Generalized Ratio-Type Estimator For Population Variance Using Auxiliary Information In Simple Random Sampling

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Abstract: This paper suggests a new generalized ratio-type estimator for population variance of study variable utilizing information obtained from two auxiliary variables. Empirically, the estimator proves more efficient than the usual unbiased estimator and the previously existing estimators of Isaki (1983), Upadhyaya and Singh (1999), Kadilar and Cingi (2006), Yadav et al. (2013). Efficiency of the new generalized estimators has been compared mathematically with that of Yadav et al. (2013). The empirical study supports the new estimators against above mentioned estimators.

Keywords ratio-type variance estimator, ratio-product-type variance estimator, transformed sample variances.

1. Introduction

A statistical population can be described by several characteristics one of which is variance. It is usual practice that population variance is not known. So far, remarkable attempts have been made to estimate this characteristic in the best possible way. Objective is to minimize the dispersion of the desired estimator if obtained from different possible samples. Unbiasedness is one of the ideal properties of an estimator but it may be sacrificed if a biased estimator provides less scattered results in repeated sampling. Moreover, need for the use of information available from one or more auxiliary variables has also been emphasized to raise the efficiency of estimator. Numerous efforts have already been made in this connection. These efforts include ratio, ratio-type, transformed ratio-product-type estimators for variance.

Consider y , the study variable in a finite population of size N from which a sample of size n is drawn without replacement. Let x and z be the auxiliary variables about which the information in the form of observations or some useful parameters is available. The available variances are transformed by using the parameters of the relevant variable.

Here are some important results and notations to be used later.

N : Population size $\qquad$ n : Sample size

the population means of the variables:

$$\bar{Y} = \sum_{i=1}^{N} y_i / N \;,\; \bar{X} = \sum_{i=1}^{N} x_i / N \;,\; \bar{Z} = \sum_{i=1}^{N} z_i / N$$

the population variances:

$$S_y^2 = \sum_{i=1}^{N} \left(y_i - \bar{Y} \right)^2 / (N-1), \; S_x^2 = \sum_{i=1}^{N} \left(x_i - \bar{X} \right)^2 / (N-1), \; S_z^2 = \sum_{i=1}^{N} \left(z_i - \bar{Z} \right)^2 / (N-1)$$

the sample mean and the unbiased sample variance

$$\bar{y} = \sum_{i=1}^{n} y_i / n \;,\; \bar{x} = \sum_{i=1}^{n} x_i / n \;,\; \bar{z} = \sum_{i=1}^{n} z_i / n \;,\; s_y^2 = \sum_{i=1}^{n} \left(y_i - \bar{y} \right)^2 / (n-1),$$

$$s_x^2 = \sum_{i=1}^{n} \left(x_i - \bar{x} \right)^2 / (n-1), \; s_z^2 = \sum_{i=1}^{n} \left(z_i - \bar{z} \right)^2 / (n-1)$$

the population correlation coefficients:

$$\rho_{xy} = S_{xy} / S_x S_y \;,\; \rho_{zy} = S_{zy} / S_z S_y \;,\; \rho_{zx} = S_{zx} / S_z S_x$$

where

$$S_{xy} = \sum_{i=1}^{N} \left(x_i - \bar{X} \right)\left(y_i - \bar{Y} \right) / (N-1), S_{zy} = \sum_{i=1}^{N} \left(z_i - \bar{Z} \right)\left(y_i - \bar{Y} \right) / (N-1), S_{zx} = \sum_{i=1}^{N} \left(z_i - \bar{Z} \right)\left(x_i - \bar{X} \right) / (N-1)$$

coefficients of variation

$$C_y = S_y / \bar{Y} \;,\; C_x = S_x / \bar{X} \;,\; C_z = S_z / \bar{Z}$$

coefficients of kurtosis

$$\beta_2(y) = \mu_{400}/\mu_{200}^2, \, \beta_2(x) = \mu_{040}/\mu_{020}^2, \, \beta_2(z) = \mu_{004}/\mu_{002}^2$$

also $h = \mu_{220}/\mu_{200}\mu_{020}$, $k = \mu_{202}/\mu_{200}\mu_{002}$, $l = \mu_{022}/\mu_{020}\mu_{002}$

where $\mu_{rst} = \dfrac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r (x_i - \bar{X})^s (z_i - \bar{Z})^t$

$\upsilon' = h'/\beta_2'(x)$, $\mu' = k'/\beta_2'(z)$ where $h' = h-1$, $k' = k-1$, $l' = l-1$

$\beta_2'(y) = \beta_2(y)-1$, $\beta_2'(x) = \beta_2(x)-1$, $\beta_2'(z) = \beta_2(z)-1$

$$\lambda' = \frac{1}{n}\left(1 - \frac{n}{N}\right)$$

$$e_0 = (S_y)^{-2}(s_y^2 - S_y^2), \, e_1 = (S_x)^{-2}(s_x^2 - S_x^2), \, e_2 = (S_z)^{-2}(s_z^2 - S_z^2)$$

the expected values of e_0 e_1 and e_2 are all zero as:

$$E(s_y^2) = S_y^2, \, E(s_x^2) = S_x^2, \, E(s_z^2) = S_z^2$$

$$E(e_0^2) = \lambda'\beta_2'(y), \, E(e_1^2) = \lambda'\beta_2'(x), \, E(e_2^2) = \lambda'\beta_2'(z)$$

$$E(e_0 e_1) = \lambda' h', \, E(e_0 e_2) = \lambda' k', \, E(e_1 e_2) = \lambda' l'$$

$$\theta_0 = \frac{S_y^2}{S_y^2 + d}, \, \theta_1 = \frac{b}{b+c}, \, \theta_2 = \frac{m}{m+p}, \, w_1 = \frac{\theta_1}{\theta_0}, \, w_2 = \frac{\theta_2}{\theta_0}$$

$$W_1 = \omega\tau w_1, \, W_2 = (1-\omega)\varphi w_2, \, W_{11} = (1-\omega-\psi)\chi w_1, \, M_1 = W_1 - W_{11}, \, M_2 = \psi\varphi w_2,$$

$D = \beta_2'(x)\beta_2'(z) - l'^2$ and $N-1 \cong N$ for sufficiently large population

Isaki (1983) proposed an estimator of population variance using auxiliary information that has been cited by the successors. Some other references are Ahmed, Raman and Hossain (2000), Arcos et al. (2005), Kadilar and Cingi (2006), Gupta and Shabbir (2008), Subramani and Kumarapandiyan (2012a), Subramani and Kumarapandiyan (2012b), Singh and Solanki (2013a), Subramani and Kumarapandiyan (2013), Singh and Solanki (2013b), Yadav et al. (2013), Ismail et al. (2016).

Rohini Yadav, Lakshmi N. Upadhyaya, Housila P. Singh, and S. Chatterjee (2013) developed a class of estimators of variance as the combination of ratio and product-type estimators based on some parametric information of an auxiliary as well as study variable and transforming the variances of both the variables. Moreover, it described usual unbiased estimator s_y^2, the estimators of Isaki (1983) , Upadhyaya and Singh (1999), Kadilar and Cingi (2006) as special cases for certain values of the scalars and constants and under certain conditions surpassed all these estimators by obtaining smaller mean-squared error.

Some biased variance estimators of the study variable y, utilizing information of an auxiliary variable along with their bias, MSE and minimum MSE are as under:

$t_{Isaki} = s_y^2 (S_x^2/s_x^2)$ is the ratio estimator of Isaki (1983) along with bias $Bias(t_{Isaki}) \cong \lambda' S_y^2 \{\beta_2'(x) - h'\}$ and mean-squared error $MSE(t_{Isaki}) \cong \lambda' S_y^4 \{\beta_2'(y) + \beta_2'(x) - 2h'\}$.

$t_{US} = s_y^2 \{s_x^2 + \beta_2(x)\}^{-1} \{S_x^2 + \beta_2(x)\}$ is the ratio-type estimator given by Upadhyaya and Singh (1999) and has

bias as $Bias(t_{US}) \cong \lambda' S_y^2 \dfrac{S_x^2}{S_x^2 + \beta_2(x)}\left\{\dfrac{S_x^2}{S_x^2 + \beta_2(x)}\beta_2'(x) - h'\right\}$, mean-squared error as

$MSE(t_{US}) \cong \lambda' S_y^4 \left[\beta_2'(y) - 2\dfrac{S_x^2}{S_x^2 + \beta_2(x)}h' + \left\{\dfrac{S_x^2}{S_x^2 + \beta_2(x)}\right\}^2 \beta_2'(x)\right]$ and minimum mean-squared error as

$MSE_{\min}(t_{US}) \cong \lambda' S_y^4 \{\beta_2'(y) - h'\upsilon'\}$

Following are the ratio-type estimators suggested by Kadilar and Cingi (2006):

$$t_{CH1} = s_y^2 \left\{ s_x^2 - \beta_2(x) \right\}^{-1} \left\{ S_x^2 - \beta_2(x) \right\}$$

$$t_{CH2} = s_y^2 \left\{ C_x s_x^2 - \beta_2(x) \right\}^{-1} \left\{ C_x S_x^2 - \beta_2(x) \right\}$$

$$t_{CH3} = s_y^2 \left\{ s_x^2 - C_x \right\}^{-1} \left\{ S_x^2 - C_x \right\}$$

$$t_{CH4} = s_y^2 \left\{ \beta_2(x) s_x^2 - C_x \right\}^{-1} \left\{ \beta_2(x) S_x^2 - C_x \right\}$$

Their biases and mean-squared errors are

$$Bias(t_{CHi}) \cong \lambda' S_y^2 \beta_2'(x) A_i (A_i - \upsilon')$$

$$MSE(t_{CHi}) \cong \lambda' S_y^4 \left\{ \beta_2'(y) + \beta_2'(x) A_i (A_i - 2\upsilon') \right\} \qquad i = 1, 2, 3, 4$$

where $A_1 = \dfrac{S_x^2}{S_x^2 - \beta_2(x)}$, $A_2 = \dfrac{C_x S_x^2}{C_x S_x^2 - \beta_2(x)}$, $A_3 = \dfrac{S_x^2}{S_x^2 - C_x}$, $A_4 = \dfrac{\beta_2(x) S_x^2}{\beta_2(x) S_x^2 - C_x}$,

Yadav et al. (2013) defined the following transformed ratio-product-type estimator

$$t_{YrpG} = (s_y^2 + d) \left[\omega \left(\frac{S_u^2}{s_u^2} \right)^{\tau} + (1-\omega) \left(\frac{s_u^2}{S_u^2} \right)^{\chi} \right] - d \qquad (1.1)$$

$$\text{where } s_u^2 = b s_x^2 + c S_x^2 \text{ and } S_u^2 = (b+c) S_x^2$$

b, c and d are the known constants or parameters of the variables

$$Bias(t_{YrpG}) \cong \tfrac{1}{2} \lambda' S_y^2 \beta_2'(x) \theta_1 \left[w_1 \left\{ \omega \tau^2 + (1-\omega) \chi^2 \right\} + \left\{ \omega \tau - (1-\omega) \chi \right\} \left\{ w_1 - 2\upsilon' \right\} \right] \qquad (1.2)$$

$$MSE(t_{YrpG}) \cong \lambda' S_y^4 \left[\beta_2'(y) + \beta_2'(x) A_{YrpG} (A_{YrpG} - 2\upsilon') \right] \qquad (1.3)$$

$$\text{where } A_{YrpG} = w_1 \left\{ \omega \tau - (1-\omega) \chi \right\}$$

and the corresponding ratio-type estimator of Yadav et al. (2013) at $\omega = 1$ is:

$$t_{YrG} = (s_y^2 + d) \left(\frac{S_u^2}{s_u^2} \right)^{\tau} - d \qquad (1.4)$$

All the variance estimators with one auxiliary variable are minimized at the same point.

2. The Suggested Family of Estimators

2.1 New Ratio-Type Generalized Estimator using Two Auxiliary Variables Information

Taking motivation from Yadav et al. (2013), here is being presented a new generalized estimator based on the information obtained from two auxiliary variables and using the concept of transformation of variances.

$$t_{rG} = (s_y^2 + d) \left[\omega \left(\frac{S_u^2}{s_u^2} \right)^{\tau} + (1-\omega) \left(\frac{S_w^2}{s_w^2} \right)^{\varphi} \right] - d \qquad (2.1)$$

$$\text{where } s_w^2 = m s_z^2 + p S_z^2 \text{ and } S_w^2 = (m+p) S_z^2$$

where m and p are the known constants or parameters in usual, and τ and φ are suitably chosen values.

To find bias and mean of the squared sampling error, the ratio-type generalized estimator can be simplified as:

$$t_{rG} = \left\{ (1+e_0)S_y^2 + d \right\} \left[\omega \left\{ \frac{(b+c)S_x^2}{b(1+e_1)S_x^2 + cS_x^2} \right\}^{\tau} + (1-\omega) \left\{ \frac{(m+p)S_z^2}{m(1+e_2)S_z^2 + pS_z^2} \right\}^{\varphi} \right] - d$$

$$= (S_y^2 + d)(1+\theta_0 e_0) \left[\omega(1+\theta_1 e_1)^{-\tau} + (1-\omega)(1+\theta_2 e_2)^{-\varphi} \right] - d$$

expanding the above expression using Taylor series expansion assuming that $|\theta_1 e_1| < 1$ and $|\theta_2 e_2| < 1$ and omitting throughout the higher-order terms in e

$$t_{rG} \cong (S_y^2 + d)(1+\theta_0 e_0) \left[\omega \left\{ 1 - \tau\theta_1 e_1 + \frac{\tau(\tau+1)}{2}(\theta_1 e_1)^2 \right\} + (1-\omega) \left\{ 1 - \varphi\theta_2 e_2 + \frac{\varphi(\varphi+1)}{2}(\theta_2 e_2)^2 \right\} \right] - d$$

$$t_{rG} - S_y^2 \cong \frac{S_y^2}{\theta_0} \left[\begin{array}{l} \theta_0 e_0 + \omega \left\{ -\tau\theta_1 e_1 + \frac{\tau(\tau+1)}{2}(\theta_1 e_1)^2 \right\} + (1-\omega) \left\{ -\varphi\theta_2 e_2 + \frac{\varphi(\varphi+1)}{2}(\theta_2 e_2)^2 \right\} \\ -\omega\tau\theta_0\theta_1 e_0 e_1 - (1-\omega)\varphi\theta_0\theta_2 e_0 e_2 \end{array} \right]$$

$$Bias(t_{rG}) \cong \tfrac{1}{2}\lambda' \frac{S_y^2}{\theta_0} \left[\omega\tau(\tau+1)\theta_1^2 \beta_2'(x) + (1-\omega)\varphi(\varphi+1)\theta_2^2 \beta_2'(z) - 2\omega\tau\theta_0\theta_1 h' - 2(1-\omega)\varphi\theta_0\theta_2 k' \right]$$

$$Bias(t_{rG}) \cong \tfrac{1}{2}\lambda' S_y^2 \theta_0 \left[\omega\tau(\tau+1)w_1^2 \beta_2'(x) + (1-\omega)\varphi(\varphi+1)w_2^2 \beta_2'(z) - 2\omega\tau w_1 h' - 2(1-\omega)\varphi w_2 k' \right]$$

$$Bias(t_{rG}) \cong \tfrac{1}{2}\lambda' S_y^2 \theta_0 \left[\begin{array}{l} \beta_2'(x)\omega\tau^2 w_1^2 + \beta_2'(x)\omega\tau w_1(w_1 - 2\upsilon') + \beta_2'(z)(1-\omega)\varphi^2 w_2^2 \\ + \beta_2'(z)(1-\omega)\varphi w_2(w_2 - 2\mu') \end{array} \right] \qquad (2.2)$$

Now

$$(t_{rG} - S_y^2)^2 \cong \frac{S_y^4}{\theta_0^2} \left[\begin{array}{l} (\theta_0 e_0)^2 + (\omega\tau\theta_1 e_1)^2 - 2\omega\tau\theta_0\theta_1 e_0 e_1 + (1-\omega)^2(\varphi\theta_2 e_2)^2 \\ -2(1-\omega)\varphi\theta_0\theta_2 e_0 e_2 + 2\omega\tau\theta_1 e_1(1-\omega)\varphi\theta_2 e_2 \end{array} \right]$$

$$\cong S_y^4 \left[e_0^2 + W_1^2 e_1^2 - 2W_1 e_0 e_1 + W_2^2 e_2^2 - 2W_2 e_0 e_2 + 2W_1 W_2 e_1 e_2 \right]$$

$$MSE(t_{rG}) \cong \lambda' S_y^4 \left[\beta_2'(y) + W_1^2 \beta_2'(x) - 2W_1 h' + W_2^2 \beta_2'(z) - 2W_2 k' + 2W_1 W_2 l' \right]$$

$$MSE(t_{rG}) \cong \lambda' S_y^4 \left[\beta_2'(y) + \beta_2'(x)W_1(W_1 - 2\upsilon') + \beta_2'(z)W_2(W_2 - 2\mu') + 2W_1 W_2 l' \right] \qquad (2.3)$$

$MSE(t_{rG})$ is minimized at

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} (\beta_2'(z)h' - k'l')/D \\ (\beta_2'(x)k' - h'l')/D \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \beta_2'(z)(h' - \mu'l')/D \\ \beta_2'(x)(k' - \upsilon'l')/D \end{bmatrix} \qquad (2.4)$$

3. Special Cases

Following are the previously existing estimators and are the special cases of t_{rG}:

Table 1

Existing Estimators as special cases of t_{rG}

S.No.	Estimator	ω	d	b	c	m	p	τ	φ

1.	s_y^2	1	0	0	c	m	p	1	φ
2.	t_{Isaki}	1	0	1	0	m	p	1	φ
3.	t_{US}	1	0	1	$\dfrac{\beta_2(x)}{S_x^2}$	m	p	1	φ
4.	t_{CH1}	1	0	1	$\dfrac{-\beta_2(x)}{S_x^2}$	m	p	1	φ
5.	t_{CH2}	1	0	C_x	$\dfrac{-\beta_2(x)}{S_x^2}$	m	p	1	φ
6.	t_{CH3}	1	0	1	$\dfrac{-C_x}{S_x^2}$	m	p	1	φ
7.	t_{CH4}	1	0	$\beta_2(x)$	$\dfrac{-C_x}{S_x^2}$	m	p	1	φ

Several further estimators can be proposed from t_{rG}, two of which are as under:

Table 2

Estimators from t_{rG}

S. No.	Estimator	ω	d	b	c	m	p	τ	φ
8.	$t_{rG1} = \left\{ s_y^2 - \beta_2(y) \right\} \left[\begin{array}{c} \omega \left\{ \dfrac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} \\ + (1-\omega) \\ \left\{ \dfrac{S_z^2 + \beta_2(z)}{s_z^2 + \beta_2(z)} \right\}^{\varphi} \end{array} \right] + \beta_2(y)$ (3.1)	ω	$\beta_2(y)$	1	$\dfrac{-\beta_2(x)}{S_x^2}$	1	$\dfrac{\beta_2(z)}{S_z^2}$	1	1
9.	$t_{rG2} = s_y^2 \left[\begin{array}{c} \omega \left\{ \dfrac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} \\ + (1-\omega) \left\{ \dfrac{S_z^2 + \beta_2(z)}{s_z^2 + \beta_2(z)} \right\}^{\varphi} \end{array} \right]$ (3.2)	ω	0	1	$\dfrac{\beta_2(x)}{S_x^2}$	1	$\dfrac{\beta_2(z)}{S_z^2}$	1	1

The corresponding ratio-product-type generalized estimators, for the known constants and parameters, from t_{YrpG} are given below:

$$t_{YrpG1} = \left\{ s_y^2 - \beta_2(y) \right\} \left[\omega \left\{ \frac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} + (1-\omega) \left\{ \frac{C_x s_x^2 + \beta_2(x)}{C_x S_x^2 + \beta_2(x)} \right\}^{\chi} \right] + \beta_2(y) \qquad (3.1.1)$$

$$t_{YrpG2} = s_y^2 \left[\omega \left\{ \frac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} + (1-\omega) \left\{ \frac{C_x s_x^2 + \beta_2(x)}{C_x S_x^2 + \beta_2(x)} \right\}^{\chi} \right] \qquad (3.2.1)$$

and the parallel ratio-type generalized estimators of t_{YrG} are:

$$t_{YrG1} = \left\{ s_y^2 - \beta_2(y) \right\} \left\{ \frac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} + \beta_2(y) \qquad (3.1.2)$$

$$t_{YrG2} = s_y^2 \left\{ \frac{C_x S_x^2 + \beta_2(x)}{C_x s_x^2 + \beta_2(x)} \right\}^{\tau} \qquad (3.2.2)$$

4. Efficiency Comparison

Comparison of the performance of estimators (2.1) and (1.1) by MSE

$$MSE\left(t_{rG}\right) < MSE\left(t_{YrpG}\right)$$

$$\lambda' S_y^4 \left[\beta_2'(y) + \beta_2'(x)W_1\left(W_1 - 2\upsilon'\right) + \beta_2'(z)W_2\left(W_2 - 2\mu'\right) + 2W_1W_2 l' \right] <$$

$$\lambda' S_y^4 \left[\beta_2'(y) + \beta_2'(x) A_{YrpG}\left(A_{YrpG} - 2\upsilon'\right) \right]$$

if $\beta_2'(z)W_2\left\{ W_2 - 2\mu' + 2W_1 \,{l'}\big/{\beta_2'(z)} \right\} - \beta_2'(x)(1-\omega)\chi \dfrac{\theta_1}{\theta_0}\left\{ -2W_1 + (1-\omega)\chi \dfrac{\theta_1}{\theta_0} + 2\upsilon' \right\} < 0$ (4.1)

and comparison of the biases of the estimators (2.1) and (1.1) is:

$$\beta_2'(x)w_1^2\left\{ \omega\tau^2 \right\} + \beta_2'(x)\left\{ \omega\tau \right\}w_1\left\{ w_1 - 2\upsilon' \right\}$$
$$+ \beta_2'(z)(1-\omega)\varphi^2 w_2^2 + \beta_2'(z)(1-\omega)\varphi w_2\left(w_2 - 2\mu' \right) <$$
$$\beta_2'(x)w_1\left[w_1\left\{ \omega\tau^2 + (1-\omega)\chi^2 \right\} + \left\{ \omega\tau - (1-\omega)\chi \right\}\left\{ w_1 - 2\upsilon' \right\} \right]$$

$$-\beta_2'(x)\psi(1-\omega)^2 w_1^2 + \beta_2'(x)(1-\omega)\chi w_1^2 - 2\beta_2'(x)(1-\omega)\chi w_1\upsilon'$$
$$+\beta_2'(z)(1-\omega)\varphi^2 w_2^2 + \beta_2'(z)(1-\omega)\varphi w_2^2 - 2\beta_2'(z)(1-\omega)\varphi w_2\mu' < 0$$

$$-\beta_2'(x)\chi(\chi-1)w_1^2 - 2\chi w_1 h' + \beta_2'(z)\varphi(\varphi+1)w_2^2 - 2\varphi w_2 k' < 0 \qquad (4.2)$$

holds if (1.1) is positively biased otherwise

$$-\beta_2'(x)\chi(\chi-1)w_1^2 - 2\chi w_1 h' + \beta_2'(z)\varphi(\varphi+1)w_2^2 - 2\varphi w_2 k' > 0 \qquad (4.3)$$

(4.3) is the required condition.

Therefore, (2.1) is more efficient than (1.1) if (4.1) and (4.2) / (4.3) are satisfied accordingly.

It is important to note that percentage relative efficiency (PRE) has been used to make empirical analysis easier.

$$\mathrm{PRE}\left(t_{..}\right) = \frac{Var\left(s_y^2\right)}{MSE\left(t_{..}\right)} \times 100$$

The estimator $t_{..}$ is preferred over the other estimator if PRE $\left(t_{..}\right)$ is higher than the PRE of the other estimator under consideration and has smaller bias irrespective of algebraic sign**.**

5. Empirical Study

To conduct the empirical analysis, secondary data have been taken from an authentic source:
Data 1
Source: Basic Econometrics, 4th edition Gujarati and Sangeetha (2007) Page: 189
Data collected from 64 countries regarding fertility and other factors:

y : number of children died in a year under age 5 per one thousand live-births.

x : literacy rate of females in percentage.

z : total fertility rate during 1980-1985

Data 2
Source: Basic Econometrics, 4th edition Gujarati and Sangeetha (2007) Page: 189
Data collected from 64 countries regarding fertility and other factors

y : number of children died in a year under age 5 per one thousand live-births.

x : total fertility rate during 1980-1985,

z : literacy rate of females in percentage

Table 3

Data	C_y	C_x	C_z	S_y^2	S_x^2	S_z^2
1	0.536947	0.50809	0.271906	5772.6667	676.40873	2.27706
2	0.536947	0.271906	0.50809	5772.6667	2.27706	676.40873
Data	$\beta_2(y)$	$\beta_2(x)$	$\beta_2(z)$	ρ_{xy}	ρ_{zy}	ρ_{xz}
1	2.378336	1.657521	2.816908	-0.818285	0.671135	-0.625954
2	2.378336	2.816908	1.657521	0.671135	-0.818285	-0.625954
Data	h	k	l			
1	1.437668	1.481965	1.0868699			
2	1.481965	1.437668	1.0868699			

5.1. Empirical Analysis (Data 1)

Table 4

Percentage Relative Efficiency of the Estimators

Estimators	s_y^2	t_{Isaki}	t_{US}	t_{CH1}	t_{CH2}	t_{CH3}	t_{CH4}
PRE	100	118.76878	118.87848	118.65794	118.54952	118.73492	118.74837

Table 5

PRE and Bias of t_{rG} for different values of ω, τ and φ ; and the corresponding t_{YrpG} for χ also:

S. No.	Constants				PRE		Bias	
	ω	τ	χ	φ	t_{rG}	t_{YrpG}	t_{rG}	t_{YrpG}
8	0.5	1	0.2	0.5	133.784222	121.531857	0.24365	0.25032
	0.55	1	0.2	0.5	134.176776	123.566106	0.26235	0.26835

S. No.	Constants				PRE		Bias	
	ω	τ	χ	φ	t_{rG}	t_{YrpG}	t_{rG}	t_{YrpG}
	0.6	1	0.2	0.5	134.111550	125.135099	0.28104	0.28638
	0.65	1	0.2	0.5	133.589875	126.200772	0.29974	0.30441
	1	1			119.001849		0.43060	
9	0.5	1	0.2	0.5	128.848573	121.538043	0.19436	0.25067
	0.55	1	0.2	0.5	129.546687	123.571819	0.21805	0.26874
	0.6	1	0.2	0.5	129.853354	125.139821	0.24175	0.28680
	0.65	1	0.2	0.5	129.762980	126.203972	0.26544	0.30486
	1	1			118.983606		0.43131	

Table 6

PRE and Bias of t_{rG} for different values of ω, τ and φ; and the corresponding t_{YrG}

S. No.	Constants			PRE		Bias	
	ω	τ	φ	t_{rG}	t_{YrG}	t_{rG}	t_{YrG}
8	0.5	1	1	137.797731		0.36270	
	0.55	1	1	138.527037	119.001849	0.36949	0.43060
	0.6	1	1	138.608534		0.37628	
	0.65	1	1	138.039937		0.38307	
	1	1		119.001849		0.43060	
9	0.5	1	1	132.371164		0.1878	
	0.55	1	1	132.825662	118.983606	0.21215	0.43131
	0.6	1	1	132.847126		0.23651	
	0.65	1	1	132.435137		0.26086	
	1	1		118.983606		0.43131	

Table 7

PRE and Bias of t_{rG} for few more values of ω, τ and φ; and the corresponding t_{YrG}

S. No.	Constants			PRE		Bias	
	ω	τ	φ	t_{rG}	t_{YrG}	t_{rG}	t_{YrG}
8	0.4	1	0.5	131.666623		0.20627	
	0.5	1	0.5	133.784222	119.001849	0.24365	0.43060
	0.6	1	0.5	134.111550		0.28104	

S. No.	Constants			PRE		Bias	
	ω	τ	φ	t_{rG}	t_{YrG}	t_{rG}	t_{YrG}
	0.7	1	0.5	132.622331		0.31843	
	1	1		119.001849		0.43060	
9	0.4	1	0.5	126.334795	118.983606	0.14696	0.43131
	0.5	1	0.5	128.848573		0.19436	
	0.6	1	0.5	129.853354		0.24175	
	0.7	1	0.5	129.277216		0.28914	
	1	1		118.983606		0.43131	

Table 8

Percentage Relative Efficiency of the Ratio-Type Estimators for minimized MSE

Estimators	s_y^2	t_{CHi}	t_{YrG}	t_{rG}
PRE	100	126.80077	126.80077	139.59701

5.2. Empirical Analysis (Data 2)

Table 9

Percentage Relative Efficiency of the Estimators

Estimators	s_y^2	t_{Isaki}	t_{US}	t_{CH1}	t_{CH2}	t_{CH3}	t_{CH4}
PRE	100	-	105.176214	-	-	-	-

The blank cells show that estimators for this data set are not efficient.

Table 10

PRE and Bias of t_{rG} for different values of ω, τ and φ; and the corresponding t_{YrpG} for χ also:

S. No.	Constants				PRE		Bias	
	ω	τ	χ	φ	t_{rG}	t_{YrpG}	t_{rG}	t_{YrpG}
8	0.55	1	0.2	0.5	120.840287	105.140763	0.00518	0.01927
	0.65	1	0.2	0.5	118.752862	106.23297	0.01642	0.02737
	0.75	1	0.2	0.5	116.325858	107.206367	0.02765	0.03548
	0.85	1	0.2	0.5	113.604609	108.053776	0.03888	0.04358
	1	1		1	109.07487		0.05573	
9	0.55	1	0.2	0.5	120.751016	105.142578	0.00686	0.01925
	0.65	1	0.2	0.5	118.678816	106.23503	0.01771	0.02735
	0.75	1	0.2	0.5	116.270598	107.208559	0.02857	0.03545
	0.85	1	0.2	0.5	113.570895	108.055979	0.03942	0.04355

S. No.	Constants				PRE		Bias	
	ω	τ	χ	φ	t_{rG}	t_{YrpG}	t_{rG}	t_{YrpG}
	1	1		1	109.076852		0.05571	

Table 11

PRE and Bias of t_{rG} for different values of ω, τ and φ ; and the corresponding t_{YrG}

S. No.	Constants			PRE		Bias	
	ω	τ	φ	t_{rG}	t_{YrG}	t_{rG}	t_{YrG}
8	0.55	1	0.5	120.840287	109.074870	0.00518	**0.05573**
	0.65	1	0.5	118.752862		0.01642	
	0.75	1	0.5	116.325858		0.02765	
	0.85	1	0.5	113.604609		0.03888	
	1	1	1	109.07487		0.05573	
9	0.55	1	0.5	120.751016	109.076852	0.00507	**0.05571**
	0.65	1	0.5	118.678816		0.01632	
	0.75	1	0.5	116.270598		0.02757	
	0.85	1	0.5	113.570895		0.03883	
	1	1	1	109.076852		0.05571	

Table 12

Percentage Relative Efficiency of the Estimators for minimized MSE

Estimators	s_y^2	t_{CHi}	t_{YrG}	t_{rG}
PRE	100	110.223930	110.223930	139.597011

6. Conclusion

In this paper, a new estimator termed as ratio-type generalized variance estimator utilizing two auxiliary variables information has been proposed. The bias and mean of the squared sampling error of the estimator have been derived. The efficiency of these two estimators has also been compared with Yadav et al. (2013) estimator mathematically. Hence, several new estimators can be generated of which two are depicted here and proved more efficient mathematically under two conditions; and empirically for different values of scalars and constants, with two sets of data, as well. Even it can also be noticed, for the Data 2, where the other biased estimators under consideration cannot attain efficiency over the unbiased estimator, the proposed estimator still proves its efficiency. It can also be observed that at $\omega = 1$; t_{YrpG}, t_{YrG} and t_{rG} produce similar estimator.

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