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New types of filters in Q-algebra

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Abstract: *In this paper, we studied some of the new types of filters, called (Q-filter, complete Q-filter, S-filter, and complete S-filter). Also, we presented and proved several propositions that determine the relationships among them that are discussed in Q-algebra.*

Keywords: Q-algebra, Q-filter, S-filter, C-Q-filter ,C-S-filter.

1.Introduction

Iseki and Imai presented two notions of abstract algebras: BCI-algebras and BCK-algebras [6]. The notion of BCK-algebras is proper subclass of the notion of BCI-algebras. In 2001, Kim, Neggers and Ahn defined generalization of BCI/ BCH/BCK-algebras as a new notion called Q-algebra[1].Also they generalized some propositions discussed in BCI-algebras. Y.B.Jun, satisfactory filter of BCK-algebras[2].J.Meng BCK- filter[3]. Abdullah.H.K., Radhi.K.T at (2016) introduced the connotation of T-filter in BCK- algebra,[4]. Abdullah.H.K., Jawad.H.K. at (2018) introduce a new types of Ideal in a Q-algebra,[5].The main purpose of this paper is to define new types of filter and also some of theorems which explain relationships among them in bounded Q-algebra.

2. Basic concept and Notations

In this part, we provide the definition of Q-algebra, bounded, an involutory, Q-filter and some of their properties **.**

Definition (2.1) [1]
A Q-algebra is a set D with a binary operation * and constant 0 which satisfied the following axioms:
(1)$d * d = 0$
(2)$d * 0 = d$
(3)$(d * k) * m = (d * m) * k, \forall d, k, m \in D$
Remark (2.1) [1]
In a Q-algebra D, we can describe a partial order $relation \leq$ by which $d \leq k \; if \; and \; only \; if$ $d * k = 0, \forall d, k \in D.$

Definition (2.2) [4]
If (D,*,0) is a Q-algebra, we call X is bounded if there is an element $e \in D$ satisfying $d \leq e$ for all $d \in D,$ then e is said to be a unit of X. For every $d \in D$ in bounded Q-algebra D, we denoted $e * d$ by d^*.
Example (2.1)
A binary operation * with D={0,1,2,3}can be shown in table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

Thereafter (D,*,0) be a Q-algebra [1] . Notice that D is bounded with unit 3.
Remark (2.2) [4]
The unit in bounded Q-algebra not be a unique as explain in the following example.
Example (2.2)
A binary operation * with D={0,1,2}[1], can be shown in table:

*	0	1	2
0	0	0	0
1	1	0	0
2	2	0	0

Notice that D is a bounded with two units 1,2.

Remark (2.3)

In Q-algebra, we will study the bounded with one unit only.

Proposition(2.1) [4]

In a bounded Q-algebra $D, a, b \in D$, the following are hold

(1) $e^* = 0, 0^* = e$

(2) $a^* * b = b^* * a$

(3) $0 * b = 0$

(4)$e^* * a = 0$.

Definition(2.3) [4]

For bounded Q-algebra D ,if the element d of D satisfy $d^{**} = d$, *then* d *is called an involution.*
If every element of D *is an involution, we call* D *is an involutory* $Q - algebra$.

Example(2.3)

A binary operation * with D={0,1,2,3,4}, can be shown in table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	2	1	0	4
4	4	0	0	0	0

Thereafter (D,*,0) is a bounded Q-algebra with unit 3. Notice D is an involutory **.**

Proposition(2.2)

In bounded an involutory Q-algebra D ,if

(1) If $a \le b^*$then $b^{**} \le a^*$

(2) $a * b = b^* * a^*$

(3) $a * b^* = b * a^*$

(4) $a^* * b^* \le b * a$

Definition (2.4)

Let (D,*,0) be a Q-algebra , if F is a nonempty subset of D that satisfy two condition

(1) $e \in F$ (2) $(a^* * b^*)^* \in F$ and $b \in F$ implies $a \in F$., $for\ all\ a, b \in D$, then F is said to be a Q-filter of D.

Example(2.4):

A binary operation * with D={0,1,2,3} can be shown in table :

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	0	3	0

Thereafter (D,*,0) be a bounded Q-algebra with unit 3. If F={1,3} be a Q-filter.

Proposition(2.3):

If F is a Q - filter and 0 belong to filetr, then $F = D$.

Proof:

It is clear $(a^* * 0^*)^* = (a^* * e)^* = 0^* = e \in F$,for all a $\in D$

Proposition(2.4):

The intersection collection of Q-filter is a Q –filter**.**

Proof:

Let { $F_i, i \in \Delta$}be a collection of Q-filter in bounded Q-algebra D , so e$\in F_i, \forall i \in \Delta$,
e$\in \cap_{i\in\Delta} F_i$
Now, Let ($a^* * b^*)^* \in \cap\, i_{i\in\Delta} F_i$,y$\in \cap_{i\in\Delta} F_i$ $then$
$(a^* * b^*)^* \in F_i$, b$\in F_i, \forall i \in \Delta$,
because F_i is a Q-filter, $\forall i \in \Delta$,
so $a \in F_i, \forall i \in \Delta$. Thus $a \in \cap_{i\in\Delta} F_i$.

Remark(2.4):.

By seeing, the union of the two C-Q-filter, it is unnecessary to be a C-Q-filter, which can be shown in the next example

Example(2.5):

A binary operation * with D={0,1,2,3,4}can be shown in table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	3	2	0

Thereafter (D,*,0) is a bounded Q-algebra with unit 4 .if F_1={2,4}and F_2={3,4}, so we can easily show that both F_1, F_2 are a Q-filter in D, then $F_1 \cup F_2$={2,3,4} is not a Q-filter in D ,because
$(0^* \ * \ 2^*)^* = (4 * 3)^* = 2^* = 3 \in F_1 \cup F_2, but\ 0 \notin F_1 \cup F_2.$

Proposition (2.5):

Let X be bounded Q-algebra and F is a Q-filter of X. If $x^* \leq y^*, y \in F$ implies $x \in F$.

Proof:

Let$F$ is a Q_filter , $x^* \leq y^*$, [by Remark(2.1)].

So $(x^* \ * \ y^*)^* = (0)^* = e,$ thus$x \in F$.

Definition (2.5) [1]:
If f is a mapping of a Q-algebra N into Q-algebraV, then mapping f will called
(1) homomorphism if $f(a * b) = f(a) * f(b), \forall a, b \in X.$
(2) epimorphism if f is a surjective homomorphism.
(3) monomorphism if f is an injective homomorphism.
(4) isomorphism if f is a surjective and injectiive homomorphism.
Proposition (2.6) [4]:
If f is an epimorphisim from bounded Q-algebra N into bounded Q-algebra$V$, then
(1) $f(e) = e'$, both e and $e'$are the units of N and V, respectively.
(2) $f(a^*) = \left(f(a)\right)^*, \forall a \in N$
(3) If f is isomorphism, then $f^{-1}(b^*) = \left(f^{-1}(b)\right)^*$ for all $b \in V$.

3. Complete Q-filter

In this part, we provide the definition of complete Q-filter, and study its relationship with Q-filter in Q-algebra**.**

Definition(3.1):
Let (D,*,0) be a Q-algebra, if F is a non empty subset of D that satisfy
(1) $e \in F$ (2) $(a^* * b^*)^* \in F$,$\forall\ b \in F$ implies $a \in F., for\ all\ a, b \in D,$ then F is said to be a complete -Q-filter of D.
Example(3.1):
A binary operation * with D={0,1,2,3,4} can be shown in table :

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	0	0	0	0
3	3	0	0	0	0
4	4	3	3	2	0

Thereafter (D,*,0) be a bounded Q -algebra with unit .
(1) Let F={1,4} is C-Q-filter, since
$(2^* * 4^*)^*$ =$(3 * 0)^*$ =$3^* = 2 \notin F$
$(0^* * 1^*)^*$ =$(4 * 3)^*$ =$2^* = 3 \notin F$
$(3^* * 4^*)^*$ =$(2 * 0)^*$ =$2^* = 3 \notin F.$
(2) Let F_2={2,4}then F is not a C-Q –filter ,because

$(1^* \ * \ 2^*)^* = (3 * 3)^*$=4$\in F$
$(1^* \ * \ 4^*)^* = (3 * 0)^* = 2 \in F\ but\ 1 \notin F$.

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Proposition(3.1):
Every Q-filter is a C-Q-filter.
Proof:
Assume F is a Q-filter in bounded Q-algebra,$(a^* * b^*)^* \in F, \forall b \in F$,
because F is a Q-filter, so a ∈ F. Hence F is a C-Q-filter.

Remark(3.1):
In a converse way of Proposition (3.1)is not correct like in the Example (3.1)(1), a subset F is not a Q-filter, because $(2^* * 1^*)^* =(3 * 3)^* =0^* = 4 \in F, 1 \in F \; but \; 2 \notin F.$
Remark(3.2):
By seeing, the intersection and the union of the two C-Q-filter, it is unnecessary to be a C-Q-filter. For e. g
Example(3.3):
Assume F_1={0,2,4} and F_2, ={0,3,4},By example(2.5).we can show easily that F_1,F_2 are a C-Q-filter in D ,but $F_1 \cap F = \{0,4\}_2 does\ not\ C$-Q-filter in D, because
$(1^* * 0^*)^* =(4 * 4)^* =0^* = 4 \in F_1 \cap F_2, (1^* * 4^*)^* =(4 * 0)^*=4^* = 0 \in F_1 \cap F_2$, but
$1 \notin F \cap_1 F_{2.}$
And also $F_1 \cup F_2$={0,2,3,4} $do\ not$ C-Q-filter in D, because
$(1^* * 0^*)^* = 4 \in F_1 \cup F_2, (1^* * 2^*)^* = 3 \in F_1 \cup F_2, (1^* * 3^*)^* = 2 \in F_1 \cup F_2$ and
$(1^* * 4^*)^* = 0 \in F_1 \cup F_2$,but $1 \notin F_1 \cup F_2$.
Proposition(3.2):
Consider g is an isomorphism mapping from a bounded Q-algebra N into a bounded Q-algebra V. Let P be a C-Q-filter in N ,then g(P) be a C-Q-filter in V.
Proof:
Suppose that g is an isomorphism mapping from a bounded Q-algebra N into a bounded Q-algebra V, P is a C-Q-filter in N,
If $e_n \in$ P , then g(e_n) =$e_V \in g$(P) [by Proposition(2.6)(1)].
Now let $(a^* * b^*)^* \in g$(P) ,$\forall b \in g$(P),
thus $g^{-1}(a^* * b^*) \in$ P , $\forall g^{-1}(b) \in$ P (because g is surjective)
But $g^{-1}((a^* * b^*)^*) =((g^{-1}(a))^* * (g^{-1}(b))^*)^*$[by Proposition(2.6)(3)]
There for$(((g^{-1}(a))^* * (g^{-1}(b))^*)^*) \in$ P ,$\forall (g^{-1}(b)) \in$ P
Because P is C-Q-filter in N , then $g^{-1}(a) \in$ P .
And so on a ∈ g(P), which means g(P) is a C-Q-filter in V.
Proposition(3.3):
Consider h is an epimorphism mapping from a bounded Q-algebra N into a bounded Q-algebra V. Let σ be a C-Q-filter in V. Then $h^{-1}(\sigma)$ be a C-Q-filter in N.
Proof:
Suppose that h is an epimorphism mapping from a bounded Q-algebra N into a bounded Q-algebra V, σ is a C-Q-filter in V, so $e_v \in \sigma$, then h(e_n)= $e_v \in \sigma$,
so $e_n \in h^{-1}(\sigma)$, [by Proposition(2.6)(1)])
Now let $(a^* * b^*)^* \in h^{-1}(\sigma) , \forall b \in h^{-1}(\sigma)$, so $h(a^* * b^*)^* \in \sigma, \forall h(b) \in \sigma,$
$but\ h((a^* * b^*)^*) = \left(\left(\left(h(a) \right)^* * \left(h(b) \right)^* \right)^* \right) \in \sigma, \forall (h(b)) \in \sigma, [by\ proposition(2.6)(2)]$
Because σ is a C-Q-filter, so h(a) ∈ σ, therefore a ∈ $h^{-1}(\sigma)$.
Hence $h^{-1}(\sigma)$ is a C-Q-filter in N.

4. S-filter

In this part, we provide the definition of S-filter and study its relationships with Q-filter in Q-algebra.

Definition(4.1):
Let (D,*,0) be a Q-algebra ,if F is a nonempty subset of D that satisfy two condition
(1) e∈F
(2) $(b^{**} * a^*)^* \in F, \; b \in F \; implies \; a^* \in F$. Then F is said to be a complete -Q-filter of D.
Example(4.1):
A binary operation * with D={0,1,2,3} can be shown in table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	1	3	0

Thereafter (D,*,0)be a bounded Q-algebra with unit 3, F={0,1,3} is an S-filter,

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but F_1={2,3} is not S-filter ,because($2^{**} * 1^*$)* =3∈ F_1, but $1^* = 1 \notin F_1$.
And also, F_2={0,3}is not S-filter ,because $(0^{**} * 1^*)^*$ =3∈ F_2, but $1^* = 1 \notin F_2$.

Proposition(4.1):
(D,*,0) is a bounded Q-algebra. If F is a subset of D and $a^* \in F$, for any $a \in D$, then F *is an* S-filter.

Proof:
(1)$0^* = e \in F$ [by proposition(2.1)(1)]
(2)Let $(b^{**} * a^*)^* \in F, b \in F , a \in D,$ because $c^* \in F, \forall c \in D$. It follows that $a^* \in F$.

Proposition(4.2):
Every Q-filter is an S-filter.

Proof:
Suppose F is a Q-filter in bounded Q-algebra, $((b^{**} * a^*)^*) \in F$, $b \in F$.
Because $a^{**} * b^*$ =$b^{**} * a^*$ [by Proposition(2.1)(2)] .
And so on $(a^{**} * b^*)^* \in F$, $b \in F$, because F be a Q-filter, Thereafter $a^* \epsilon F$**.** Hence F is an S-filter**.**

Remark (4.1):
Conversely, Proposition (4.3) is unnecessary to be valid as shown in the next example.

Example (4.2):
By Example *(4.1)*,let *F={0,1,3}* be a *S*-filter, then *F* be not Q-filter**.** Because
$(2^* * 1^*)^*$= $(3 * 1)^*$ =$1^* = 1 \epsilon F$ but $2 \notin F$.

Proposition(4.3):
Every S-filter in an involutory Q-algebra D is a Q-filter.

Proof:
Suppose *F* is a *S*-filter in *D* ,$(a^* * b^*)^* \in F$, $b \in F$.
Then$(b^{**} * a^{**})^*$ =$(a^{***} * b^*)^*$=$(a^* * b^*)^* \in F$.[because *D* is an involutory].
So *F* is an S-filter ,*b*∈*F*, then a^{**} ∈F.
But $a^{**}=a$, thereafter $a \in F$, and so on *F* is a *Q*-filter.

Proposition(4.4):
If *F* is an *S*-filter in a bounded Q-algebra *D* and *a*∈*F* then $a^{**} \in F$

Proof:
Suppose *F* is an *S*-filter in *D* , $a \in F$ implies $(a^{**} * a^{**})^* = 0^* = e \in F$**,** then $a^{**} \in F$**.**

Remark(4.2):
Conversely, Proposition *(4.4)* is unnecessary to be valid as shown in the next example.

Example(4.3):
A binary operation * with the set D={0,1,2,3,4} can be defined in following table :

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	0	0
3	3	3	0	0	0
4	4	3	1	1	0

Thereafter *(D,*,0)* be bounded Q-algebra with unit *4***.** If *F={1,3,4}* then a^{**} ∈F is hold, ∀*a* ∈*F***.**
But *F* is not S-filter, because $(1^{**} * 4^*)^*$ = $(1 * 0)^*$ =$1^* = 3$ ∈*F*, but $4^* = 0 \notin F$.

Proposition(4.5)
Let *F* be *S*-filter of involuntary Q-algebra *D*, $b \le a^*$ and *b*∈ *F* then $a^* \in F$.

Proof:
Assume *F* be *S*-filter ,$b \le a^*$, $b \in F$ [by Proposition(2.1)(1)] .
So $(b^{**} * a^*)^* = 0^* = e \in F$ thus $a^* \in F$.

Proposition(4.6):
Let *F* be an *S*-filter and *0*∈ *F* then $a^* \in F$, for all $a \in D$.

Proof:
It is clear by $(0^{**} * a^*)^*$=e $\in F$.

Proposition(4.7):
The intersection of family of S-filter is S-filter.

Proof:
Let $\{ F_\propto, \propto \in \Delta \}$ be a family of S-filter in bounded Q-algebra D, So e∈ $F_\propto, \forall \propto \in \Delta$ then $e \in \cap_{\propto \in \Delta} F_\propto$
Now, Let$(b^{**} * a^*)^* \in \cap_{\propto \in \Delta} F_\propto$,y$\in \cap_{\propto \in \Delta} F_\propto$ *then*
$(b^{**} * a^*)^* \in F_\propto, y \in F_\propto, \forall \propto \in \Delta$, because $F_\propto$ is *S*-filter, $\forall \propto \in \Delta$,
then $a^* \in F_\propto, \forall \propto \in \Delta$. Hence $a^* \in \cap_{\propto \in \Delta} F_\propto$.

Remark(4.4):
Seeing that the union of two *S*-filters are unnecessary to be S-filter as the following example show.

Example(4.4):

In example (4.3). A subset F_1={1,4} and F_2={2,3,4} are S-filter, while $F_1 \cup F_2$={1,2,3,4} is not S-filter because $(1^{**} * 4^*)^* = 1^* = 3 \in F_1 \cup F_2, 1 \in F_1 \cup F_2$ but $4^* = 0 \notin F_1 \cup F_2$.

Proposition(4.8):

Consider h is an isomorphism mapping from a bounded Q-algebra N into a bounded Q-algebra V, if G is an S-filter, then the image of S-filter is an S-filter.

Proof:

Assume G be S-filter in N, then $e' \in G$ .To prove that $h(G)$ is S-filer in V.

(1) Since $h(e) = e'$, $e \in G$ then $e' \in h(G)$

(2)Let $(b^{**} * a^*)^* \in h(G), b \in h(G)$, then

$h^{-1}((b^{**} * a^*)^*) \in G, h^{1-}(b) \in G$ [$since\ h\ is\ surjective$].

But $h^{-1}((b^{**} * a^*)^*) = \left(h^{-1}(b^{**} * a^*)\right)^* = \left(h^{-1}(b^{**}) * h^{-1}(a^*)\right)^*$

$= \left(\left(\left(\left(h^{-1}(b)\right)^*\right)^* * \left(h^{-1}(a)\right)^*\right)^*\right)$ [$by\ lemma(2.6)(3)$]

So $\left(\left(\left(\left(h^{-1}(b)\right)^*\right)^* * \left(h^{-1}(a)\right)^*\right)^*\right) \in G, h^{1-}(a) \in G$,because G is S-filter in N,

then $\left(h^{-1}(a)\right)^* = h^{-1}(a^*) \in G$.Thus $a^* \in h(G)$, that means $h(G)$ is an S-filter in V.

Proposition(4. 9):

Consider h is an epimorphism mapping from bounded Q-algebra N into bounded Q-algebra V, if A be S-filter ,then the inverse image of S-filter be S-filter.

Proof:

Suppose that A is an S-filter in V, then $e' \in A$ .To show that $h^{-1}(A)$ is S-filer in N.

(1)because $h(e) = e'$ and h is a one to one, then $e = h^{-1}(e') \in h^{-1}(A)$

(2)assume that $(b^{**} * a^*)^* \in h^{-1}(A), y \in h^{-1}(A)$.Because (h is onto), then

$h(b^{**} * a^*)^* = ((h(b^{**}) * h(a^*))^*) \in A$ and $h(b) \in A$, but

$h(b^{**} * a^*)^* = \left(\left(h(b^{**}) * h(a^*)\right)^*\right) = \left(\left(\left(\left(h(b)\right)^*\right)^* * \left(h(a)\right)^*\right)^*\right) \in A,$

$h(b) \in$ A. By $lemma(2.6)(2)$],because A is S-filter in V, then $\left(h(a)\right)^* = h(a^*) \in A$.

And so on $a^* \in h^{-1}(A)$, which means $h^{-1}(A)$ is a S-filter in N.

Definition(4.2):

Let (D,*,0) be a bounded Q-algebra, a subset F of D is a proper S-filter if F≠X.

Proposition(4.10):

Consider (D,*,0) be an involutory Q-algebra, a subset F of D be a proper S-filter if and only if $0 \notin F$.

Proof:

Assume that $0 \notin F$,so; follow clearly from F is a proper.

In a Converse way ,if F is a proper S-filter.

Let $0 \in F$, then for some $a \in D$, $(0^{**} * a^{**})^* = e \in F$.(Since F is S-filter). Thus $a^{**} \in F$ i.e $a \in F$ (because D is involution). It gets F=D, is a contradiction.

Therefore $0 \notin$ F.

5. complete S-filter

In this part ,we provide the definition of complete S-filter, and study its relationship with s-filter in Q-algebra.

Definition(5.1):

A nonempty subset F of a bounded Q-algebra D is called complete S-filter (C-$S - filter$) if,

(1) $e \in F$;

(2) $(b^{**} * a^*)^* \in F$, $\forall\ b \in F\ implies\ a^* \in F$;$for\ all\ a, b \in D$

Example(5.1):

A binary operation * with the set D={0,1,2,3} can be defined in following table :

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	3	3	0

Thereafter (D,*,0) be bounded Q-algebra with the unit 3, and F={0,2,3} is a C-S filter. Because $(b^{**} * a^*)^* \in F$, $\forall\ b \in F\ implies\ a^* \in F$ is hold.

Example(5.2):

A binary operation * with the set D={0,1,2,3,4,5} can be defined in following table:

*	0	1	2	3	4	5

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0	0	0	0	0	0	0
1	1	0	1	0	0	0
2	2	2	0	2	0	0
3	3	3	0	0	0	0
4	4	4	3	0	0	0
5	5	5	4	0	3	0

Thereafter(D,*,0) be bounded Q-algebra with the unit 5, F={0,1,5} is not a C-S-filter, because
$(0^{**} * 4^{*})^{*}$ =5∈ F, $(1^{**} * 4^{*})^{*}$ =5∈ F and $(5^{**} * 4^{*})^{*}$ =5∈ F , but 4^{*}=3$\notin F$. .

Proposition(5.1):
In bounded Q-algebra, every S-filter is a C-S-filter.

Proof:
Suppose F be S-filter , $(b^{**} * a^{*})^{*} \in F, \forall b \in F$, *since F is* S-filter, then $a^{*} \in F$, and so on F is a *C*-*S*-filter.

Remark(5.1):
Conversely, Proposition *(5.1)* is unnecessary to be valid as shown in the next example**.**

Example(5.3):
Let F={1,2,3} be a C-S-filter, in Example (5.1),but F unnecessary to be S-filter ,because
$(1^{**} * 3^{*})^{*} = 3 \in F,\ 3^{*} = 0 \notin F$.

Proposition(5.2):
Every Q-filter is C-S- filter**.**

Proof:
By Proposition(4.2) and Proposition (5.1).

Remark(5.2):
In general, the converse of Proposition (5.2) unnecessary to be true as in the example(5.1) F={1,2,3} is a c-s-filter, but it is not a Q-filter, because $(0^{*} * 1^{*})^{*} = 3 \in F,\ but\ 0 \notin F$.

Proposition(5.3):
Every C-Q-filter is C-S-filter.

Proof:
Suppose F be a *C*-*Q*-filter of bounded *Q*-algebra *D*, $(b^{**} * a^{*})^{*} \in F, \forall b \in F$.
Because $a^{**} * b^{*}$ =$b^{**} * a^{*}$ [by proposition(2.1)(2)]
Implies $(a^{**} * b^{*})^{*} \in F, \forall b \in F$.if F is *a C*-*Q*-filter from *D* ,then $a^{*} \in F$. Hence F is a *C*-*S*-filter.

Remark(5.3):
Conversely, Proposition *(5.3)* is unnecessary to be valid as shown in the next example**.**

Example(5.4):
Let *F={0,2,3} be C*-*S*-filter from *D* , in Example (5.1), then *F* is not *C*-*Q*-filter ,because
$(1^{*} * 0^{*})^{*} = 3 \in F, (1^{*} * 2^{*})^{*} = 3 \in F\ and\ (1^{*} * 3^{*})^{*} = 0 \in F$,
but *1*∉ *F*.

Proposition(5.4):
If F is a C-S-filter in involuntary Q-algebra D, thereafter *F* is a C-Q-filter.

Proof:
Let *F* be a *C*-*S*-filter of *D* ,$(a^{*} * b^{*})^{*} \in F$, ,$\forall b \in F$,
Then$(a^{*} * b^{*})^{*} = (b^{**} * a)^{*}$ [byproposition(2.1) (2)],
because D is an involuntary. Then $(a^{*} * b^{*})^{*}$ =$(b^{**} * a^{**})$,
of supposed *F* be a *C*-*S*-filter. Then $a^{**} \in F$.But $a^{**}=a$, $\forall a \in D$, then a∈ *F*. Hence *F* is a *C*-*Q*-filter.

Proposition(5.5):
Consider g is an isomorphism mapping from bounded Q-algebra N into bounded Q-algebra V ,if U is a c-S-filter, then the image of C-S-filter is a C-S-filter.

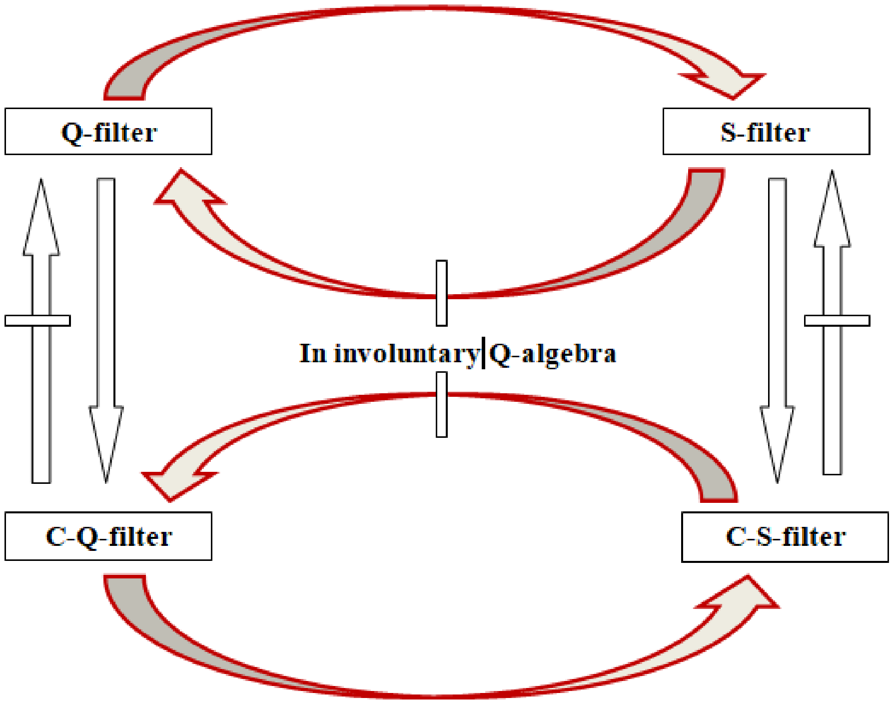
Proof:
Suppose that g is an isomorphism function from bounded Q-algebra N into bounded Q-algebra V . If U is a C-S-filter in N ,then e ∈ U. So g(e)=$e' \in g(U)$ [by proposition(2.5)(1)].
Now let $(b^{**} * a^{*})^{*} \in g(u), \forall y \in g(U)$, thereafter $(g^{-1}(b^{**} * a^{*})^{*}) \in U, \forall g^{-1}(b) \in U$,
because [f is a surjective].Let $g^{-1}(b^{**} * a^{*})^{*} = (g^{-1}(b^{**}) * g^{-1}(a^{*}))^{*}$, then
$g^{-1}(b^{**} * a^{*})^{*} = (((g^{-1}(b))^{*})^{*} * (g^{-1}(a))^{*})^{*}$ [by proposition(2.5)(3)].

Therefore $(((g^{-1}(b))^*)^* * (g^{-1}(a))^*)^* \in U,, \forall g^{-1}(b) \in U$.Because U is a C-S-filter in N, then $(g^{-1}(a))^* = g^{-1}(a^*) \in U$.Thus $a^* \in g(U)$, which means g(U) is a C-S-filter in V.

Proposition(5.6):
Consider h is an epimorphism mapping from bounded Q-algebra N into bounded Q-algebra V, if G is C-S-filter in V, then inverse image of C-S-filter is C-S-filter .

Proof:
Assume that f is an epimorphism from bounded Q-algebra N into bounded Q-algebra V, and G is a C-S-filter in V.
If $\acute{e} \in G$ then $h(e)= \acute{e}$, thus $e \in h^{-1}(G)$ [by proposition(2.5)(1)].
Now let $(b^{**} * a^*)^* \in h^{-1}(G), \forall b \in h^{-1}(G)$,So $h((b^{**} * a^*)^*) \in G, \forall h(b) \in G$,
because[h is an onto].If $h((b^{**} * a^*)^*) = ((((h(b))^*)^* * (h(a))^*)^*) \in G, \forall h(b) \in G$.
[by proposition(2.5)(2)], because G is a C-S-filter then $((h(a))^*)=h(a^*) \in G$.
Thus $a^* \in h^{-1}(G)$, which mean $h^{-1}(G)$ is a C-S-filter in N.

Remark(5.4):
The following diagram shows the relation among Q-filter, S-filter, C-Q-filter and C-S-filter in bounded Q-algebra .



References

[1] Neggers J, Ahn SS, kim HS. on Q-algebra. International Journal of Mathematics and Mathematical Sciences (IJMMS). 2001; 27(12):749-757.
[2] Y.B.Jun, "satisfactory filter of BCK-algebra", Scientiae MathematicaeJaponicae online, Vol.g, 2003,PP.1-7.
[3] Abdullah.H.K,Radhi. K.T, T-filter.in BCK- algebra, Jounrnal university of kerbala. 2017; 15(2): 36-43
[4] Abdullah.H.K, Jawad. H.K, New types of Ideals in Q-algebra, Journal university of kerbala, Vol.16 No.4 scientific. 2018
[5] J.Meng. "Bck-filter", Math. Japan, Vol,44,1996,PP. 199-129.
[6] Y.Lmai and K.Iseki , On axiom system of propositional calculi XIV, proc, Japan . Academy . 42 (1966), pp. 19 - 22.
[7] Y.Huang, On Involutory BCK-algebras, Soochow Journal of Mathematics, Vol. 32, No. 1, (2006), pp. 51-57.
[8] Q.P. Hu and X. Li, On BCH-algebras, Math. Seminar Notes 11 (1983), pp. 313-320.
[9] E.Y.Deeba, Filter theory of BCK-algebra, Math. Japon, Vol. 25(1980), pp. 631-639.
[10]Y.B.Jun, Satisfactory Filters of BCK-algebras, Scientiae Mathematicae Japonicae Online Vol. 9, (2003) pp. 1-7.
[11]A.H.Kareem. and H.Z.Ahmed, Complete BCK-ideal, European Journal of Scientific Research, Vol. 137, No. 3 (2016), pp.302-314.