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Expansivity on the G-Spaces

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Abstract *: This paper is review about expansivity on G- spaces. We recall the general definition and some general properties with important propositions.*

Keywords: expansive, positive expansive, G- spaces

1. Introduction

Expansivity is one of the important properties for dynamical systems. The homeomorphism maps which is expansive, have many applications in lots of branches like Symbolic Dynamics , Ergodic Theory , Topological Dynamic, etc.[4] . Let $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ be an action , where $\mathfrak{K}$ be a metric space with metric ς and G be a topological group. Let $\mathfrak{H} \neq \emptyset$ be subset of G . $\mathfrak{H}$ is syndetic set, if there is compact set $\mathfrak{A}$ such that $G = \mathfrak{A}\mathfrak{H}$. The actions $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ and $\Theta : G \times \mathfrak{Z} \to \mathfrak{Z}$ is topologically conjugate if there is a homeomorphism $\mathfrak{h} : \mathfrak{K} \to \mathfrak{Z}$ such that $\mathfrak{h} \circ \Xi_g = \Theta_g \circ \mathfrak{h}$, for all $g \in G$ [2]. Let $\mathfrak{H} \neq \emptyset$ and subset of G. $\mathfrak{H}$ is satisfy Property P, if for all $\varepsilon > 0$ there is a compact subset $\mathfrak{A} \subseteq \mathfrak{H}$ of $\mathfrak{K}$ such that $(\mathfrak{H} \times \mathfrak{H} \cap \varsigma^{-1}([0,\varepsilon)) \cup \mathfrak{A} \times \mathfrak{A} = \mathfrak{H} \times \mathfrak{H})$. A continuous action $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ is an orbit expansive action if there exists a finite open cover r of $\mathfrak{K}$ such that if for every $g \in G$, the set $\{\Xi_g(n), \Xi_g(m)\} \prec r$, then $n = m$, and r is called an orbit expansive covering of Ξ [1].

2. Review

Definition 2.1. [2]
The action $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ is expansive if there exists a constant $\mathfrak{d} > 0$ such that for every $n, m \in \mathfrak{K}$ with $n \neq m$ there is $g \in G$ satisfying $d(\Xi_g(n), \Xi_g(m)) > c$. The constant $\mathfrak{d}$ is expansive constant. Equivalently, Ξ is expansive if for every $g \in G$, $d(\Xi_g(n), \Xi_g(m)) \leq \mathfrak{d}$, then $n = m$.

Proposition 2.2. [3]
The notion of positive $G -$ expansivity under the trivial action coincides with the notion of positive expansivity for a continuous onto map $\eta: \mathfrak{K} \to \mathfrak{K}$.,
Proposition 2.3. [2]
Suppose $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ is an expansive action. Let Y be a closed $\Xi -$invariant subset of $\mathfrak{K}$. Then $\Xi: G \times Y \to Y$ is expansive.

Proposition 2.4. [3]
Let G is compact and d is invariant and $\mathfrak{K}/G$ is considered as a metric space with metric $\mathfrak{d}_1$ induced by $\mathfrak{d}$. Then the induced map $\hat{\eta} : \mathfrak{K}/G \to \mathfrak{K}/G$ is positively expansive if and only if, a pseudoequivariant map $\eta: \mathfrak{K} \to \mathfrak{K}$ is positively expansive.

proposition 2.5. [3]
Let $\eta: \mathfrak{K} \to \mathfrak{K}$ be a positively expansive equivariant map with $\mathfrak{K}$ and G are compact. Let $\mathfrak{K}_\eta$ be the inverse limit space and suppose G acts diagonally on $\mathfrak{K}_\eta$. Then the shift map $\sigma((n_1)) = (\eta(n_1))$ is an expansive homeomorphism.

proposition 2.6 [1]
let $\mathfrak{H}$ be any subgroup of G . The equicontinuous Ξ is expansive if and only if $\Xi|_{\mathfrak{H}}$ is expansive.

Proposition 2.7. [2]
let $\mathfrak{H}$ be a syndetic subgroup of G on compact space $\mathfrak{K}$. Ξ is expansive if and only if $\Xi|_{\mathfrak{H}}$ is expansive.

proposition 2.8: [4]

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Let $\eta: \mathfrak{K} \to \mathfrak{K}$ be a homeomorphism on compact T_2 space $\mathfrak{K}$ with a compact group G. Then η is expansive on $\mathfrak{K}$ if η^i not constant for some $i \geq 1$,.

Proposition 2.9 [3]
Let $\eta: \mathfrak{K} \to \mathfrak{K}$ be a positively expansive on a compact metric $G -$ space $\mathfrak{K}$. Then η^i is positively expansive, for any integer $i > 0$.

Proposition 2.10 [1]
The action Ξ is expansive if and only if Ξ^{-1} is expansive.

Proposition 2.11 [1]
For some $i \in Z \setminus \{0\}$, if Ξ^i is expansive then Ξ is expansive.

Proposition 2.12 [1]
for all $i \in Z \setminus \{0\}$, Ξ^i is expansive if Ξ is commutative expansive action.

Proposition 2.13 [4]
Let $\mathfrak{K}$ be a G_1 −space , and $\mathfrak{Z}$ be G_2 −space and , $\eta: \mathfrak{K} \to \mathfrak{K}$, $\vartheta : \mathfrak{Z} \to \mathfrak{Z}$ are homeomorphisms . If one of η and η_2 is expansive, then $\eta \times \vartheta: \mathfrak{K} \times \mathfrak{Z} \to \mathfrak{K} \times \mathfrak{Z}$ expansive.

Proposition 2.14 [3]
Let $\mathfrak{K}$ be G −space with metric ς and $\mathfrak{Z}$ be G −space with metric ρ . Suppose $\eta: \mathfrak{K} \to \mathfrak{K}$ and $\vartheta: \mathfrak{Z} \to \mathfrak{Z}$ are positively expansive maps. If G acts diagonally on the product space $\mathfrak{K} \times \mathfrak{Z}$, then the product map $\eta \times \vartheta: \mathfrak{K} \times \mathfrak{Z} \to \mathfrak{K} \times \mathfrak{Z}$ defined by $(\eta, \vartheta)(\mathcal{n}, \mathcal{m}) = (\eta(\mathcal{n}), \vartheta(\mathcal{m}))$ is positively expansive.

Proposition 2.15 [1]
The space $\mathfrak{K}$ must be a discrete space if Ξ is equicontinuous expansive.

Proposition 2.16 [1]
Let $P(\Xi)$ be the set of any commutative expansive action on a compact metric space . $P(\Xi)$ is at most countable.

Proposition 2.17 [2]
Let $\mathfrak{K}$ be compact G −space with metric ς and $\mathfrak{Z}$ be compact G −space with metric ρ . If $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ is expansive then $\Theta : G \times \mathfrak{Z} \to \mathfrak{Z}$ is also expansive if Ξ and Θ are conjugate actions .

Proposition 2.18 [1]
Let $\mathfrak{K}$ be G −space with metric ς and $\mathfrak{Z}$ be G −space with metric ρ and $\pi: \mathfrak{Z} \to \mathfrak{K}$ be locally isometric covering map. Suppose that $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ and $\Theta : G \times \mathfrak{Z} \to \mathfrak{Z}$ satisfy $\pi\Theta_g(\mathcal{m}) = \Xi_g\pi(\mathcal{m})$, for all $\mathcal{m} \in \mathfrak{Z}$ and for all $g \in G$. Suppose that there is $\delta_0 > 0$ such that for every $\mathcal{m} \in \mathfrak{Z}$ and $0 < \delta < \delta_0$, $\pi: \mathcal{r}_\delta(\mathcal{m}) \to \mathcal{r}_\delta\big(\pi(\mathcal{m})\big)$ is an isometry. Then Ξ is expansive if and only if Θ is expansive.

Proposition 2.19 [2]
Let $\mathfrak{K}$ be compact G −space with metric ς and $\mathfrak{Z}$ be compact G −space with metric ρ. Suppose that $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ is an uniformly continuous action of G on $\mathfrak{K}$ and $\Theta : G \times \mathfrak{Z} \to \mathfrak{Z}$ is an expansive action of G on $\mathfrak{Z}$. Let $\mathfrak{h} : \mathfrak{K} \to \mathfrak{Z}$ be a covering map. If for every $g \in G$, $\mathfrak{h} \circ \Xi_g = \Theta_g \circ \mathfrak{h}$, then Ξ is an expansive action.

Proposition 2.20 [3]
Let $\mathfrak{K}$ be compact G −space with metric ς and $\mathfrak{Z}$ be compact G −space with metric ρ and $\eta: \mathfrak{K} \to \mathfrak{K}$ be a positively expansive map. If $\mathfrak{h}: \mathfrak{K} \to \mathfrak{Z}$ is a pseudoequivariant homeomorphism, then $\eta_1 = \mathfrak{h}\eta\mathfrak{h}^{-1} : \mathfrak{Z} \to \mathfrak{Z}$ is a positively expansive map on $\mathfrak{Z}$.

Proposition 2.21 [2]
let $\mathfrak{B}$ be a dense subset of compact metric space $\mathfrak{K}$ with Property P. Then Ξ is expansive on $\mathfrak{B}$ if and only if Ξ is expansive on $\mathfrak{K}$.

Definition 2.22 [4]
A G −space $\mathfrak{K}$ is said to be expansive G −chaos space if $\{\eta: \mathfrak{B} \to \mathfrak{B}: \eta \text{ is expansive G} - \text{chaotic on compact subset } \mathfrak{B}\} \neq \emptyset$.

Proposition 2.23 [4]
Let $(G_1, \mathfrak{K}, \Xi) \cong_{(\mu,\psi)} (G_2, \mathfrak{Z}, \Theta)$. If $\mathfrak{K}$ is expansive G_1 −chaos space , then $\mathfrak{Z}$ is G_2 −expansive chaos space.

Definition 2.24 [4]
Let $\mathfrak{B}$ be a compact subset of $\mathfrak{K}$. A continuous map $\eta: \mathfrak{K} \to \mathfrak{K}$ is G −expansive chaotic on $\mathfrak{B}$ if the following hold :

1. $\overline{GO(\eta, n)} = \mathfrak{B}$, for some $n \in \mathfrak{B}$,
2. $\overline{GP_r(\mathfrak{K})} = \mathfrak{B}$,and
3. η is expansive on $\mathfrak{B}$.

Proposition 2.25 [4]
Let $\mathfrak{K}$ be a G_1 −space , $\mathfrak{Z}$ be G_2 −space . If equivariant maps $\eta: \mathfrak{K}_1 \to \mathfrak{K}_1$,and $\vartheta: \mathfrak{Z} \to \mathfrak{Z}$ are G_1, G_2 −expansive chaotic, then $\eta_1 = \eta \times \vartheta$ is $G_1 \times G_2$ −expansive chaotic on $\mathfrak{B}_1 \times \mathfrak{B}_2$.

Proposition 2.26 [2]
let $\Xi : G \times \mathfrak{K} \to \mathfrak{K}$ be an uniformly continuous on compact $\mathfrak{K}$. Then Ξ is orbit expansive if and only if it is expansive.

REFERENCES

[1] Abdul Gaffar Khan (2018). Pramod Das and Tarun Das, A Note on Expansivity and Shadowing for Group Actions ", Applied General Topology, vol. 20, pp. 364, 19-31.

[2] Ali Barzanouni, Mahin Sadat Divandar, and Ekta Shah (2019). On Properties Expansive Group Actons , Acta Mathematica Vietnamica, vol. 44, pp. 923–934.

[3] Ekta Shah (2005). Dynamical Topological Properties of Maps on Topological Spaces and G – Spaces , Ph. D. thesis.

[4] Salah H. Abid and Ihsan J. Kadhim (2014). On Expansive Chaotic Maps in G-Spaces , International Journal of Mathematical Research, vol. 3, pp. 225-232 .

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