} - x_{r+1,u+1,j+1} + x_{r+1,u+1,j+1}$$

Now, we introduced the p^{th} order difference triple Sequence the space as follows :

$$3c_0(\Delta^p) = \{(x_{r,u,j}) \in \Omega^3 : (\Delta^p x_{r,u,j}) \text{ is regularly null}\}.$$

$$3c(\Delta^p) = \{(x_{r,u,j}) \in \Omega^3 : (\Delta^p x_{r,u,j}) \text{ is convergent in pringsheim's sense } \}$$

$$3l_\infty(\Delta^p) = \{(x_{r,u,j}) \in \Omega^3 : (\Delta^p x_{r,u,j}) \text{ is bounded } \}.$$

By above forms of $(\Delta^p x_{r,u,j})$ the binomial explanation of generaliz difference triple sequence has the following mode:

$$\Delta^p x_{r,u,j} = \sum_{r=1}^p \sum_{u=1}^p \sum_{j=1}^p (-1)^{s+k+r} \binom{p}{r} \binom{p}{u} \binom{p}{j} x_{ru+uj+rj}, \text{ For all } r, u \text{ and } j \in N \text{ [3].}$$

When Δ^p is replaced by Δ , the above spaces becomes $3c_0(\Delta)$, $3c(\Delta)$, $3l_\infty(\Delta)$.

So, we can define the triple sequence spaces by using Kizmaz [6] as follows:

$$3l_\infty(\Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3l_\infty\}$$

$$3c_0(\Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3c_0\}$$

$$3c(\Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3c\}$$

Let U be the set of all triple sequence $\Lambda = (\Lambda_{r,u,j})$.

By Malkowsky[5] we can define difference triple sequence $3c$, $3c_0$, $3l_\infty$ as following forms

$$3l_\infty(\Lambda, \Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3l_\infty\}$$

$$3c(\Lambda, \Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3c\}, \text{ and}$$

$$3c_0(\Lambda, \Delta) = \{(x, y) \in \Omega^3 : (\Delta x, \Delta y) \in 3c\}$$

Also, we can defined these spaces $3c$, $3c_0$, $3l_\infty$ of difference triple sequence in Asma[1] as following :

$$3l_\infty(\Lambda, \Delta, F) = \{ (x, y) \in \Omega^3 : \sup_{r,u,j} [(F_1(\frac{|\Lambda_{r,u,j} \Delta x_{r,u,j}|}{\rho})) \vee (F_2(\frac{|\Lambda_{r,u,j} \Delta y_{r,u,j}|}{\rho}))] < \infty$$

For some $\rho > 0 \}$.

$$3c(\Lambda, \Delta, F) = \{ (x, y) \in \Omega^3 : p - \lim_{r,u,j \rightarrow \infty} [(F_1(\frac{|\Lambda_{r,u,j} \Delta x_{r,u,j} - l_1|}{\rho})) \vee (F_2(\frac{|\Lambda_{r,u,j} \Delta y_{r,u,j} - l_2|}{\rho}))] = 0, \text{ for some } \rho > 0, l_1, l_2 \in C \}$$

$$3c_0(\Lambda, \Delta, F) = \{ (x, y) \in \Omega^3 : p - \lim_{r,u,j \rightarrow \infty} [(F_1(\frac{|\Lambda_{r,u,j} \Delta x_{r,u,j}|}{\rho})) \vee (F_2(\frac{|\Lambda_{r,u,j} \Delta y_{r,u,j}|}{\rho}))] = 0 \text{ for some } p > 0 \}$$

Now, from idea of Orlicz sequence in Lindenstrauss and Tzafriri [10] we can construct a double Orlicz-functions as following:

$$3l_F = \{(x, y) \in \Omega^3 : \sum_{r=1}^\infty \sum_{u=1}^\infty \sum_{j=1}^\infty \left[(F_1(\frac{|x_{r,u,j}|}{\rho}) \vee F_2(\frac{|y_{r,u,j}|}{\rho})) \right] < \infty, \text{ for all } \rho > 0\}$$

$$l_{F_1} = \{ x \in \Omega^3 : \sum_{r=1}^\infty \sum_{u=1}^\infty \sum_{j=1}^\infty F_1(\frac{|x_{r,u,j}|}{\rho}) < \infty, \text{ for all } \rho > 0\}$$

And

$$l_{F_2} = \{ y \in \Omega^3 : \sum_{r=1}^\infty \sum_{u=1}^\infty \sum_{j=1}^\infty F_2(\frac{|y_{r,u,j}|}{\rho}) < \infty \text{ for all } \rho > 0\}$$

which is a Banach space with the norm :

$$\|(x, y)\|_F = \inf \left\{ \rho > 0 : \sum_{r=1}^\infty \sum_{u=1}^\infty \sum_{j=1}^\infty \left[\left(F_1(\frac{|x_{r,u,j}|}{\rho}) \vee F_2(\frac{|x_{r,u,j}|}{\rho}) \right) \right] \leq 1 \right\}$$

where

$$\|x\|_{F_1} = \inf \left\{ \rho > 0 : \sum_{r=1}^{\infty} \sum_{u=1}^{\infty} \sum_{j=1}^{\infty} F_1 \left(\frac{x_{r,u,j}}{\rho} \right) \leq 1 \right\}$$

$$\|y\|_{F_2} = \inf \left\{ \rho > 0 : \sum_{r=1}^{\infty} \sum_{u=1}^{\infty} \sum_{j=1}^{\infty} F_2 \left(\frac{y_{r,u,j}}{\rho} \right) \leq 1 \right\}$$

Bu Mursuleen [8] we define difference sequence spaces by a double Orlicz-functions as following:

$$3l_{\infty}(\Delta, F) = \left\{ (x, y) \in \Omega^3 : \sup_{r, u, j} \left[\left(F_1 \left(\frac{|\Delta x_{r,u,j}|}{\rho} \right) \right) \vee \left(F_2 \left(\frac{|\Delta y_{r,u,j}|}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0 \right\},$$

$$3c_0(\Delta, F) = \left\{ (x, y) \in \Omega^3 : \sup_{r, u, j} \left[\left(F_1 \left(\frac{|\Delta x_{r,u,j}|}{\rho} \right) \right) \vee \left(F_2 \left(\frac{|\Delta y_{r,u,j}|}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0 \right\},$$

$$3C(\Delta, F) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[\left(F_1 \left(\frac{|\Delta x_{r,u,j} - l_1|}{\rho} \right) \right) \vee \left(F_2 \left(\frac{|\Delta y_{r,u,j} - l_2|}{\rho} \right) \right) \right] = 0 \text{ for some } \rho > 0, l_{\infty}, l_{\infty} \in C \right\},$$

where $F = (F_1(x), F_2(y))$ is a double Orlicz-functions, these spaces are a Banach spaces with the norm:

$$\|(x, y)\|_{\Delta} = \inf \left\{ \rho > 0 : \sup_{r, u, j} \left[\left(F_1 \left(\frac{|\Delta x_{r,u,j}|}{\rho} \right) \right) \vee \left(F_2 \left(\frac{|\Delta y_{r,u,j}|}{\rho} \right) \right) \right] \leq 1 \right\}$$

$$\|x\|_{\Delta} = \inf \left\{ \rho > 0 : \sup_{r, u, j} F_1 \left(\frac{|\Delta x_{r,u,j}|}{\rho} \right) \leq 1 \right\}$$

$$\|y\|_{\Delta} = \inf \left\{ \rho > 0 : \sup_{r, u, j} F_2 \left(\frac{|\Delta y_{r,u,j}|}{\rho} \right) \leq 1 \right\}$$

Note.2.2. Throughout this study let $F_1 = F_{1,r,u}, F_2 = F_{2,r,u}$ and $(F_1, F_2) = F = F_{r,u} = (F_{1,r,u}, F_{2,r,u})$

Definition.2.6.

Let F, F_1 and F_2 be a double Orlicz-functions, and v be a positive integer, we use the notation $(\Delta_u^v x_{r,u,j})$ for $(\Delta_{r,u,j}^v x_{r,u,j})$, $(\Delta_u^v y_{r,u,j})$ for $(\Delta_{r,u,j}^v y_{r,u,j})$ and $(\Delta_u^v x_{r,u,j}, \Delta_u^v y_{r,u,j})$ for $(\Delta_{r,u,j}^v x_{r,u,j}, \Delta_{r,u,j}^v y_{r,u,j})$ we define :

$$3l_{\infty}(\Delta_u^v, F_{r,u}, \varphi) = \left\{ (x, y) \in \Omega^3 : \sup_{r, u, j} (ruj)^{-\varphi} \left[\left(F_{1,r,u} \left(\frac{|\Delta_u^v x_{r,u,j}|}{\rho} \right) \right) \vee \left(F_{2,r,u} \left(\frac{|\Delta_u^v y_{r,u,j}|}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0, \varphi \geq 0 \right\}.$$

$$3c_0(\Delta_u^v, F_{r,u}, \varphi) =$$

$$\left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} (ruj)^{-\varphi} \left[\left(F_{1,r,u} \left(\frac{|\Delta_u^v x_{r,u,j} - l_1|}{\rho} \right) \right) \vee \left(F_{2,r,u} \left(\frac{|\Delta_u^v y_{r,u,j} - l_2|}{\rho} \right) \right) \right] = 0 \text{ for some } \rho > 0, \varphi \geq 0 \right\}$$

And :

$$3c(\Delta_u^v, F_{r,u}, \varphi) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} (ruj)^{-\varphi} \left[\left(F_{1,r,u} \left(\frac{|\Delta_u^v x_{r,u,j} - l_1|}{\rho} \right) \right) \vee \left(F_{2,r,u} \left(\frac{|\Delta_u^v y_{r,u,j} - l_2|}{\rho} \right) \right) \right] = 0, \text{ for some } \rho > 0, \right.$$

$$\left. l_1, l_2 \in C, \varphi \geq 0 \right\}.$$

Main Results :

Theorem.2.2 $3l_{\infty}(\Delta_u^v, F_{r,u}, \varphi)$ is a Banach space with norm

$$\| (x, y) \|_{\Delta_u^v} = \inf \left\{ \rho > 0 : \sup_{r, u, \square} (ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{|\Delta_u^v x_{r, u, \square}^i|}{\rho} \right) \right) \vee \left(F_{2, r, u} \left(\frac{|\Delta_u^v y_{r, u, \square}^i|}{\rho} \right) \right) \right] \leq 1 \right\},$$

where

$$\| (x) \|_{\Delta_u^v} = \inf \left\{ \rho > 0 : \sup_{r, u, j} (ruj)^{-\varphi} F_{1, r, u} \left(\frac{|\Delta_u^v x_{r, u, \square}^i|}{\rho} \right) \leq 1 \right\}$$

$$\| (y) \|_{\Delta_u^v} = \inf \left\{ \rho > 0 : \sup_{r, u, j} (ruj)^{-\varphi} F_{2, r, u} \left(\frac{|\Delta_u^v y_{r, u, \square}^i|}{\rho} \right) \leq 1 \right\}$$

Proof . Let (x^i, y^i) be any triple Cauchy sequence in $3\mathbb{I}_\infty(\Delta_u^v, F_{r, u}, \varphi)$

Such that (x^i) and (y^i) be a Cauchy sequence in $3\mathbb{I}_\infty(\Delta_u^v, F_{1, r, u}, \varphi)$,

$3\mathbb{I}_\infty(\Delta_u^v, F_{2, r, u}, \varphi)$ respectively, where.

$$(x^i, y^i) = (x^i_{r, u, \square}, y^i_{r, u, \square}) = ((x^i_{1,1,1}, y^i_{1,1,1}), (x^i_{2,2,2}, y^i_{2,2,2}), \dots) \in$$

$3\mathbb{I}_\infty(\Delta_u^v, r, u, \varphi)$ for all $i \in \mathbb{N}$

Let $m_1, m_2 > 0$ be fixed, then for all $\frac{\epsilon}{m_1 m_2} > 0$

There exists a positive integers N such that

$$(\|x^i - x^t\|_{\Delta_u^v}, \|y^i - y^t\|_{\Delta_u^v}) < \frac{\epsilon}{m_1 m_2}, \text{ for all } i, t \geq N$$

Using the definition of norm, we have,

$$\sup_{r, u, j} (ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{|\Delta_u^v x^i_{r, u, \square} - \Delta_u^v x^t_{r, u, \square}|}{\|x^i - x^t\|_{\Delta_u^v}} \right) \right) \vee \left(F_{2, r, u} \left(\frac{|\Delta_u^v y^i_{r, u, \square} - \Delta_u^v y^t_{r, u, \square}|}{\|y^i - y^t\|_{\Delta_u^v}} \right) \right) \right]$$

≤ 1 , for all $i, t \geq N$

$$\text{Thus } (ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{|\Delta_u^v x^i_{r, u, \square} - \Delta_u^v x^t_{r, u, \square}|}{\|x^i - x^t\|_{\Delta_u^v}} \right) \right) \vee \left(F_{2, r, u} \left(\frac{|\Delta_u^v y^i_{r, u, \square} - \Delta_u^v y^t_{r, u, \square}|}{\|y^i - y^t\|_{\Delta_u^v}} \right) \right) \right] \leq 1$$

For all $r, u, j \geq 0$, and for all $i, t \geq N$.

Therefore one can find that exists $m_1, m_2 > 0$ with

$$(ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{m_1 m_2}{2} \right) \right) \vee \left(F_{2, r, u} \left(\frac{m_1 m_2}{2} \right) \right) \right] \geq 1 \text{ such that}$$

$$(ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{|\Delta_u^v x^i_{r, u, \square} - \Delta_u^v x^t_{r, u, \square}|}{\|x^i - x^t\|_{\Delta_u^v}} \right) \right) \vee \left(F_{2, r, u} \left(\frac{|\Delta_u^v y^i_{r, u, \square} - \Delta_u^v y^t_{r, u, \square}|}{\|y^i - y^t\|_{\Delta_u^v}} \right) \right) \right] \leq$$

$$(ruj)^{-\varphi} \left[\left(F_{1, r, u} \left(\frac{m_1 m_2}{2} \right) \right) \vee \left(F_{2, r, u} \left(\frac{m_1 m_2}{2} \right) \right) \right].$$

The implies that

$$\left| (\Delta_u^v x^i_{r, u, \square} - \Delta_u^v x^t_{r, u, \square}, \Delta_u^v y^i_{r, u, \square} - \Delta_u^v y^t_{r, u, \square}) \right| \leq \frac{m_1 m_2}{2} \frac{\epsilon}{m_1 m_2} = \frac{\epsilon}{2} \text{ since } \Delta_{r, u, \square} \neq 0 \text{ for all } r, u \text{ and } j \text{ we get } \left| \Delta_u^v x^i_{r, u, \square} - \Delta_u^v x^t_{r, u, \square}, \Delta_u^v y^i_{r, u, \square} - \Delta_u^v y^t_{r, u, \square} \right| \leq \frac{\epsilon}{2} \text{ for all } i, t \geq N$$

Hence $(\Delta^v x_{r,u,j}^i), (\Delta^v y_{r,u,j}^i)$ is a Cauchy sequence in R such that $(\Delta_u^v x_{r,u,j}^i, \Delta_u^v y_{r,u,j}^i)$ are triple Cauchy sequence in $R \times R \times R$.

Therefore each $0 < \epsilon < 1$, there exist a positive integer N such that

$$|(\Delta^v x_{r,u,j}^i - \Delta^v x_{r,u,j}, \Delta^v y_{r,u,j}^i - \Delta^v y_{r,u,j})| \leq \epsilon \text{ for all } i \geq N$$

Now, using the continuity of $F_{1,r,u}, F_{2,r,u}$ for each r, u we get :

$$\sup_{r,u,j \geq N} (ruj)^{-\varphi} \left[(F_{1,r,u} \left(\frac{\Delta_u^v x_{r,u,j}^i - \lim_{t \rightarrow \infty} \Delta_u^v x_{r,u,j}^i}{\rho} \right)) \vee (F_{2,r,u} \left(\frac{\Delta_u^v y_{r,u,j}^i - \lim_{t \rightarrow \infty} \Delta_u^v y_{r,u,j}^i}{\rho} \right)) \right] \leq 1$$

$$\text{Thus, } \sup_{r,u,j \geq N} (ruj)^{-\varphi} \left[(F_{1,r,u} \left(\frac{|\Delta_u^v x_{r,u,j}^i - \Delta_u^v x_{r,u,j}|}{\rho} \right)) \vee (F_{2,r,u} \left(\frac{|\Delta_u^v y_{r,u,j}^i - \Delta_u^v y_{r,u,j}|}{\rho} \right)) \right] \leq 1$$

Taking infimum of ρ^s and we have

$$\inf \left\{ \rho > 0 : \sup_{r,u,j \geq N} (ruj)^{-\varphi} \right.$$

$$\left. \left[(F_{1,r,u} \left(\frac{|\Delta_u^v x_{r,u,j}^i - \Delta_u^v x_{r,u,j}|}{\rho} \right)) \vee (F_{2,r,u} \left(\frac{|\Delta_u^v y_{r,u,j}^i - \Delta_u^v y_{r,u,j}|}{\rho} \right)) \right] \leq 1 \right\} \leq \epsilon \text{ for all } i \geq N$$

Since $(x^i, y^i) \in 3l_\infty(\Delta_u^v, F_{r,u}, \varphi)$ and F_1, F_2 be a double Orlicz-functions, then

$F = (F_1, F_2)$ is a double Orlicz-functions for each r, u and there for continuous, we get that $(x, y) \in 3l_\infty(\Delta_u^v, F_{r,u}, \varphi)$

The rest of proof $3c, 3c_0$ is like the previous case $3l_\infty$. ■

Theorem.2.3. The classes of sequences $3l_\infty(\Delta_u^v, F, \varphi), 3c(\Delta_u^v, F, \varphi), 3c_0(\Delta_u^v, F, \varphi)$ are linear spaces.

Proof. Obvious.

Lemma 2.1. Let F_1, F_2 and F be a Orlicz-functions. Then

$$i) 3l_\infty(\Delta_u^0, F_{r,u}, \varphi) \subset 3l_\infty(\Delta_u^v, F_{r,u}, \varphi)$$

$$ii) 3c(\Delta_u^0, F_{r,u}, \varphi) \subset 3c(\Delta_u^v, F_{r,u}, \varphi)$$

$$iii) 3c_0(\Delta_u^0, F_{r,u}, \varphi) \subset 3c_0(\Delta_u^v, F_{r,u}, \varphi)$$

Proof. it's clear.

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