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Ber Analysis For Downlink Massive Mimo Under Perfect Channel State Information.

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Abstract: *By equipping many antennas on the base station (BS) side, massive MIMO system has proved to achieve extraordinary spectral efficiency, but still its performance is restricted by pilot contamination due to inevitable reuse of pilot sequences from user terminals in other cells. The main aim of this paper is to investigate and analyze the performance of single-cell massive MIMO on the downlink ergodic achievable sum-rate under imperfect CSI with linear precoding schemes namely Zero-forcing (ZF) and maximum ratio transmission (MRT) with minimum mean square error (MMSE) used to give the estimation error. The analysis through simulations have been done using MATLAB software, and the results have indicated that, in the presence of large channel estimation error there is no any ergodic sum- rate achieved while if the estimation error is very small then the imperfect channel tends to be like that of the perfect channel. About linear precoding scheme, the ZF pre-coder outperforms the MRT pre-coder in the high SNR as shown in different simulations graphs in which the high ergodic sum-rate was always high for ZF compared to MRT. In the future, the same analysis using FDD transmission mode under imperfect channel state information must be conducted.*

Keywords- Bit Error Rate (BER), Linear Precoding scheme, Maximum Transmit Ratio (MRT), Zero forcing (ZF), Perfect Channel State Information (CSI), Massive MIMO, Time Division Duplex (TDD).

I. Introduction

Massive multiple-input multiple-output (MIMO) networks describes the knowledge of equipping base stations (BSs) with many antennas that serves tens of user equipment antennas, and it has shown an improvement in spectral and energy efficiency using comparatively linear processing [1]. Theoretically, the number of BS antennas (M) must be more than the number of users (K) that are served by those BS antennas i.e. M>>K or in simple words, hundreds or thousands of BS antennas should serve only tens of user terminals [2].

Having massive MIMO system, it is very simple for hundreds of antennas or more to serve tens of users and by doing this the users are then served with high data rate all together, and consequently the technology offers large network capacities in multi-user scenarios [3].

Even though the transmission systems operates in the “massive MIMO situations” they also depends on other several parameters like the number of base station (BS) antennas per user equipment, division duplex mode and precoding schemes like Maximum Ratio Transmission (MRT), Zero-forcing (ZF) and Minimum Mean Square Error (MMSE). In TDD mode the channel reciprocity is largely exploited which makes the training overhead to scale down linearly with the number of users, so adding antenna elements can come at no overhead cost making it simple to improve the system performance [4]. Maximum ratio transmission (MRT) and zero-forcing (ZF) have been proved to be the practical linear pre-coders in Massive MIMO [5-7].

Very few literatures have dealt with the analysis of BER for downlink massive MIMO like Yu *et* al in [8], stressed a little bit about the BER and outage probability for Multiuser scheduling MIMO(MUS-MIMO) by using elective transmission/selective combining (ST/SC), ST/maximum ratio combining (MRC) in which they found that their simulation results are in good agreement with that of theoretical ones.

Wang *et* al [9], proposed an Approximate Minimum Bit Error Rate (AMBER) algorithm which alleviates error propagation in the interference cancellation and as a results it performed better than normal pre-coders because it reduced the transmit processing complexity and the overheads in feedback which eventually improved the error rate performance.

There has been substantial linear analysis on downlink precoding for massive MIMO in terms of achievable sum rate, downlink transmit power, spectral efficiency and energy efficiency with respect to the number of BS antennas, number of users, SNR, and even channel estimation errors using two principal linear precoding techniques which are Zero-forcing (ZF) and Maximum Ratio Transmit (MRT), but very little emphasis has been placed on the analysis of BER on the downlink transmission with the mentioned parameters. Therefore, in this paper we will analyze the BER of massive MIMO using two linear precoding schemes (ZF and MRT) under the known or perfect CSI.

In this paper, we concentrate on the downlink TDD mode where by the uplink and downlink transmission are taken as perfectly reciprocal to each other. In TDD mode the BS transfers multiple data streams to each user concurrently and selectively with CSI by doing so all the users then estimates the channel information and feed them back to the BS antenna for CSI acquisition [10].

Contributions this paper offers are:

- We first derive the approximated relationship between BER and the Signal to Noise Ratio (SNR) under ZF and then MRT with different defined parameters. The derived approximated formula are accurate and very simple to analyze.

- We then analyze and compare the performance of the BER with respect to ZF and MRT when varying different parameters of our simulation in which theoretically BER must be low even when the number of BS antennas increases.

- By numerical and graphical simulations, we show which linear precoding scheme gives better performance in terms of BER under varying conditions of the chosen parameters.

II. SYSTEM MODEL

Consider the downlink massive MIMO systems which is equipped with *M* BS antennas that serves K users antenna in the TDD mode in which the uplink and the downlink share the same channel at various time. This makes the BS to be able to estimate the channel from known pilots from users.

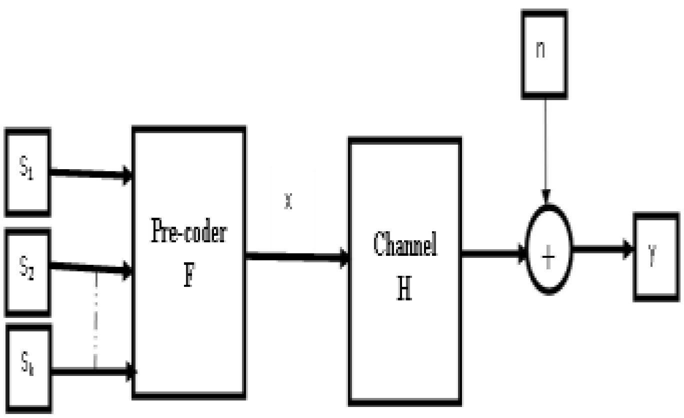

Figure 1: Diagram showing downlink massive MIMO.

If we let s_k be the symbol that is transmitted to k^{th} user where $E\left\{\left|s_k\right|^2\right\}=1$, we denote also $H \in \mathbb{C}^{M\times K}$ be the channel matrix between the user and the BS in which H is assumed to be *i.i.d* Gaussian distributed having zero mean and unit variance, let also $F \in \mathbb{C}^{M\times K}$ be the linear pre-coding matrix and is a function of channel matrix H. The $M \times 1$ transmit vector is given by:

$$x = \sqrt{P_d}\, Fs$$

Where $s = \left[s_1, s_2, s_3 \ldots, s_k\right]^T$ and P_d is the downlink transmit power from the BS, which satisfies power constraint and the channel matrix F is chosen such that $E\left\{\| x \|^2\right\} = P_d$ which is the same as $E\left\{tr\left(FF^H\right)\right\} = 1$.

The received vector signal at the K users is [11]:

$y = H^T x + n$ But $x = \sqrt{P_d}\, Fs$ so:

$$y = H^T \sqrt{P_d}\, Fs + n$$

Where n is a noise vector with n_k being the additive noise of the k^{th} user and $n_k \sim CN(0,1)$. If we let $a_{ki} = h_k^{\ T} f_i$ where h_i and f_i are the i^{th} columns of H and F respectively, then the signal received at the k^{th} user is [11]:

$$y_k = \sqrt{P_d}\, a_{kk} s_{kk} + \sqrt{P_d} \sum_{i \neq k}^{K} a_{ki} s_i + n_k \qquad (1)$$

Where $\sqrt{P_d}\, a_{kk} s_{kk}$ =desired signal, $\sqrt{P_d} \sum_{i=1, i\neq k}^{K} a_{ki} s_i$ =interference term and n_k =noise term .

i. Signal-Interference-Noise Ratio (SINR)

This is the ratio of signal to the sum of intrusion and noise of the channel, in which for the quality signal this ratio must be huge enough to cancel out any noises and intrusion associated with the channel. In this description, the SINR for MRT and ZF precoding under and imperfect perfect channel state

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information are derived from [12].

A well-known *SINR* definition is defined as following:

$$SINR_n = \frac{S_n}{I_n + N_n}$$

Where S_n , I_n and N_n are the power of the desired signal, interference and the noise respectively. . For a received signal y_k, the signal to interference plus noise ratio *(SINR)* of the user k is written as [13]:

$$SINR_k = \frac{P_d \left|a_{kk}\right|^2}{P_d \sum_{i=1, i\neq k}^{K} \left|a_{ki}\right|^2 + 1} \qquad (2)$$

ii. Linear Precoding schemes

For capacity lower bound, there are two simple (with low complexity) linear precoding techniques usually used at the BS in downlink transmission, which are ZF and MRT/MF respectively.

a) ZF Precoding

This is a linear pre-coding scheme, which cancels out inter-user- interference at each user, and it is assumed to implement a pseudo-inverse of the channel matrix.

ZF precoding at BS is given as:

$$F = \beta\left\{H^H \left(HH^H\right)^{-1}\right\} \qquad (3)$$

Where β is a scaling factor that satisfies the transmit power constraint at the BS i.e. $E\left\{tr\left(FF^H\right)\right\} = 1$ and is given as:

$$\beta = \sqrt{\frac{1}{tr(AA^H)}} \qquad (4)$$

Where $A = H^H \left(HH^H\right)^{-1}$

In order to satisfy the power control, the precoding matrix must always be normalized.

The approximated kth user $SINR_k{}^{ZF}$ under perfect CSI for large values of M , K and with lower bound vector and at low SNR is given as [5, 14]:

$$SINR_k{}^{ZF} = P_d(\alpha - 1) \qquad (5)$$

Where $\alpha = \frac{M}{K}$ then $SINR_k{}^{ZF} = \frac{P_d(M-K)}{K}$

b) Maximum ratio transmit (MRT) precoding

MRT is one of the linear precoding scheme that maximizes the signal gain of the planned user [13, 15].The MRT pre-coding at the BS is given as:

$$F = \beta\left\{H^H\right\} \qquad (6)$$

Where β is a scaling factor that satisfies the transmit power constraint at the BS i.e. $E\left\{tr\left(FF^H\right)\right\} = 1$ and is given as:

$$\beta = \sqrt{\frac{1}{tr(AA^H)^{-1}}}$$

Where . $A = H^H$

The approximated kth user $SINR_k{}^{MRT}$ under perfect CSI for large values of M , K and with lower bound vector and at low SNR is given as [5, 14]:

$$SINR_k{}^{MRT} = \frac{P_d M}{P_d(K-1) + K} \qquad (7)$$

BIT ERROR RATE (BER)

Bit Error Rate (BER) is ratio of the number of bits having errors that are received to the total number of bits transferred (or received) in any communication channel. The number of bits may usually be affected by noise, distortions or interference during transmission. BER usually helps in determination of the quality of signal, and in most cases, the lower the BER the better the quality of the signal transmitted. The idea of having massive MIMO is to improve the quality of signal that were missing in conventional MIMO, so having very low BER will prove that concept of massive MIMO.

In this paper we assume the BER for Rayleigh fading channels and for k^{th} user it is given by [16]:

$$BER_k^{PR} = \frac{1}{2}\left(1 - \sqrt{\frac{SINR_k^{PR}}{1 + SINR_k^{PR}}}\right) \qquad (8)$$

Where PR =linear precoding scheme (ZF or MRT)

For ZF the BER is:

$$BER_k^{ZF} = \frac{1}{2}\left(1 - \sqrt{\frac{SINR_k^{ZF}}{1 + SINR_k^{ZF}}}\right) \qquad (9)$$

But $SINR_k^{ZF} = \frac{P_d(M-K)}{K}$ Substituting $SINR_k^{ZF} = \frac{P_d(M-K)}{K}$ in **(9)** above we get:

$$BER_k^{ZF} = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P_d(M-K)}{K}}{1 + \frac{P_d(M-K)}{K}}}\right)$$ then

$$BER_k^{ZF} = \frac{1}{2}\left(1 - \sqrt{\frac{P_d(M-K)}{P_d(M-K)+K}}\right) \qquad (10)$$

For MRT the BER is:

$$BER_k^{MRT} = \frac{1}{2}\left(1 - \sqrt{\frac{SINR_k^{MRT}}{1 + SINR_k^{MRT}}}\right) \qquad (11)$$

But $SINR_k^{MRT} = \frac{P_d M}{P_d(K-1)+K}$ Substituting in **(11)** above we get

$$BER_k^{MRT} = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P_d M}{P_d(K-1)+K}}{1 + \frac{P_d M}{P_d(K-1)+K}}}\right)$$

$$BER_k^{MRT} = \frac{1}{2}\left(1 - \sqrt{\frac{P_d M}{K + P_d(K-1) + P_d M}}\right) \qquad (12)$$

III. SIMULATION RESULTS AND DISCUSSION

In order to give basic proof on the derived equations, simulations of basic parameters involved in this paper for BER is done in this section. Main assumption made is that the massive MIMO system in this regard operates in TDD mode in which the uplink and downlink transmission shares the same resources, but the concentration was much on the downlink transmission side as the simulations figures below.

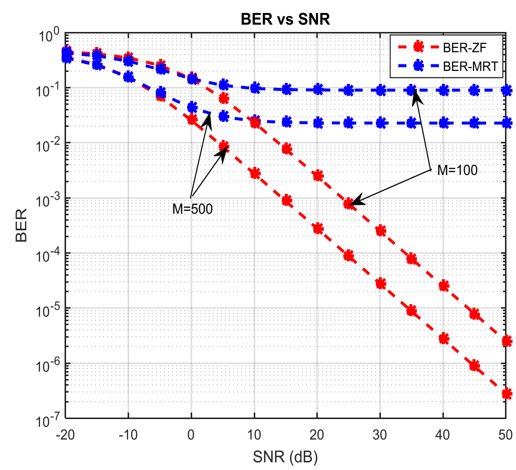

Figure 2a: Diagram showing the BER against SNR when K=50

As it can be seen in the figure above the BER for ZF linear precoding scheme is better compared to the BER of the MRT precoding scheme under the same conditions of BS antennas and the number of user terminals K, also when the number of BS antennas increases or grows large the quality of signal improves gradually depending on the number of added BS antennas for example the performance at M=500 is far better than the performance when M=100 for both linear pre-coders (MRT and ZF) when the number of users is kept constant i.e. K=50.

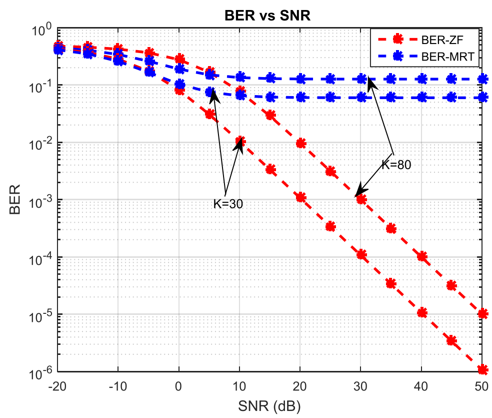

Figure 2b: Diagram showing the relationship between BER and SNR when M=100

As it can be seen in the figure above, BER for ZF pre-coding still provides better performance or gives high signal quality compared to that of MRT in both varying conditions of K when the number of BS antennas M is kept constant i.e. M=100. Also increasing the number of users K has detrimental effects on the quality of signal as it can be seen in the figure above, because adding more users while keeping all other conditions the same degrades the signal as the users have to share the available resources which was intended for the specific number of users and hence cause an increase in BER which accounts to the poor quality of signal. For example the BER for ZF precoding at K=30 is 10^{-6} while at K=80 the BER is 10^{-5} which shows there is an increase in BER for an increased number of users and for MRT at K=30 the BER is 0.08 while at K=80 the BER is 0.2.

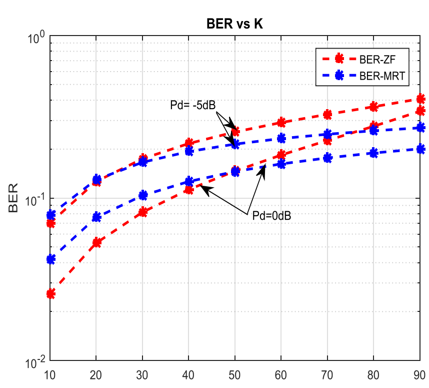

Figure 3: Diagram showing the relationship between BER and the number of users K when M=100.

As it can be seen in the figure above as the number of users increases, the performance of massive MIMO in terms of BER becomes poor due to the restricted number of BS antennas serving the intended users, also as the downlink transmit power increases at least the performance improves a little bit in order to cover for the additional number of users. But for the case of linear precoding as it can be seen, the MRT linear precoder provides good performance as the number of users increases compared to ZF which gives good BER when there are only few users for the increased downlink transmit power. So in general ZF precoder works better when the number of users is small at high downlink transmit power.

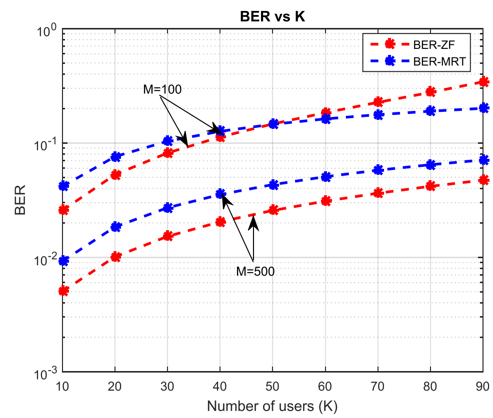

Figure 4: Diagram showing the relationship between BER and the number of users K at Pd=0dB

Figure above shows the relationship between BER and the number of users under varying condition of the number of BS antennas serving the specified number of users at Pd=0dB, it can be seen that, adding more number of BS antennas to serve the fixed number of users increases the performance and in that regard, the ZF linear pre-coding scheme works better than the MRT linear pre-coder in the sense that the BER for ZF when M=500 is far better than that of MRT but for when M=100, the BER of MRT scheme is better for a large number of users under the same condition of Pd.

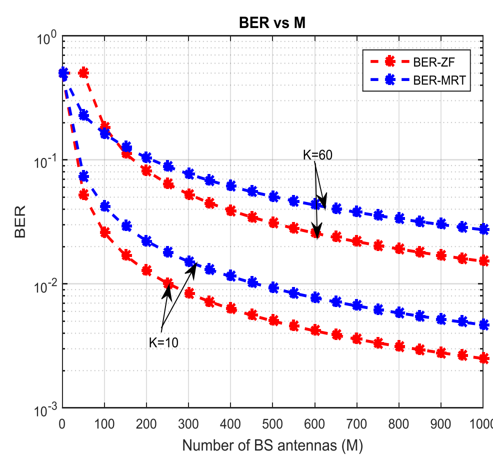

Figure 5: Showing the relationship between BER and the number of BS antennas when Pd=0dB

As shown in figure above, the BER is better for ZF precoding scheme than MRT precoding scheme for an increased number of BS antennas, but as it becomes customs now when adding

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more number of users, the performance of degrades gradually for both linear precoders schemes. For example the value of BER for ZF and MRT at K=10 is far better than the value of BER at K=60. Also when increasing the number of BS antennas M, the BER improves rapidly for both ZF and MRT as shown.

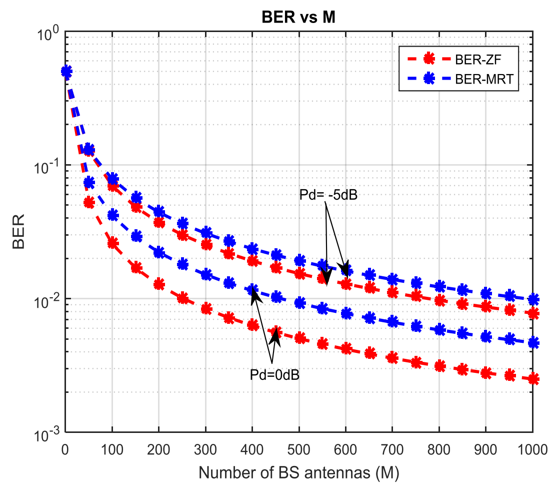

Figure 6: Showing the relationship between BER and the number of BS antennas when K=10

Figure 6 explain about the variation of BER with the number of BS antennas when the number of users is fixed at K=10 but the downlink transmit power is changed according to simulation requirements which was about to find out the impact of downlink transmit power on BER under increasing number of BS antennas under MRT and ZF linear pre-coders, it was found that, the ZF linear pre-coder gives better BER compared to MRT under the same conditions but increasing downlink transmit power improves the signal quality for both pre-coders even though the number of BS antennas keeps increasing and as it can be seen increasing more BS antennas again improves BER.

IV. CONCLUSSION

In this paper, the analysis of BER of massive MIMO with respect to various parameters under perfect CSI and using linear precoding scheme of ZF and MRT have been studied. The main parameters analyzed were SNR, number of BS antennas (**M**) and number of users (**K**) which were theoretically derived for both linear precoding schemes from equations (10) and (11). In most of the analysis done above, it was found that the ZF pre-coding scheme have an edge over MRT pre-coding scheme under the given conditions for BER, except for the case of the relationship between BER and the number of users, in which when the number of users grows high under low downlink transmit power, the BER for MRT were better than that of ZF linear pre-coder. So from the above observations, it may easily be concluded that BER for ZF linear precoding gives best performance than that of the MRT linear precoding scheme under the same conditions.

V. RECOMMENDATION AND FUTURE WORK

As per analysis made in this paper, the authors would like to recommend the same analysis to be conducted under imperfect channel state information in order to have good comparisons of the two linear precoding schemes under TDD mode of operation. In addition, as this paper analyzed the BER by assuming capacity low bound, analysis on the capacity upper bound of these two pre-coders must also be taken into considerations.

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