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On Fuzzy Semi Proper Mappings

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Abstract: *In this paper we introduce the definitions of fuzzy proper and fuzzy semi proper functions and the concept of fuzzy semi compact and fuzzy semi coercive mappings in fuzzy topological spaces. We discuss Several characterizations and some interesting properties of these mappings . Also, we explain the relation between these types*

Keywords: semi continuous, fuzzy topology, fuzzy semi proper, fuzzy semi coercive.

Introduction

The concept of fuzzy set and fuzzy set operations were first introduced by Zadeh 1965. Several other authors applied fuzzy sets to various branches of mathematics. One of these objects is a topological space. At the first time in 1968, Chang formulated the natural definition of fuzzy topology on a set and investigated how some of the basic ideas and theorems of point-set topology behave in generalized setting. Chang's definition on a fuzzy topology is very similar to the general topology by exchange all subsets of a universal set by fuzzy subset but this definition is not investigate some properties if we comparison with the general topology. For example in a general topology, any constant mapping is continuous while this idea is not true in Chang's definition on fuzzy topology. One of the very important concepts in fuzzy topology is the concept of mapping. There are several types of mapping. The purpose of this paper is to introduced and study the concept of fuzzy semi proper functions and fuzzy semi compact and fuzzy semi coercive, We discuss Several characterizations and some interesting properties of these mappings.

1.Preliminaries

First, we present some fundamental definitions and propositions which are needed in the next sections.

Definition (1.1) **[4]**

A mapping f from an fts X into an fts Y is said to be:

(a) fuzzy semi continuous if $f^{-1}(A)$ is a fuzzy semi open set in X, for each fuzzy open set A in Y.

(b) fuzzy semi irresolute if $f^{-1}(A)$ is a fuzzy semi open set in X for each fuzzy semi open set A in Y.

Definition (1.2) **[4]**

Let (X, T) be an fts, $x_r \in \mathrm{FP}\,(X)$ and let $\mathfrak{F} = \{\, x^n_{r_n} : n \in D\}$ be a fuzzy net in X. Then:

(i) x_r is called a semi $cluster$ point of a fuzzy net $\mathfrak{F}$, denoted by $\mathfrak{F} \propto^s x_r$, if for each $A \in N^{Qs}_{x_r}$, $\mathfrak{F}$ is frequently with A.

(ii) $\mathfrak{F}$ is said to be semi convergent to x_r and x_r is called a semi limit point of $\mathfrak{F}$, denoted by $\mathfrak{F} \xrightarrow{s} x_r$, if for each $A \in N^{Qs}_{x_r}$, $\mathfrak{F}$ is eventually with A.

Proposition (1.3) **[4]**

Let (X, T) be an fts. A fuzzy point x_r is $semi\ cluster$ point of a fuzzy net $\mathfrak{F} = \{x^n_{r_n} : n \in D\}$ if and only if it has a fuzzy subnet which is semi convergent to x_r.

Corollary (1.4)

Let (X, T) be an fts, $x_r \in \mathrm{FP}(X)$ and A be a fuzzy set in X. there exists a fuzzy net $\mathfrak{F} = \{x^n_{r_n} : n \in D\}$in A, such that $\mathfrak{F} \propto^s x_r$. then $x_r \in \overline{A}^s$.

Proof **:**

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Let $\mathfrak{F}$ be a fuzzy net in A, such that $\mathfrak{F} \propto^s x_r$, and let B be a fuzzy semi open set in X, such that $x_r qB$, then $\forall\, m \in D$, $\exists\, n \in D$ such that $x_{r_n}^n qB$ with $n \geq m$ and so BqA for each fuzzy semi open set B in X, such that $x_r qB$. Thus $x_r \in \overline{A}^s$.

***Definition* (*1.5*)**

An fts (X, T) is called fuzzy semi Hausdorff or fuzzy semi T_2- space if and only if for any pair of distinct fuzzy points x_r, y_s in X, there exists $A \in N_{x_r}^{Qs}$, $B \in N_{y_s}^{Qs}$ such that $A \wedge B = 0_X$.

***Definition* (*1.6*) [1]**

A family Ω of fuzzy sets is a cover of a fuzzy set A if and only if $A \leq \vee\{G_i: G_i \in \Omega\}$ and it is called a fuzzy open cover if and only if Ω is a cover of A and each member of Ω is a fuzzy open set. A subcover of Ω is a subfamily of Ω which is also a cover of A.

***Definition* (*1.7*) [2]**

Let A be a fuzzy set in an fts X. Then A is said to be fuzzy compact if for every fuzzy open cover of A has a finite subcover of A. Also, an fts X is called fuzzy compact if every fuzzy open cover of X has a finite sub cover.

***Definition* (*1.8*) [3]**

Let X be an fts. A fuzzy subset V of X is called fuzzy compactly closed if for every fuzzy compact set K in X, $V \wedge K$ is fuzzy compact.

***Definition* (*1.9*) [4]**

A family Ω of fuzzy sets is called a fuzzy semi open cover if Ω is a cover of A and each member of Ω is a fuzzy semi open set. A subcover of Ω is a subfamily of Ω which is also a cover of A.

***Definition* (*1.10*) [4]**

Let A be a fuzzy set in an fts X. Then A is said to be fuzzy semi compact if for every fuzzy semi open cover of A has a finite subcover of A. Also, an fts X is called fuzzy semi compact if every fuzzy semi open cover of X has a finite sub cover.

***Definition* (*1.11*)**

A mapping f from an fts X into an fts Y is called fuzzy semi compact if the inverse image of each fuzzy semi compact set in Y, is a fuzzy compact set in X.

***Definition* (*1.12*)**

A mapping f from an fts X into an fts Y is called fuzzy semi coercive if for every fuzzy semi compact set B in Y, there exists a fuzzy compact set A in X such that $f(1_X \backslash A) \leq (1_Y \backslash B)$.

***Proposition* (*1.13*)**

If $f: X \longrightarrow Y$ is a fuzzy closed function and $g: Y \longrightarrow Z$ is a fuzzy semi closed function, then $g \circ f: X \longrightarrow Z$ is a fuzzy semi closed function.

***Proof* :**

Let A be a fuzzy closed subset of X. Then $f(A)$ is a fuzzy closed set in Y. But g is a fuzzy semi closed function, then $g(f(A))$ is a fuzzy semi closed set in Z, hence $(g \circ f)(A)$ is a fuzzy semi closed set in Z. Thus $g \circ f: X \longrightarrow Z$ is a fuzzy semi closed function.

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***Corollary* (*1.14*) [5]**

A fuzzy closed subset of a fuzzy compact space is fuzzy compact.

***Proposition* (*1.15*) [5]**

A fuzzy compact subset of a fuzzy T_2-space is fuzzy closed.

***Proposition* (*1.16*) [5]**

Let X be a fuzzy T_2-space. A fuzzy subset A of X is fuzzy compactly closed if and only if A is fuzzy closed.

***Proposition* (*1.17*)**

Let X be a fuzzy semi compact and fuzzy T_2-space and A be a fuzzy set in X. Then A is fuzzy compact if and only if A is fuzzy semi compact.

***Proof* :**

Let A be a fuzzy compact set in X. Since X is a fuzzy T_2-space, then by Proposition (1.15), A is a fuzzy closed set in X, and then it is a fuzzy semi closed set in X. Since X is a fuzzy semi compact space, then A is a fuzzy semi compact set in X.

***Proposition* (*1.18*)**

Every fuzzy semi compact function is fuzzy semi coercive.

***Proof* :**

Let $f: X \longrightarrow Y$ be a fuzzy semi compact function, and A be a fuzzy semi compact set in Y, since f is fuzzy compact, then $f^{-1}(A)$ is a fuzzy compact set in X, such that $f(1_X \backslash f^{-1}(A)) \leq 1_Y \backslash A$. Hence f is a fuzzy semi coercive function.

***Proposition* (*1.19*)**

Let X and Y be fuzzy spaces, such that Y is a fuzzy T_2-space, and $f: X \longrightarrow Y$ be a fuzzy continuous function. Then f is fuzzy semi compact if and only if f is fuzzy semi coercive.

***Proof* :**

$\Longrightarrow$ By Proposition (1.18).

$\Longleftarrow$ Let B be a fuzzy semi compact set in Y, so it is fuzzy compact. To prove that $f^{-1}(B)$ is a fuzzy compact set in X. Since Y is a fuzzy T_2-space and f is a fuzzy continuous function, then by Proposition (1.15) $f^{-1}(B)$ is a fuzzy closed set in X. Since f is a fuzzy semi coercive function, then there exists a fuzzy compact set A in X, such that $f(1_X \backslash A) \leq 1_Y \backslash B$. Then $f(A^c) \leq B^c \Longrightarrow f^{-1}\big(f(A^c)\big) \leq f^{-1}(B^c) \Longrightarrow A^c \leq f^{-1}(B^c) = (f^{-1}(B))^c \Longrightarrow f^{-1}(B) \leq A$,then by corollary (1.14), $f^{-1}(B)$ is a fuzzy compact set in X. Hence , f is a fuzzy semi compact function.

***Remark* (*1.20*)**

Every fuzzy compact function is fuzzy semi compact.

***Proposition* (*1.21*)**

Let X and Y be fuzzy spaces, such that Y is a fuzzy semi compact and fuzzy T_2-space. A function $f: X \longrightarrow Y$ is fuzzy compact if and only if it is fuzzy semi compact.

***Proof* :**

$\Longrightarrow$ By Remark (1.20).

$\Leftarrow$ Let A be a fuzzy compact set in Y. Since Y is a fuzzy semi compact and fuzzy T_2-space, then by Proposition (1.17), A is a fuzzy semi compact set in Y, then $f^{-1}(A)$ is a fuzzy compact set in X. Hence , f is a fuzzy compact function.

***Proposition* (*1.22)* [6]**

The fuzzy continuous image of a fuzzy compact set is fuzzy compact.

***Proposition* (*1.23*) [5]**

In any fts X, the intersection of any fuzzy closed set with any fuzzy compact set is fuzzy compact.

***Proposition* (*1.24*) [6]**

Let X and Y be fuzzy spaces. A bijective function $f: X \longrightarrow Y$ is fuzzy open if and only if f^{-1} is fuzzy continuous.

***Proposition* (*1.25*)**

If $f: X \longrightarrow Y$ is a fuzzy closed function and $g: Y \longrightarrow Z$ is a fuzzy semi closed function, then $g \circ f: X \longrightarrow Z$ is a fuzzy semi closed function.

***Proof* :**

Let A be a fuzzy closed subset of X. Then $f(A)$ is a fuzzy closed set in Y. But g is a fuzzy semi closed function, then $g(f(A))$ is a fuzzy semi closed set in Z, hence $(g \circ f)(A)$ is a fuzzy semi closed set in Z. Thus $g \circ f: X \longrightarrow Z$ is a fuzzy semi closed function.

2. Fuzzy semi Proper Function

In this section, we introduce the definitions of fuzzy proper and fuzzy semi proper functions and explain the relation between these types. Also, we study the relations between fuzzy semi proper function and certain types of functions, such as (fuzzy semi closed function, fuzzy semi compact function and fuzzy semi coercive function).

***Proposition* (*2.1*) [6]**

Let X and Y be fuzzy spaces, such that Y be a fuzzy ssc-space. If $f: X \longrightarrow Y$ be a fuzzy homeomorphism function, then f is a fuzzy proper function.

***Definition* (*2.2*)**

Let X and Y be fuzzy spaces. A fuzzy continuous function $f: X \longrightarrow Y$ is called fuzzy semi proper if:

1. f is fuzzy semi closed.
2. $f^{-1}(\{y_r\})$ is fuzzy compact, for each $y_r \in \mathrm{FP}(Y)$.

***Remark* (*2.3*)**

Every fuzzy proper function is fuzzy semi proper.

***Proposition* (*2.4*)**

Let $f: X \longrightarrow Y$ be a fuzzy proper function and $g: Y \longrightarrow Z$ be a fuzzy semi proper function, then $g \circ f: X \longrightarrow Z$ is a fuzzy semi proper function.

***Proof* :**

Since f and g are fuzzy continuous functions, then$g \circ f$ is fuzzy continuous. Also, since f is a fuzzy proper function, then f is fuzzy closed. Similarly, since g is a fuzzy semi proper function, then g is fuzzy semi closed. Thus by Proposition (1.25), $g \circ f$ is

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a fuzzy semi closed function. Let $z_r \in \mathrm{FP}(Z)$, since g is a fuzzy semi proper function, then $g^{-1}(\{z_r\})$ is a fuzzy compact set in Y, since f is a fuzzy proper function, then $f^{-1}(g^{-1}(\{z_r\}))$ is a fuzzy compact set in X. But $(g \circ f)^{-1}(\{z_r\}) = f^{-1}(g^{-1}(\{z_r\}))$, hence $(g \circ f)^{-1}(\{z_r\})$ is a fuzzy compact set in X, then $g \circ f$ is a fuzzy semi proper function.

***Proposition* (*2.5*)**

Let $f: X \longrightarrow Y$ be a fuzzy proper function and $g: Y \longrightarrow Z$ be a fuzzy semi proper function, then $g \circ f: X \longrightarrow Z$ is a fuzzy semi proper function.

***Proof* :**

Since f and g are fuzzy continuous functions, then $g \circ f$ is fuzzy continuous. Also, since f is a fuzzy proper function, then f is fuzzy closed. Similarly, since g is a fuzzy semi proper function, then g is fuzzy semi closed. Thus by Proposition (1.25), $g \circ f$ is a fuzzy semi closed function. Let $z_r \in \mathrm{FP}(Z)$, since g is a fuzzy semi proper function, then $g^{-1}(\{z_r\})$ is a fuzzy compact set in Y, since f is a fuzzy proper function, then $f^{-1}(g^{-1}(\{z_r\}))$ is a fuzzy compact set in X. But $(g \circ f)^{-1}(\{z_r\}) = f^{-1}(g^{-1}(\{z_r\}))$, hence $(g \circ f)^{-1}(\{z_r\})$ is a fuzzy compact set in X, then $g \circ f$ is a fuzzy semi proper function.

***Proposition* (*2.6*) [6]**

Let X and Y be fuzzy spaces, such that Y be a fuzzy ssc-space. If $f: X \longrightarrow Y$ be a fuzzy homeomorphism function, then f is a fuzzy proper function.

***Corollary* (*2.7*)**

Let X, Y and Z be fuzzy spaces, such that Y is a fuzzy ssc-space. If $f: X \longrightarrow Y$ be a fuzzy homeomorphism function and $h: Y \longrightarrow Z$ be a fuzzy semi proper function, then $h \circ f: X \longrightarrow Z$ is a fuzzy semi proper function.

***Proof* :**

Since f is a fuzzy homeomorphism function, then by Proposition (2.6), f is a fuzzy proper function. Since h is a fuzzy semi proper function, then by Proposition (2.5), $h \circ f$ is a fuzzy semi proper function.

***Proposition* (*2.8*)**

Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be fuzzy continuous functions, such that $g \circ f: X \longrightarrow Z$ is a fuzzy semi proper function. If f is onto, then g is a fuzzy semi proper function.

***Proof* :**

Let A be a fuzzy closed set in Y, since f is fuzzy continuous, then $f^{-1}(A)$ is a fuzzy closed set in X. Since $g \circ f$ is a fuzzy semi closed function, then $(g \circ f)(f^{-1}(A))$ is a fuzzy semi closed set in Z. But f is onto. Thus g is a fuzzy semi closed function. Let $z_r \in \mathrm{FP}(Z)$, since $g \circ f$ is a fuzzy semi proper function, then $(g \circ f)^{-1}(\{z_r\}) = f^{-1}(g^{-1}(\{z_r\}))$ is a fuzzy compact set in X. Now, since f is fuzzy continuous and onto, then $f(f^{-1}(g^{-1}(\{z_r\}))) = g^{-1}(\{z_r\})$ is fuzzy compact in Y, for each $z_r \in \mathrm{FP}(Z)$. So g is a fuzzy semi proper function.

***Proposition* (*2.9*)**

Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be functions, such that $g \circ f: X \longrightarrow Z$ is a fuzzy semi proper function. If f is a fuzzy continuous function g is a one to one and fuzzy semi irresolute function. Then f is a fuzzy semi proper function.

***Proof* :**

Let A be a fuzzy closed set in X, since $g \circ f$ is a fuzzy semi proper function, then $(g \circ f)(A)$ is a fuzzy semi closed set in Z, since g is fuzzy semi irresolute, then $g^{-1}(g \circ f)(A)$ is a fuzzy semi closed set in Y. But g is one to one, then $g^{-1}(g \circ f)(A) = f(A)$. Hence $f(A)$ is a fuzzy semi closed set in Y. Thus f is a fuzzy semi closed function. Let $y_r \in \mathrm{FP}\ (Y)$, $g(y_r) \in \mathrm{FP}\ (Z)$. Now, since $g \circ f: X \longrightarrow Z$ is fuzzy semi proper and g is a one to one function, then the fuzzy set$(g \circ f)^{-1}(g(\{y_r\})) = f^{-1}(g^{-1}(g(\{y_r\}) = f^{-1}(\{y_r\})$ is fuzzy compact in X. Thus the function $f: X \longrightarrow Y$ is fuzzy semi proper.

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Proposition (2.10)

Let X and Y be fuzzy spaces, such that X be a fuzzy ssc-space. If $f: X \longrightarrow Y$ be a fuzzy homeomorphism function, then $f^{-1}: Y \longrightarrow X$ is a fuzzy semi proper function.

***Proof* :**

Since f is a bijective and fuzzy open function, then by Proposition (1.24), f^{-1} is fuzzy continuous. To prove that f^{-1} is a fuzzy semi closed function. Let A be a fuzzy closed subset of Y, since f is fuzzy continuous, then $f^{-1}(A)$ is a fuzzy closed set in X, so $f^{-1}(A)$ is a fuzzy semi closed set in X. Hence f^{-1} is a fuzzy semi closed function. Let $x_r \in$ FP (X), then $\{x_r\}$ is a fuzzy compact set in X. Since f is fuzzy continuous, then $f(\{x_r\}) = (f^{-1})^{-1}(\{x_r\})$ is a fuzzy compact set in Y, then f^{-1} is a fuzzy semi proper function.

Proposition (2.11)

If $f: X \longrightarrow Y$ is a fuzzy semi proper function, then f is a fuzzy semi compact function.

***Proof* :**

Let A be a fuzzy semi compact subset of Y and let $\{V_\lambda\}_{\lambda\in\Lambda}$ be a fuzzy open cover of $f^{-1}(A)$. Put $B = \{x_r \in$FP $(Y): A(x) = r\}$. Since f is a fuzzy semi proper function, then $f^{-1}(\{x_r\})$ is a fuzzy compact set, $\forall\, x_r \in B$.
But $f^{-1}(x_r) \leq f^{-1}(A) \leq \vee_{\lambda\in\Lambda} V_\lambda$, thus there exists $\lambda_1,\lambda_2,\ldots,\lambda_n$, such that $f^{-1}(x_r) \leq \vee_{i=1}^{n} V_{\lambda_i}$, let $V_{\lambda_{x_r}} = \vee_{i=1}^{n} V_{\lambda_i}$, so for all $x_r \in B$, $f^{-1}(x_r) \leq V_{\lambda_{x_r}}$. Notice that, for all $x_r \in B$, $f^{-1}(x_r)\tilde{q}(1_X - V_{\lambda_{x_r}})$, hence $x_r\tilde{q} f(1_X - V_{\lambda_{x_r}})$, therefore $x_r \in 1_Y - (f(1_X - V_{\lambda_{x_r}}))$, then $A \leq \vee_{x_r\in B}(1_Y - (f(1_X - V_{\lambda_{x_r}})))$. Since $V_{\lambda_{x_r}}$ is a fuzzy open set, then $1_X - V_{\lambda_{x_r}}$ is a fuzzy closed set, thus $f(1_X - V_{\lambda_{x_r}})$ is a fuzzy semi closed set, then $1_Y - (f(1_X - V_{\lambda_{x_r}}))$ is a fuzzy semi open set. Since A is a fuzzy semi compact set in Y and $\vee_{x_r\in B}(1_Y - (f(1_X - V_{\lambda_{x_r}}))$ is a fuzzy semi open cover of A, i.e., $A \leq \vee_{x_r\in B}(1_Y - (f(1_X - V_{\lambda_{x_r}})))$, then there exists $y_{r_1}^1, y_{r_2}^2, \ldots, y_{r_m}^m$ in A, such that: $A \leq \vee_{i=1}^{m}(1_Y - (f(1_X - V_{\lambda_{y_{r_i}^i}})))$, so $f^{-1}(A) \leq \vee_{i=1}^{m} V_{\lambda_{y_{r_i}^i}}$. Therefore , $f^{-1}(A)$ is a fuzzy compact set in X. Hence the function $f: X \longrightarrow Y$ is a fuzzy semi compact function.

Proposition (2.12)

Let X and Y be fuzzy spaces, such that Y be a fuzzy T_2-space and fuzzy ssc-space. If $f: X \longrightarrow Y$ is a fuzzy continuous function. Then f is fuzzy semi proper if and only if f is fuzzy semi compact.

***Proof* :**

$\Longrightarrow$ By Proposition (2.11).

$\Longleftarrow$ Let F be a fuzzy closed subset of X , and let K be a fuzzy semi compact set in Y, then $f^{-1}(K)$ is a fuzzy compact set in X, thus by Proposition (1.23), $F \wedge f^{-1}(K)$ is a fuzzy compact set. Since f is fuzzy continuous , then by Proposition (1.22), $f(F \wedge f^{-1}(K))$ is a fuzzy compact set in Y. But $f(F \wedge f^{-1}(K)) = f(F) \wedge K$, then $f(F) \wedge K$ is a fuzzy compact set in Y. Therefore $f(F)$ is a fuzzy compactly closed set in Y. Since Y is a fuzzy T_2-space, then by Proposition (1.16), $f(F)$ is a fuzzy closed set in Y and so it is fuzzy semi closed. Hence f is a fuzzy semi closed function. Let $y_r \in$ FP (Y), then $\{y_r\}$ is fuzzy semi compact in Y. Since f is a fuzzy semi compact function, then $f^{-1}(\{y_r\})$ is fuzzy compact in X. Therefore f is a fuzzy semi proper function.

Proposition (2.13)

Let X and Y be fuzzy spaces, such that Y is a fuzzy semi compact space, fuzzy T_2-space and fuzzy ssc-space. If $f: X \longrightarrow Y$ is a fuzzy continuous function. Then the following statements are equivalent:

(a) f is a fuzzy compact function.

(b) f is a fuzzy proper function.

(c) f is a fuzzy semi proper function.

(d) f is a fuzzy semi compact function.

***Proof* :**

(a)$\Longrightarrow$(b) Let F be a fuzzy closed subset of X. To prove that $f(F)$ is a fuzzy closed set in Y, let K be a fuzzy compact set in Y, then $f^{-1}(K)$ is a fuzzy compact set in X, thus by Proposition (1.23), $F \wedge f^{-1}(K)$ is a fuzzy compact set. Since f is fuzzy continuous , then by Proposition (1.22), $f(F \wedge f^{-1}(K))$ is a fuzzy compact set in Y. But $f(F \wedge f^{-1}(K)) = f(F) \wedge K$, then $f(F) \wedge K$ is a fuzzy compact set in Y. Therefore $f(F)$ is a fuzzy compactly closed set in Y. Since Y is a fuzzy T_2-space, then by Proposition (1.16), $f(F)$ is a fuzzy closed set in Y. Hence f is a fuzzy closed function. Let $y_r \in$ FP (Y), then $\{y_r\}$ is fuzzy compact in Y. Since f is a fuzzy compact function, then $f^{-1}(\{y_r\})$ is fuzzy compact in X. Therefore f is a fuzzy proper function.

(b)⟹(c) By Remark (2.3).

(c)⟹(d) By Proposition (2.11).

(d)⟹(a) By Proposition (1.21)

The following diagram shows the relations among certain types of fuzzy compact functions and certain types of fuzzy proper functions.

***Theorem* (*2.14*)**

Let X and Y be fuzzy spaces, such that Y is a fuzzy T_2-space and fuzzy ssc-space. If $f: X \longrightarrow Y$ is a fuzzy continuous function. Then the following statements are equivalent:

(a) f is a fuzzy semi coercive function.

(b) f is a fuzzy semi compact function.

(c) f is a fuzzy semi proper function.

***Proof* :**

(a)⟹(b) By Proposition (1.19).

(b)⟹(c) By Proposition (2.12).

(c)⟹(a) Since f is fuzzy semi proper function, then by Proposition (2.11), f is a fuzzy semi compact function, hence by Proposition (1.18), $f: X \longrightarrow Y$ is a fuzzy semi coercive function.

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