

Exponential Functions of Al-Tememe Acceleration Methods for Improving the Values of Integrations Numerically of First Kind

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Abstract: The main aim of this work is to introduce acceleration methods called an exponential acceleration methods which are of series of numerated methods. In general, these methods named as AL-Tememe's acceleration methods of first kind to his discoverer "Ali Hassan Mohammed". These are very beneficial to acceleration the numerical results for definite integrations with continuous integrands which are of 2nd order main error, with respect to the accuracy and the number of the used subintervals and the fasting obtaining results. Especially, for acceleration the results which are obviously obtained by trapezoidal and midpoint methods. Moreover, these methods could be enhancing the results of the ordinary differential equations numerically which are of 2nd order main error.

1. INTRODUCTION

There are numerical methods for calculating single integrals that are bounded in their integration intervals such as:

1. Trapezoidal Rule
2. Midpoint Rule
3. Simpson's Rule

Which are called “Newton-cotes formulas”.

In this paper, we introduce two methods which are trapezoidal and midpoint methods for finding an approximate values of single integrals which integrands are continuous in interval of integration using exponential acceleration methods which are part of a series of AL-Tememe's acceleration methods of first kind. We will make a comparison of these methods as an accuracy and fasting reaching of its values with the real values (analytic) for those integrals.

Consider the integral J defined as:

J=

$$\int_{x_0}^{x_m} f(x) dx$$

Such that, f(x) is a continuous function defined on [x₀, x_m]. We need to calculate the integral J approximately. In general we can write Newton-cotes formula as:

$$J = \int_{x_0}^{x_m} f(x) dx = f(x) dx = (h) + E_{G(h)} + R_G \quad \dots (2)$$

Here, G(h) is Lagrangian approximation to the value of the integral J, (the letter G symbolizes the rule type), E_G is the remainder and related to amputation after the use of certain terms of E_G(h) and h = $\frac{x_m - x_0}{m}$; m is number of sub intervals used and the general form of G(h) is :

$$G(h) = h(w_0 f_0 + w_1 f_1 + w_2 f_2 + \dots + w_{m-2} f_{m-2} + w_1 f_{m-1} + w_0 f_m) \quad \dots (3)$$

Where f_r = f(x_r) and x_r = x₀ + rh ; r = 0,1,2,...,m and weight coefficients w_r take the sequence

(w₀, w₁, w₂, ..., w₂, w₁, w₀). To simplify the formula (3), we write weights by w₀ such that w₁=2(1-w₀), w₂=2w₀, we note that when w₀ = $\frac{1}{2}$ we get the trapezoidal rule and refer for G(h) by T(h) where T(h) = $\frac{h}{2}(f_0 + 2f_1 + 2f_2 + \dots + 2f_{m-1} + f_m)$. When w₀=0, we get the midpoint rule and we refer to it by M(h) where M(h)=h(f₁+f₃+...+f_{2i-1}); i=1,2,...,m. The general formula of E_G(h) is the following:

1- For Trapezoidal rule:

$$E_T(h) = \frac{-1}{12} h^2 (f_m' - f_0') + \frac{1}{720} h^4 (f_m^{(3)} - f_0^{(3)}) - \frac{1}{30240} h^6 (f_m^{(5)} - f_0^{(5)}) + \dots \dots (4)$$

2- Midpoint rule:

$$E_M(h) = \frac{1}{6} h^2 (f_m' - f_0') - \frac{7}{360} h^4 (f_m^{(3)} - f_0^{(3)}) + \frac{31}{15120} h^6 (f_m^{(5)} - f_0^{(5)}) \dots \dots (5)$$

In these methods when the integrands is a continuous function and their derivatives are in each point of integration points interval [x₀, x_m] the error formula can be written as:

$$\begin{aligned} J - T(h) &= A_1 h^2 + A_2 h^4 + A_3 h^6 + \dots \\ J - M(h) &= B_1 h^2 + B_2 h^4 + B_3 h^6 + \dots \\ \dots (6) (1) \end{aligned}$$

Where A₁, A₂, A₃, ... and B₁, B₂, B₃, ... are constants that do not depend on h but on the values of their derivatives at the end of integration interval.

[1]

2. DERIVATION AL-TEMEME'S ACCELERATION OF EXPONENTIAL FUNCTIONS:

A series of acceleration methods of Al-Tememe's are introduced and we will call it exponential accelerations. Due to the similar error of both trapezoidal and midpoint methods regarding h basics, we will deal with the error for trapezoidal method to derive our acceleration methods following the same way to derive these methods as for the midpoint method.

In Trapezoidal rule:

$$J = \int_{x_0}^{x_m} f(x)dx = h \left[\frac{1}{2} f_0 + f_1 + f_2 + \dots + f_{m-1} + \frac{1}{2} f_m \right] + E(h) \quad \dots(7)$$

Where

$$E(h) = A_1 h^2 + A_2 h^4 + A_3 h^6 + \dots ; A_1, A_2, \dots \text{ are constants} \quad \dots(8)$$

$$= h^2 (A_1 + A_2 h^2 + A_3 h^4 + \dots) \cong h^2 e^{h^2}; e^{h^2} = 1 + h^2 + \frac{h^4}{2} + \dots \quad \dots(9)$$

So, we will have:

$$\begin{aligned} J & \cong \frac{h}{2} [f_0 + f_1 + \dots + f_m] \\ & + h^2 e^{h^2} \end{aligned} \quad \dots(10)$$

Therefore,

$$E = J - T(h) \cong h^2 e^{h^2} \quad \dots(11)$$

We assume that $T_1(h_1)$ represents the value of the above mentioned integration numerically when $h=h_1$, also, $T_2(h_2)$ represents the value of integration numerically when $h=h_2$, So;

$$\begin{aligned} J - T_1(h_1) & \cong h_1^2 e^{h_1^2} \\ J - T_2(h_2) & \cong h_2^2 e^{h_2^2} \end{aligned} \quad \dots(12)$$

From the equations (12) and (13) we get:

$$\begin{aligned} A_{e^{h^2}}^F & \cong \frac{h_2^2 e^{h_2^2} T_1(h_1) - h_1^2 e^{h_1^2} T_2(h_2)}{h_2^2 e^{h_2^2} - h_1^2 e^{h_1^2}} \end{aligned} \quad \dots(14)$$

The formula (14) is called the first exponential acceleration rule for Al-Tememe of the first kind. We refer to it by $(A_{e^{h^2}}^F)$. Also is possible to write the equation (11) by the formula:

$$E = J - T(h) \cong h^2 e^{-h^2} \quad [2]$$

Following the same above method, we get;

$$\begin{aligned} A_{e^{-h^2}}^F & \cong \frac{h_2^2 e^{-h_2^2} T_1(h_1) - h_1^2 e^{-h_1^2} T_2(h_2)}{h_2^2 e^{-h_2^2} - h_1^2 e^{-h_1^2}} \end{aligned}$$

The formula (15) is called the second exponential acceleration rule for Al-Tememe of the first kind, that referred to by $(A_{e^{-h^2}}^F)$. In the same way, we can conclude the third exponential acceleration rule, that referred to as $(A_{(e^{h^2}-1)}^F)$ and the fourth exponential acceleration rule, that referred to by $(A_{(e^{-h^2}-1)}^F)$, respectively:

$$\begin{aligned} A_{(e^{h^2}-1)}^F & \cong \frac{(e^{h^2}-1) T_1(h_1) - (e^{h^2}-1) T_2(h_2)}{e^{h_2^2} - e^{h_1^2}} \end{aligned} \quad \dots(16)$$

$$\begin{aligned} A_{(e^{-h^2}-1)}^F & \cong \frac{(e^{-h_2^2}-1) T_1(h_1) - (e^{-h_1^2}-1) T_2(h_2)}{e^{-h_2^2} - e^{-h_1^2}} \end{aligned} \quad \dots(17)$$

This is possible to write the equation (11) in the following both formulas:

$$J - T(h) \cong e^{h^2} - 1$$

$$J - T(h) \cong e^{-h^2} - 1$$

3. EXAMPLES:

We will review some integrals that have continuous integrands on the interval of integration using exponential acceleration methods of Al-Tememe to improve the results numerically:

3.1: $\int_0^1 \frac{dx}{1+e^{-x}}$ and its analytic value is 0.62011450695828 is rounded to 14 decimal .

3.2: $\int_1^2 \sqrt{x} \sqrt{1+x} dx$ and its analytic value is 1.93499144475889 is rounded to 14 decimal.

3.3: $\int_3^4 \ln(x) dx$ and its analytic value is 1.24934057847523 is rounded to 14 decimal .

We will compare the values of the acceleration methods with values of trapezoidal rule and midpoint rule. The priority of the acceleration methods can be calculated based on n values, n=1,2,3,..., and the results we adopted in Mat lab program through putting Eps=10⁻¹⁰ that represents (the absolute error of the subsequent value- previous value).

4.THE RESULTS:

The integrands of integration $\int_0^1 \frac{dx}{1+e^{-x}}$ is continuous in the integration interval [0,1] and the formula of the correction errors for the (trapezoidal and midpoint rules) identical for the formula in the equation (8).

1.Regarding trapezoidal rule with the exponential acceleration methods, we note in table (1) the following: when n=42,...,50 the values in the $A_{e^{h^2}}^F$ method are valid for nine decimal order, while the trapezoidal value without acceleration was valid for only five decimal order when n=51, also the same accuracy is obtained for all remaining ... (15) acceleration methods with a simple variation in n values.

2-For the base midpoint rule with the exponential acceleration methods, we observe from table (2): When n=39,...,44 the values in the method $A_{e^{h^2}}^F$ are correct for nine decimal order but we note that the value in the midpoint method without acceleration was valid for only five decimal order when n=45, also the same accuracy is obtained for all remaining acceleration methods with a simple variation in n values.

To find the value of integration $\int_1^2 \sqrt{x} \sqrt{1+x} dx$ numerically, we note that the integrand is continuous in the integration interval [1,2] and that the formula of the correction terms for the two (trapezoidal, midpoint) rules, respectively, is as in formula equation(8).

1-For the base trapezoidal rule with exponential acceleration methods, we observe the following in Table (3): when $n = 26, \dots, 39$ the value of the $A^F_{e^{h^2}}$ acceleration is valid for eight decimal order, but note that the trapezoidal value without acceleration was valid for only five decimal order when $n=39$, and we obtained the same accuracy for the other exponential acceleration rules with a simple variation in n values.

2-For the base midpoint with the exponential acceleration methods we observe from table(4): that when $n=19, \dots, 29$ the values of the $A^F_{e^{h^2}}$ acceleration is correct for eight decimal order, but note that the value in the midpoint method without acceleration was correct for only five decimal order when $n=34$, and we obtained the same accuracy for the other exponential acceleration laws with simple variations in n values.

To find the value of integration $\int_3^4 \ln(x)dx$ numerically, where that the integrand is continuous in the integration interval [3,4] and that the formula of the correction terms for

5.Tables:

n	Values of trapezoidal rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	0.61552928931500				
2	0.61899431025843	0.61945829270521	0.62288992866497	0.61968048743652	0.62085953321274
3	0.61961862151438	0.62001244862199	0.62027014655894	0.62005925623077	0.62018440768970
4	0.61983595514451	0.62008682145636	0.62014937572799	0.62010030060460	0.62013136623690
5	0.61993634681867	0.62010421050621	0.62012657756954	0.62010933771904	0.62012049467356
6	0.61999082715453	0.62010985574143	0.62011977252244	0.62011219746687	0.62011715068328
7	0.62002365896822	0.62011211173953	0.62011716569858	0.62011332515668	0.62011585080012
8	0.62004496080112	0.62011315180194	0.62011599296314	0.62011384097298	0.62011526113291
9	0.62005956196185	0.62011368399273	0.62011540211085	0.62011410358676	0.62011496249238
10	0.62007000443790	0.62011397883111	0.62011507817584	0.62011424858069	0.62011479819035
11	0.62007772980485	0.62011415272113	0.62011488853154	0.62011433388789	0.62011470176504
12	0.62008360507662	0.62011426059635	0.62011477151902	0.62011438671489	0.62011464216271
13	0.62008817711874	0.62011433036828	0.62011469614675	0.62011442083625	0.62011460371856
14	0.62009180470690	0.62011437710272	0.62011464582109	0.62011444366729	0.62011457802274
15	0.62009473113914	0.62011440935258	0.62011461118009	0.62011445940911	0.62011456032075
16	0.62009712612925	0.62011443218550	0.62011458670368	0.62011447054684	0.62011454780470
17	0.62009911098373	0.62011444871633	0.62011456901217	0.62011447860605	0.62011453875322
18	0.62010077427289	0.62011446092160	0.62011455596771	0.62011448455373	0.62011453207632
19	0.62010218188723	0.62011447009106	0.62011454617898	0.62011448902035	0.62011452706401
20	0.62010338366579	0.62011447708731	0.62011453871741	0.62011449242727	0.62011452324212
21	0.62010441786976	0.62011448250010	0.62011453294936	0.62011449506237	0.62011452028687
22	0.62010531426626	0.62011448674064	0.62011452843370	0.62011449712630	0.62011451797274
23	0.62010609629469	0.62011449010078	0.62011452485773	0.62011449876140	0.62011451613982
24	0.62010678261484	0.62011449279102	0.62011452199623	0.62011450007028	0.62011451467284
25	0.62010738823049	0.62011449496541	0.62011451968449	0.62011450112802	0.62011451348753
26	0.62010792531652	0.62011449673823	0.62011451780048	0.62011450199029	0.62011451252139
27	0.62010840383560	0.62011449819526	0.62011451625262	0.62011450269888	0.62011451172755
28	0.62010883200363	0.62011449940166	0.62011451497143	0.62011450328552	0.62011451107040
29	0.62010921664476	0.62011450040743	0.62011451390363	0.62011450377455	0.62011451052264

the two (trapezoidal, midpoint) rules, respectively, is as it he formula in equation(8).

1- For the base trapezoidal rule with exponential acceleration methods, we observe the following in table (5): When $n = 28, \dots, 48$ the values of the $A^F_{e^{h^2}}$ acceleration is valid for eight decimal order, but note that the trapezoidal value without acceleration was valid for only five decimal order when $n=48$, and we obtained the same accuracy for the other exponential acceleration rules with a simple variation in n values.

2- For the base midpoint with the exponential acceleration methods we observe from table(6): when $n = 40, 41, 42$ the values of the $A^F_{e^{h^2}}$ acceleration is correct for eight decimal order, but note that the value in the midpoint method without acceleration was correct for only five decimal order when $n=42$, and we obtained the same accuracy for the other exponential acceleration rules with simple variations in n values.

30	0.62010956346499	0.62011450125130	0.62011451300794	0.62011450418483	0.62011451006314
31	0.62010987726507	0.62011450196357	0.62011451225211	0.62011450453110	0.62011450967536
32	0.62011016210750	0.62011450256811	0.62011451161074	0.62011450482497	0.62011450934628
33	0.62011042144869	0.62011450308389	0.62011451106363	0.62011450507568	0.62011450906554
34	0.62011065824434	0.62011450352611	0.62011451059464	0.62011450529062	0.62011450882489
35	0.62011087503400	0.62011450390700	0.62011451019076	0.62011450547574	0.62011450861762
36	0.62011107400946	0.62011450423649	0.62011450984144	0.62011450563588	0.62011450843835
37	0.62011125707037	0.62011450452268	0.62011450953805	0.62011450577496	0.62011450828264
38	0.62011142586976	0.62011450477223	0.62011450927355	0.62011450589623	0.62011450814688
39	0.62011158185147	0.62011450499062	0.62011450904210	0.62011450600235	0.62011450802809
40	0.62011172628115	0.62011450518241	0.62011450883886	0.62011450609555	0.62011450792377
41	0.62011186027199	0.62011450535140	0.62011450865981	0.62011450617766	0.62011450783187
42	0.62011198480625	0.62011450550074	0.62011450850157	0.62011450625022	0.62011450775063
43	0.62011210075334	0.62011450563313	0.62011450836130	0.62011450631454	0.62011450767863
44	0.62011220888498	0.62011450575084	0.62011450823663	0.62011450637174	0.62011450761463
45	0.62011230988812	0.62011450585575	0.62011450812549		0.62011450755758
46	0.62011240437585	0.62011450594952	0.62011450802617		
47	0.62011249289674	0.62011450603353	0.62011450793720		
48	0.62011257594282	0.62011450610897	0.62011450785731		
49	0.62011265395643	0.62011450617687	0.62011450778539		
50	0.62011272733612	0.62011450623813	0.62011450772055		
51	0.62011279644177		0.62011450766191		

Table (1) to calculate integration $\int_0^1 \frac{dx}{1+e^{-x}} = 0.62011450695828$ by using trapezoidal rule with the exponential acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	0.62245933120186				
2	0.62067760003060	0.62043901792623	0.61867445348124	0.62032476427948	0.61971849301990
3	0.62036303279469	0.62016459795391	0.62003475354225	0.62014101333971	0.62007795417150
4	0.62025396645772	0.62012807213716	0.62009668000855	0.62012130778448	0.62010571786074
5	0.62020366205713	0.6201954868562	0.62010834096622	0.62011697953464	0.62011138899226
6	0.62017638299870	0.62011678373735	0.62011181826780	0.62011561120300	0.62011313105897
7	0.62015995044558	0.62011567921287	0.62011314967090	0.62011507188908	0.62011380778681
8	0.62014929145739	0.62011517011790	0.62011374846072	0.62011482527124	0.62011411465314
9	0.62014198658393	0.62011490965531	0.62011405009109	0.62011469973493	0.62011427002958
10	0.62013676289368	0.62011476537050	0.62011421543997	0.62011463043236	0.62011435549846
11	0.62013289872767	0.62011468027980	0.62011431223339	0.62011458966165	0.62011440565248
12	0.62012996015306	0.62011462749515	0.62011437195218	0.62011456441573	0.62011443665100
13	0.62012767351430	0.62011459335617	0.62011441041752	0.62011454810996	0.62011445664410
14	0.62012585930037	0.62011457048995	0.62011443609966	0.62011453719996	0.62011447000669
15	0.62012439579083	0.62011455471110	0.62011445377712	0.62011452967782	0.62011447921188
16	0.62012319808574	0.62011454353988	0.62011446626724	0.62011452435584	0.62011448572014
17	0.62012220550495	0.62011453545213	0.62011447529490	0.62011452050496	0.62011449042672
18	0.62012137374602	0.62011452948075	0.62011448195116	0.62011451766306	0.62011449389850
19	0.62012066985229	0.62011452499467	0.62011448694604	0.62011451552886	0.62011449650469
20	0.62012006889650	0.62011452157185	0.62011449075339	0.62011451390102	0.62011449849189
21	0.62011955174274	0.62011451892373	0.62011449369658	0.62011451264197	0.62011450002845
22	0.62011910350371	0.62011451684913	0.62011449600071	0.62011451165583	0.62011450123166
23	0.62011871245700	0.62011451520526	0.62011449782534	0.62011451087459	0.62011450218466

24	0.62011836927080	0.62011451388912	0.62011449928541	0.62011451024922	0.62011450294738
25	0.62011806644176	0.62011451282536	0.62011450046496	0.62011450974384	0.62011450356366
26	0.62011779788138	0.62011451195806	0.62011450142626	0.62011450933186	0.62011450406597
27	0.62011755860751	0.62011451124526	0.62011450221604	0.62011450899331	0.62011450447871
28	0.62011734451160	0.62011451065506	0.62011450286974	0.62011450871302	0.62011450482037
29	0.62011715218108	0.62011451016302	0.62011450341458	0.62011450847938	0.62011450510516
30	0.62011697876259	0.62011450975019	0.62011450387158	0.62011450828335	0.62011450534406
31	0.62011682185546	0.62011450940174	0.62011450425723	0.62011450811791	0.62011450554567
32	0.62011667942822	0.62011450910599	0.62011450458448	0.62011450797751	0.62011450571676
33	0.62011654975246	0.62011450885366	0.62011450486363	0.62011450785773	0.62011450586272
34	0.62011643135020	0.62011450863732	0.62011450510292	0.62011450775503	0.62011450598783
35	0.62011632295154	0.62011450845098	0.62011450530899	0.62011450766659	0.62011450609559
36	0.62011622346050	0.62011450828980	0.62011450548723	0.62011450759008	0.62011450618880
37	0.62011613192715	0.62011450814979	0.62011450564203	0.62011450752363	0.62011450626975
38	0.62011604752494	0.62011450802770	0.62011450577698	0.62011450746569	0.62011450634032
39	0.62011596953187	0.62011450792087	0.62011450589507		0.62011450640209
40	0.62011589731509	0.62011450782704	0.62011450599877		
41	0.62011583031795	0.62011450774438	0.62011450609013		
42	0.62011576804930	0.62011450767131	0.62011450617086		
43	0.62011571007441	0.62011450760654	0.62011450624243		
44	0.62011565600739	0.62011450754896	0.62011450630604		
45	0.62011560550475		0.62011450636275		

Table (2) to calculate integration $\int_0^1 \frac{dx}{1+e^{-x}} = 0.62011450695828$ by using midpoint rule with the exponential acceleration methods of AL-Tememe of the first kind

n	Values of trapezoidal rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	1.93185165257814				
2	1.93417166284092	1.93448232302332	1.93677998031739	1.93463109385772	1.93542052582531
3	1.93462365557782	1.93490878097086	1.93509535072225	1.93494266903390	1.93503327696369
4	1.93478385141368	1.93496876406844	1.93501487259671	1.93497869950005	1.93500159786571
5	1.93485836979783	1.93498297107436	1.93499957362065	1.93498677688414	1.93499505842969
6	1.93489894956583	1.93498760820503	1.93499499473572	1.93498935244309	1.93499304185450
7	1.93492345255438	1.93498946649023	1.93499323835315	1.93499037208585	1.93499225702016
8	1.93493936999620	1.93499032459215	1.93499244760280	1.93499083956379	1.93499190075478
9	1.93495028936698	1.93499076413013	1.93499204901214	1.93499107792045	1.93499172024670
10	1.93495810314529	1.93499100780877	1.93499183041406	1.93499120965395	1.93499162090967
11	1.93496388618825	1.93499115159719	1.93499170240899	1.93499128721472	1.93499156259961
12	1.93496828565385	1.93499124083105	1.93499162341534	1.93499133526994	1.93499152655197
13	1.93497171006351	1.93499129856201	1.93499157252609	1.93499136632153	1.93499150329838
14	1.93497442759613	1.93499133723949	1.93499153854430	1.93499138710496	1.93499148775457
15	1.93497662020052	1.93499136393405	1.93499151515159	1.93499140143847	1.93499147704566
16	1.93497841484464	1.93499138283640	1.93499149862190	1.93499141158180	1.93499146947363
17	1.93497990230967	1.93499139652304	1.93499148667366	1.93499141892262	1.93499146399737
18	1.93498114889518	1.93499140662930	1.93499147786352	1.93499142434087	1.93499145995763
19	1.93498220393414	1.93499141422241	1.93499147125203	1.93499142841035	1.93499145692493
20	1.93498310474827	1.93499142001630	1.93499146621222	1.93499143151464	1.93499145461245
21	1.93498387999359	1.93499142449910	1.93499146231616	1.93499143391585	1.93499145282429
22	1.93498455196731	1.93499142801124	1.93499145926598	1.93499143579674	1.93499145142404
23	1.93498513822878	1.93499143079431	1.93499145685048	1.93499143728690	1.93499145031493

24	1.93498565275813	1.93499143302261	1.93499145491755	1.93499143847982	1.93499144942726
25	1.93498610679733	1.93499143482371	1.93499145335598	1.93499143944391	1.93499144871002
26	1.93498650946936	1.93499143629220	1.93499145208330	1.93499144022985	1.93499144812539
27	1.93498686824005	1.93499143749914	1.93499145103769	1.93499144087574	1.93499144764500
28	1.93498718926661	1.93499143849850	1.93499145017222	1.93499144141050	1.93499144724735
29	1.93498747766346	1.93499143933168	1.93499144945089	1.93499144185629	1.93499144691589
30	1.93498773770725	1.93499144003076	1.93499144884581	1.93499144223030	1.93499144663782
31	1.93498797299629	1.93499144062082	1.93499144833523	1.93499144254597	1.93499144640316
32	1.93498818657565	1.93499144112163	1.93499144790194	1.93499144281386	1.93499144620401
33	1.93498838103620	1.93499144154895	1.93499144753236	1.93499144304244	1.93499144603414
34	1.93498855859355	1.93499144191529	1.93499144721552	1.93499144323838	
35	1.93498872115142	1.93499144223086	1.93499144694268	1.93499144340716	
36	1.93498887035289	1.93499144250381	1.93499144670667		
37	1.93498900762206	1.93499144274096	1.93499144650176		
38	1.93498913419817	1.93499144294767	1.93499144632304		
39	1.93498925116368	1.93499144312861			

Table (3) to calculate integration $\int_1^2 \sqrt{x} \sqrt{1+x} dx = 1.93499144475889$ by using trapezoidal rule with the exponential acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	1.93649167310371				
2	1.93539603998644	1.93524932960782	1.93416425275971	1.93517907205092	1.93480625998995
3	1.93517424355384	1.93503433025466	1.93494277899308	1.93501770111247	1.93497323908034
4	1.93509488857872	1.93500328982374	1.93498044939787	1.93499836818565	1.93498702519894
5	1.93505783649276	1.93499588215409	1.93498762702377	1.93499398982654	1.93498987207035
6	1.93503762174187	1.93499345657301	1.93498977698376	1.93499258768342	1.93499074980863
7	1.93502540263788	1.93499248293319	1.93499060198772	1.93499203133248	1.93499109135700
8	1.93501745969307	1.93499203289726	1.93499097349609	1.93499177592183	1.93499124637811
9	1.93501200842337	1.93499180223620	1.93499116078546	1.93499164558288	1.93499132491480
10	1.93500810635124	1.93499167430453	1.93499126350900	1.93499157350637	1.93499136813204
11	1.93500521774637	1.93499159879283	1.93499132366473	1.93499153105246	1.93499139349890
12	1.93500301986242	1.93499155192063	1.93499136078926	1.93499150474087	1.93499140918024
13	1.93500130887521	1.93499152159103	1.93499138470640	1.93499148773536	1.93499141929564
14	1.93499995093709	1.93499150126876	1.93499140067770	1.93499147635122	1.93499142605709
15	1.9349985521399	1.93499148724124	1.93499141167240	1.93499146849893	1.93499143071530
16	1.93499795830666	1.93499147730756	1.93499141944157	1.93499146294151	1.93499143400897
17	1.93499721487743	1.93499147011439	1.93499142505747	1.93499145891917	1.93499143639098
18	1.93499659181061	1.93499146480264	1.93499142919844	1.93499145595006	1.93499143814814
19	1.93499606446221	1.93499146081160	1.93499143230603	1.93499145371993	1.93499143946726
20	1.93499561418661	1.93499145776612	1.93499143467490	1.93499145201863	1.93499144047310
21	1.93499522666631	1.93499145540972	1.93499143650618	1.93499145070259	1.93499144125087
22	1.93499489076011	1.93499145356350	1.93499143793989	1.93499144967169	1.93499144185991
23	1.93499459769363	1.93499145210049	1.93499143907527	1.93499144885491	1.93499144234232
24	1.93499434048064	1.93499145092909	1.93499143998382	1.93499144820103	1.93499144272841
25	1.93499411350300	1.93499144998225	1.93499144071783	1.93499144767258	1.93499144304038
26	1.93499391220134	1.93499144921025	1.93499144131605	1.93499144724176	1.93499144329466
27	1.93499373284433	1.93499144857572	1.93499144180752	1.93499144688769	1.93499144350359
28	1.93499357235461	1.93499144805035	1.93499144221434	1.93499144659456	1.93499144367657
29	1.93499342817588	1.93499144761232	1.93499144255341	1.93499144635019	

30	1.93499329817055	1.93499144724478	1.93499144283781	1.93499144614514	
31	1.93499318054007	1.93499144693457	1.93499144307783	1.93499144597211	
32	1.93499307376233	1.93499144667126	1.93499144328149		
33	1.93499297654226	1.93499144644659	1.93499144345520		
34	1.93499288777236	1.93499144625399			

Table (4) to calculate integration $\int_1^2 \sqrt{x} \sqrt{1+x} dx = 1.93499144475889$ by using midpoint rule with the exponential acceleration methods of AL-Tememe of the first kind

n	Values of trapezoidal rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	1.24245332489400				
2	1.24760814669468	1.24829840134852	1.25340355717475	1.24862895468820	1.25038299051381
3	1.24856970445007	1.24917627289809	1.24957317660191	1.24924836549421	1.24944112248007
4	1.24890678242833	1.24929586859582	1.24939288840540	1.24931677435272	1.24936495622186
5	1.24906289570324	1.24932393079624	1.24935871252029	1.24933190382780	1.24934925336124
6	1.24914772308084	1.24933305385616	1.24934849455627	1.24933669998692	1.24934441228076
7	1.24919887987022	1.24933670228342	1.24934457709415	1.24933859296561	1.24934252828916
8	1.24923208604227	1.24933838497515	1.24934281389392	1.24933945928326	1.24934167308689
9	1.24925485365389	1.24933924621701	1.24934192528115	1.24933990049061	1.24934123978341
10	1.24927113997609	1.24933972343593	1.24934143799889	1.24934014414350	1.24934100132717
11	1.24928319043340	1.24934000492524	1.24934115268327	1.24934028751923	1.24934086135448
12	1.24929235603410	1.24934017956706	1.24934097662179	1.24934037631577	1.24934077482206
13	1.24929948916996	1.24934029253026	1.24934086320468	1.24934043367513	1.24934071900154
14	1.24930514918416	1.24934036819895	1.24934078747197	1.24934047205760	1.24934068168828
15	1.24930971545236	1.24934042041756	1.24934073533977	1.24934049852342	1.24934065598124
16	1.24931345264653	1.24934045738961	1.24934069850311	1.24934051724947	1.24934063780430
17	1.24931654996773	1.24934048415777	1.24934067187675	1.24934053080001	1.24934062465833
18	1.24931914557124	1.24934050392206	1.24934065224385	1.24934054080058	1.24934061496074
19	1.24932134224336	1.24934051877067	1.24934063751072	1.24934054831104	1.24934060768060
20	1.24932321773582	1.24934053010026	1.24934062628003	1.24934055403976	1.24934060212934
21	1.24932483174148	1.24934053886575	1.24934061759818	1.24934055847076	1.24934059783677
22	1.24932623070340	1.24934054573297	1.24934061080127	1.24934056194137	1.24934059447538
23	1.24932745119353	1.24934055117452	1.24934060541874	1.24934056469091	1.24934059181292
24	1.24932852232733	1.24934055553123	1.24934060111154	1.24934056689191	1.24934058968199
25	1.24932946751638	1.24934055905259	1.24934059763185	1.24934056867062	1.24934058796020
26	1.24933030575857	1.24934056192362	1.24934059479594	1.24934057012063	1.24934058655676
27	1.24933105260025	1.24934056428325	1.24934059246602	1.24934057131222	1.24934058540357
28	1.24933172086224	1.24934056623702	1.24934059053750	1.24934057229875	1.24934058444897
29	1.24933232119375	1.24934056786587	1.24934058893017	1.24934057312114	1.24934058365327
30	1.24933286249928	1.24934056923253	1.24934058758189	1.24934057381108	1.24934058298576
31	1.24933335227070	1.24934057038606	1.24934058644416	1.24934057439339	1.24934058242243
32	1.24933379684793	1.24934057136512	1.24934058547870	1.24934057488759	1.24934058194437
33	1.24933420162507	1.24934057220045	1.24934058465514	1.24934057530921	1.24934058153655
34	1.24933457121486	1.24934057291664	1.24934058394918	1.24934057567068	1.24934058118695
35	1.24933490958063	1.24934057353350	1.24934058334121	1.24934057598199	1.24934058088585
36	1.24933522014302	1.24934057406713	1.24934058281537	1.24934057625129	1.24934058062541
37	1.24933550586676	1.24934057453062	1.24934058235868	1.24934057648519	1.24934058039921
38	1.24933576933170	1.24934057493478	1.24934058196051	1.24934057668913	1.24934058020200
39	1.24933601279125	1.24934057528847	1.24934058161210	1.24934057686760	1.24934058002942
40	1.24933623822069	1.24934057559908	1.24934058130616	1.24934057702433	1.24934057987787

41	1.24933644735741	1.24934057587276	1.24934058103663	1.24934057716242	1.24934057974435
42	1.24933664173446	1.24934057611464	1.24934058079842	1.24934057728445	1.24934057962634
43	1.24933682270872	1.24934057632908	1.24934058058730		
44	1.24933699148467	1.24934057651969	1.24934058039960		
45	1.24933714913446	1.24934057668961	1.24934058023230		
46	1.24933729661496	1.24934057684146	1.24934058008280		
47	1.24933743478238	1.24934057697753	1.24934057994887		
48	1.24933756440465	1.24934057709970	1.24934057982859		

Table (5) to calculate integration $\int_3^4 \ln(x)dx = 1.24934057847523$ by using trapezoidal rule with the exponential acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	$A^F_{e^{h^2}}$	$A^F_{e^{-h^2}}$	$A^F_{(e^{h^2}-1)}$	$A^F_{(e^{-h^2}-1)}$
1	1.25276296849537				
2	1.25020541816198	1.24986295025751	1.24733004155761	1.24969894714803	1.24882868720811
3	1.24972574171162	1.24942315291919	1.24922515611793	1.24938718927539	1.24929103177862
4	1.24955738965620	1.24936306234842	1.24931460624802	1.24935262106370	1.24932855684940
5	1.24947938424894	1.24934895236381	1.24933157291577	1.24934496846446	1.24933629939117
6	1.24943698898735	1.24934436385946	1.24933664686362	1.24934254158613	1.24933868711457
7	1.24941141849810	1.24934252858966	1.24933859240122	1.24934158354074	1.24933961648711
8	1.24939481925080	1.24934168207282	1.24933946812543	1.24934114504296	1.24934003839706
9	1.24938343748859	1.24934124878201	1.24933990949041	1.24934092170394	1.24934025217775
10	1.24937529549555	1.24934100868448	1.24934015152597	1.24934079836087	1.24934036983051
11	1.24936927097340	1.24934086705845	1.24934029324673	1.24934072577803	1.24934043889404
12	1.24936468862057	1.24934077918907	1.24934038070062	1.24934068082433	1.24934048159064
13	1.24936112234718	1.24934072235191	1.24934043703827	1.24934065178530	1.24934050913388
14	1.24935829254033	1.24934068427896	1.24934047465729	1.24934063235331	1.24934052754539
15	1.24935600954620	1.24934065800478	1.24934050055332	1.24934061895424	1.24934054023016
16	1.24935414104932	1.24934063940188	1.24934051885160	1.24934060947356	1.24934054919938
17	1.24935259246198	1.24934062593311	1.24934053207806	1.24934060261310	1.24934055568615
18	1.24935129471480	1.24934061598839	1.24934054183061	1.24934059754990	1.24934056047138
19	1.24935019642004	1.24934060851704	1.24934054914924	1.24934059374741	1.24934056406374
20	1.24934925870556	1.24934060281633	1.24934055472807	1.24934059084698	1.24934056680301
21	1.24934845172744	1.24934059840578	1.24934055904077	1.24934058860358	1.24934056892118
22	1.24934775226595	1.24934059495038	1.24934056241714	1.24934058684641	1.24934057057986
23	1.24934714203639	1.24934059221234	1.24934056509092	1.24934058545432	1.24934057189366
24	1.24934660648196	1.24934059002015	1.24934056723053	1.24934058433995	1.24934057294517
25	1.24934613389757	1.24934058824830	1.24934056895909	1.24934058343939	1.24934057379481
26	1.24934571478476	1.24934058680366	1.24934057036783	1.24934058270524	1.24934057448734
27	1.24934534137076	1.24934058561634	1.24934057152522	1.24934058210192	1.24934057505638
28	1.24934500724544	1.24934058463326	1.24934057248323	1.24934058160245	1.24934057552744
29	1.24934470708444	1.24934058381366	1.24934057328167	1.24934058118607	1.24934057592008
30	1.24934443643568	1.24934058312599	1.24934057395144	1.24934058083674	1.24934057624947
31	1.24934419155335	1.24934058254554	1.24934057451660	1.24934058054191	1.24934057652744
32	1.24934396926762	1.24934058205291	1.24934057499621	1.24934058029170	1.24934057676335
33	1.24934376688151	1.24934058163258	1.24934057540531	1.24934058007822	1.24934057696459
34	1.24934358208873	1.24934058127221	1.24934057575600	1.24934057989521	1.24934057713710
35	1.24934341290767	1.24934058096182	1.24934057605802	1.24934057973759	1.24934057728569
36	1.24934325762806	1.24934058069331	1.24934057631924	1.24934057960124	1.24934057741421
37	1.24934311476757	1.24934058046009	1.24934057654610	1.24934057948282	1.24934057752583

38	1.24934298303630	1.24934058025672	1.24934057674389		
39	1.24934286130758	1.24934058007876	1.24934057691697		
40	1.24934274859379	1.24934057992246	1.24934057706895		
41	1.24934264402625	1.24934057978475	1.24934057720284		
42	1.24934254683845	1.24934057966304	1.24934057732116		

Table (6) to calculate integration $\int_3^4 \ln(x)dx = 1.24934057847523$ by using midpoint rule with the exponential acceleration methods of AL-Tememe of the first kind

6. CONCLUSION:

We can say that these methods are working with the same efficiency in improving the results of the integrals we reviewed in terms of accuracy and the number of sub intervals used and the speed of obtaining their values with a simple variation in n values.

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