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Estimation of the Bi-response Poisson Regression Model Based on Local Linear Approach

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Abstract: *Bi-response Poisson regression model based on the bivariate Poisson distribution and the response variable is a count data that correlate. The function of the bi-response Poisson regression model can be estimate using the parametric approach and the nonparametric approach. In this paper, we discuss the nonparametric approach using local linear estimator. The parameters of the bi-response Poisson regression model are estimated by the method of maximum likelihood.*

Keywords: bi-response, Poisson regression, local linear.

1. INTRODUCTION

The regression model can be used to describe the relation between the response variable and the predictor variable. The response variable of the regression model allows for continue data and count data. Some the regression model of the count data are Poisson regression model, *Generalized* Poisson regression model, and Negative Binomial regression model. According to Zamani, et al (2015), Poisson regression model has been widely used for modeling count data with covariates. The Poisson regression model is the basic framework for count data analysis and it arises from the evidence that the number of occurrences of a given event over a specific time period depends on some covariates (Santos & Neves, 2008). The Poisson regression model based on the Poisson distribution. The Poisson distribution satisfies the equal-dispersion property because the conditional mean of the response variable equals its conditional variances (Famoye, 1993).

Poisson regression modeling often involves not only one response variable but also can two or more response variables because of a phenomenon involving multiple response variables. Poisson regression analysis involving one response variable with one or more predictor variables is called Poisson regression. Poisson regression involving two response variables correlated with one or more predictor variables is called bi-response Poisson regression. The extension of the bi-response Poisson regression is the multi-response Poisson regression, the Poisson regression involving more than two response variables with one or more predictor variables.

The function of the Poisson regression model can be estimated using two approaches, there are the parametric approach and the nonparametric approach. The parametric approach used if the function of the regression model is known except for finitely many unknown parameters and the relation between the response variable and the predictor variable is assumed to have a specific type curve. If we used the parametric regression approach to this condition then consequently, giving misleading inference about the regression model (Chamidah & Saifuddin, 2013). One collection of procedures that can be used for this purpose are nonparametric regression approach. These approaches give estimate of regression function that allow great flexibility in the possible form of the regression curve because the estimate get from data. A nonparametric regression model generally only assumes that the regression curve satisfies smooth properties that are continuity and differentiability (Eubank, 1999).

In this research, we will be studied about bi-response Poisson regression estimation based on local linear approach.

2. BI-RESPON POISSON REGRESSION MODEL

Let the three random variables V_1, V_2, and U to follow three independent Poisson distributions with the positive parameters θ_1, θ_2, and γ respectively. According to Jung & Winkelmann (1993), new random variables Y_1 and Y_2 can be constructed by $Y_1 = V_1 + U$ and $Y_2 = V_2 + U$ where Y_1 and Y_2 are Poisson random variables. The mean Y_1 and Y_2 are $E(Y_1) = \theta_1 + \gamma$ and $E(Y_2) = \theta_2 + \gamma$.

The density probability function of Y_1 and Y_2 given by:

$$\begin{aligned} P(Y_1 = y_1, Y_2 = y_2) &= f(y_1, y_2) \\ &= \exp\left[-(\theta_1 + \theta_2 + \gamma)\right] \sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\theta_1^{y_1-k}}{(y_1-k)!} \frac{\theta_2^{y_2-k}}{(y_2-k)!}; \quad (1) \end{aligned}$$

where $y_1, y_2 = 0, 1, 2, \ldots$ and $s = \min(y_1, y_2)$.

The covariance between Y_1 and Y_2 is:

$$\begin{aligned} Cov(Y_1, Y_2) &= Cov(V_1 + U, V_2 + U) \\ &= Var(U) \end{aligned}$$

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$$= \gamma \qquad (2)$$

The correlation between Y_1 and Y_2 is:

$$Corr(Y_1, Y_2) = \frac{\gamma}{\sqrt{(\theta_1+\gamma)(\theta_2+\gamma)}} \qquad (3)$$

The parameter γ is non-negative (Berchout & Flug, 2004). Following the standard approach in univariate Poisson regression we model the marginal expectation of Y_1 and Y_2, respectively, as a loglinear function of exogenous variables. (Jung and Winkelmann, 1993).

$$\theta_{ri} + \gamma = \exp(\underset{\sim}{x}_{ri}^T \underset{\sim}{\beta}_r);\ r = 1,2 \text{ dan } i = 1,\ 2,\ \ldots,\ n \qquad (4)$$

The likelihood function for the observed random sample is given by:

$$\ell(\beta_1, \beta_2, \gamma) = \prod_{i=1}^{n} \exp(\gamma - \sum_{r=1}^{2} \exp(\underset{\sim}{x}_{ri}^T \underset{\sim}{\beta}_r)) B_i$$

and the log likelihood function is:

$$\text{L} = \log \ell(\beta_1, \beta_2, \gamma)$$

$$= n\gamma - \sum_{i=1}^{n} \exp(\underset{\sim}{x}_{1i}^T \underset{\sim}{\beta}_1) - \sum_{i=1}^{n} \exp(\underset{\sim}{x}_{2i}^T \underset{\sim}{\beta}_2) + \sum_{i=1}^{n} \log B_i; \qquad (5)$$

where

$$B_i = \sum_{k=0}^{s_i} \frac{\gamma^k}{k!} \frac{[\exp(\underset{\sim}{x}_{1i}^T \underset{\sim}{\beta}_1) - \gamma]^{y_{1i}-k}}{(y_{1i}-k)!} \frac{[\exp(\underset{\sim}{x}_{2i}^T \underset{\sim}{\beta}_2) - \gamma]^{y_{2i}-k}}{(y_{2i}-k)!};$$

$$s_i = \min(y_{ri}),\ r = 1,2$$

The maximum likelihood estimates of the parameters can be obtained by solving equations:

$$\frac{\partial \log L}{\partial \gamma} = 0 \text{ and } \frac{\partial \log L}{\partial \beta_r} = 0;\ \text{r} = 1,2.$$

3. Local Linear Estimator in Single Respon

Local linear estimator is special case in local polynomial estimator. Local polynomial estimator is one of the smoothing methods that can be used as a regression function approach $m(.)$ (Fan & Gijbels, 1996). Local polynomial estimators can estimate regression functions $m(x_0)$ and their derivatives:

$$m'(x_0), m''(x_0), \cdots, m^{(p)}(x_0)$$

Suppose the regression function $m(.)$ has a derivative (p + 1) at the point x_0, the regression function $m(.)$ can be approached locally with the degree of polynomial p. By Taylor expansion, for x in a neighborhood of, we have:

$$m(x) \approx m(x_0) + m'(x_0)(x-x_0) + \frac{m''(x_0)(x-x_0)^2}{2!} + \cdots + \frac{m^{(p)}(x_0)(x-x_0)^p}{p!} \qquad (6)$$

or

$$m(x) \approx \sum_{j=0}^{p} \frac{m^{(j)}(x_0)}{j!}(x-x_0)^j \equiv \sum_{j=0}^{p} \beta_j(x_0)(x-x_0)^j;$$

$$x \in (x_0 - h, x_0 + h)$$

where

$$\beta_j(x_0) = \frac{m^{(j)}(x_0)}{j!};\ j = 0,1,2,\cdots,p. \qquad (7)$$

Suppose we taking *n*-pairs data sample (x_i, y_i), the estimates of $\underset{\sim}{\beta}$ based on *Weighted Least Square* (WLS) procedure by minimizing function:

$$\sum_{i=1}^{n} \{y_i - \sum_{j=0}^{p} \beta_j(x_0)(x_i - x_0)^j\}^2 K_h(x_i - x_0);\ i = 1,2,\cdots,n \qquad (8)$$

where $K(.)$ is Kernel function and h is *bandwidth*. Local polynomial estimators can estimate regression functions $m(x_0)$ and their derivatives. According to Equation (7), local polynomial estimators of derivatives *j*th in regression function $m^{(j)}(x_0)$ is:

$$\hat{m}^{(j)}(x_0) = j!\hat{\beta}_j(x_0);\ j = 0,1,2,\cdots,p; \qquad (9)$$

For *j=0*, the estimate of regression function at the point x_0 is:

$$\hat{m}(x_0) = \hat{\beta}_0(x_0). \qquad (10)$$

Local linear estimator we obtain if the degree of polynomial (*p*) equal one (*p=1*). Local linier estimator can be written is:

$$\sum_{i=1}^{n} \{y_i - \beta_0(x_0) - \beta_1(x_0)(x_i - x_0)\}^2 K_h(x_i - x_0) \qquad (11)$$

4. Study of Bi-Response Poisson Regression Using Local Linear Estimator

Suppose we have pair observational data (x_i, y_{ri}), $r = 1,2$ dan $i = 1,\ 2,\ \ldots,\ n$ which is distribute independently with x is vector of covariates and y_{ri} is the count bi-response that follows the Poisson distribution. The probability density function of y_{ri} given by equation (1).

Generally, equation (4) can be wrote:

$$\theta_{ri} + \gamma = \exp[m_r(x_i)];\ r = 1,2 \text{ dan } i = 1,\ 2,\ \ldots,\ n \qquad (12)$$

The function $m_r(.)$ in equation (5) is a smooth function. Assume that the function $m_r(.)$ has a $(p+1)^{th}$ continuous derivative at the point x_0. We approximate the function $m_r(.)$ by Taylor expansion with order one or $p = 1$, for data point x_i in a neighborhood of x_0 with $x_i \in (x_0 - h, x_0 + h)$ and h is a bandwidth:

$$m_r(x_i) \approx \beta_{0i}^{(r)}(x_0) + \beta_{1i}^{(r)}(x_0)(x_i - x_0);\ r = 1,2.$$

or

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$$m_r(x_i) = \underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(r)}(x_0); r = 1,2.$$

where

$\underset{\sim}{x}_i^T = \left[1 \quad (x_i - x_0)\right]$ and $\underset{\sim}{\beta}^{(r)} = \left[\beta_{0i}^{(r)}(x_0) \quad \beta_{1i}^{(r)}(x_0)\right]^T$

Let $K_h(x_i - x_0)$ is a Kernel weight and h is bandwidth, the local likelihood function can be obtained from Eq (1):

$$\ell(\theta_{1i}, \theta_{2i}, \gamma, x_0) = \prod_{i=1}^{n} f(y_{1i}, y_{2i})^{K_h(x_i - x_0)} \qquad (13)$$

The log local likelihood function is:

$$\mathrm{L}(\theta_{1i}, \theta_{2i}, \gamma, x_0) = \ln \ell(\theta_{1i}, \theta_{2i}, \gamma, x_0)$$

$$= \sum_{i=1}^{n}\left(K_h(x_i - x_0)\left\{\left[-(\theta_{1i} + \theta_{2i} + \gamma)\right] + \ln \sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\theta_{1i}^{y_{1i}-k}}{(y_{1i}-k)!} \frac{\theta_{2i}^{y_{2i}-k}}{(y_{2i}-k)!}\right\}\right) \qquad (14)$$

Next, by subtituting Equation (12) into Equation (14), we have:

$$\mathrm{L}(m_{1i}, m_{2i}, \gamma, x_0) = \sum_{i=1}^{n} K_h(x_i - x_0)\Big\{\gamma - \exp\left[m_1(x_i)\right] - \exp\left[m_2(x_i)\right]$$

$$+ \ln\left(\sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\left(\exp\left[m_1(x_i)\right] - \gamma\right)^{y_{1i}-k}}{(y_{1i}-k)!} \frac{\left(\exp\left[m_2(x_i)\right] - \gamma\right)^{y_{2i}-k}}{(y_{2i}-k)!}\right)\Big\} \qquad (15)$$

Let $D_i = \sum_{k=0}^{s} D_{ik}^{(1)} D_{ik}^{(2)}$;

Where

$$D_{ik}^{(1)} = \frac{\gamma^k}{k!} \frac{[\exp(\underset{\sim}{x}_{1i}^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)) - \gamma]^{y_{1i}-k}}{(y_{1i}-k)!}$$

and $$D_{ik}^{(2)} = \frac{[\exp(\underset{\sim}{x}_{2i}^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)) - \gamma]^{y_{2i}-k}}{(y_{2i}-k)!}$$

The Equation (15) can be written :

$$L(\underset{\sim}{\beta}^{(1)}(x_0)\underset{\sim}{\beta}^{(2)}(x_0), \gamma, x_0) = \sum_{i=1}^{n} K_h(x_i - x_0) \times$$

$$\left\{\gamma - \sum_{r=1}^{2} \exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(r)}(x_0)\right] + \ln D_i\right\} \qquad (16)$$

Parameter estimation γ, $\underset{\sim}{\beta}^{(1)}(x_0)$, dan $\underset{\sim}{\beta}^{(1)}(x_0)$ in bi-response Poisson regression model can be obtained by local likelihood maximum method given by following Theorem 1.

Theorem 1

By assuming bi-variate Poisson distribution for response variable y_{ri}, these considerations yield the conditional local weighted log-likelihood in Equation (16), the maximum likelihood estimator for parameter can be found from a solution of maximum likelihood equation:

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} K_h(x_i - x_0)\left\{1 - \sum_{k=0}^{s}\left[\left(\frac{\exp\left[\beta_{0i}^{(1)}(x_0) + \beta_{1i}^{(1)}(x_i - x_0)\right] - \gamma y_{1i}}{\exp\left[\beta_{0i}^{(1)}(x_0) + \beta_{1i}^{(1)}(x_i - x_0)\right] - \gamma^2}\right)\right.\right.$$

$$\left.\left.\frac{k}{\gamma} + \left(\frac{k - y_{2i}}{\exp\left[\beta_{0i}^{(2)}(x_0) + \beta_{1i}^{(2)}(x_i - x_0)\right] - \gamma}\right)\right]\right\} \qquad (17)$$

and $$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(1)}} = \sum_{i=1}^{n} K_h(x_i - x_0)\Big\{-\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right]\underset{\sim}{x}_i$$

$$+ \sum_{k=0}^{s} \frac{(y_{1i} - k)\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right]\underset{\sim}{x}_i}{\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right] - \gamma}\Big\} \qquad (18)$$

$$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(2)}} = \sum_{i=1}^{n} K_h(x_i - x_0)\Big\{-\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right]\underset{\sim}{x}_i$$

$$+ \sum_{k=0}^{s} \frac{(y_{1i}-k)(y_{2i}-k)\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right]\underset{\sim}{x}_i}{\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right] - \gamma}\Big\} \qquad (19)$$

Proof:

The likelihood function in Equation (16) will have maximum value when the first derivative of parameter γ, $\underset{\sim}{\beta}^{(1)}(x_0)$, and $\underset{\sim}{\beta}^{(2)}(x_0)$ equal to zero. The partial derivatives of equation (11) is:

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} K_h(x_i - x_0)\left\{1 - \left(\frac{1}{D_i}\frac{\partial D_i}{\partial \gamma}\right)\right\} \qquad (20)$$

$\frac{\partial D_i}{\partial \gamma}$ in equation (20) can be obtained using equation (15) and equation (16) as follows:

$$\frac{\partial D_i}{\partial \gamma} = \sum_{k=0}^{s}\left(\frac{\partial D_{ik}^{(1)}}{\partial \gamma} D_{ik}^{(2)} + \frac{\partial D_{ik}^{(2)}}{\partial \gamma} D_{ik}^{(1)}\right) \qquad (21)$$

where

$$\frac{\partial D_{ik}^{(1)}}{\partial \gamma} = \frac{\gamma^{k-1}\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right] - \gamma\right)^{y_{1i}-k-1}}{k!(y_{1i}-k)!} \times$$

$$\left(k\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right] - \gamma y_{1i}\right) \qquad (22)$$

$$\frac{\partial D_{ik}^{(2)}}{\partial \gamma} = \frac{\left(k - y_{2i}\right)\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right] - \gamma\right)^{y_{2i}-k-1}}{(y_{2i}-k)!} \qquad (23)$$

By substituting equation (15), equation (16), equation (22), and equation (23) to equation (21), we have:

$$\frac{\partial D_i}{\partial \gamma} = \sum_{k=0}^{s}\left\{\left[\frac{\gamma^k\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right] - \gamma\right)^{y_{1i}-k}}{k!(y_{1i}-k)!(y_{2i}-k)!}\right.\right.$$

$$\times\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma\right)^{y_{2i}-k}\Bigg]$$

$$\times\left[\frac{k\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma y_{1i}}{\gamma\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma^2}+\frac{k-y_{2i}}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma}\right]\Bigg\} \quad (24)$$

According to equation (14) and equation (23), equation (20) can be written:

$$\frac{\partial L}{\partial\gamma}=\sum_{i=1}^{n}K_h(x_0-x_0)\left\{1-\sum_{k=0}^{s}\left(\frac{k\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma y_{1i}}{\gamma\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma^2}+\frac{k-y_{2i}}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma}\right)\right\} \quad (25)$$

$$\frac{\partial L}{\partial\underset{\sim}{\beta}^{(r)}(x_0)}=\sum_{i=1}^{n}K_h(x_i-x_0)\left\{-\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(r)}(x_0)\right]\underset{\sim}{x}_i\times+\frac{1}{D_i}\frac{\partial D_i}{\partial\underset{\sim}{\beta}^{(r)}(x_0)}\right\}; \quad (26)$$

$\frac{\partial D_i}{\partial\underset{\sim}{\beta}^{(1)}(x_0)}$ and $\frac{\partial D_i}{\partial\underset{\sim}{\beta}^{(1)}(x_0)}$ in equation (26) can be obtained based on equation (14) :

$$\frac{\partial D_i}{\partial\underset{\sim}{\beta}^{(1)}(x_0)}=\sum_{k=0}^{s}\left\{\frac{\gamma^k}{k!}\frac{(y_{1i}-k)\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma\right)^{y_{1i}-k-1}}{(y_{1i}-k)!}\right.$$
$$\left.\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]\underset{\sim}{x}_i(x_0)\times\frac{\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma\right)^{y_{2i}-k}}{(y_{2i}-k)!}\right\} \quad (27)$$

$$\frac{\partial D_i}{\partial\underset{\sim}{\beta}^{(2)}(x_0)}=\sum_{k=0}^{s}\left\{\frac{\gamma^k}{k!}\frac{(y_{1i}-k)\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma\right)^{y_{1i}-k}}{(y_{1i}-k)!}\right.$$
$$\times\frac{(y_{2i}-k)\left(\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}$$
$$\left.\times\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]\underset{\sim}{x}_i(x_0)\right\} \quad (28)$$

According to equation (27), equation (26) can be expressed as:

$$\frac{\partial L}{\partial\underset{\sim}{\beta}^{(1)}(x_0)}=\sum_{i=1}^{n}K_h(x_i-x_0)\left\{-\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]\underset{\sim}{x}_i(x_0)+\sum_{k=0}^{s}\frac{(y_{1i}-k)\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]\underset{\sim}{x}_i(x_0)}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma}\right\} \quad (29)$$

According to equation (28), equation (26) can be expressed as:

$$\frac{\partial L}{\partial\underset{\sim}{\beta}^{(2)}(x_0)}=\sum_{i=1}^{n}K_h(x_i-x_0)\left\{-\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]\underset{\sim}{x}_i(x_0)+\sum_{k=0}^{s}\frac{(y_{1i}-k)(y_{2i}-k)\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]\underset{\sim}{x}_i(x_0)}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma}\right\} \quad (30)$$

The likelihood function in equation (16) has maximum value when equation (25), equation (29), and equation (30) equals zero, we can be written as :

$$\sum_{i=1}^{n}K_h(x_i-x_0)\left\{1-\sum_{k=0}^{s}\left(\frac{k\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma y_{1i}}{\gamma\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma^2}+\frac{k-y_2i}{\exp\left[\underset{\sim}{x}_i^T(x0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma}\right)\right\}=0 \quad (31)$$

$$\sum_{i=1}^{n}K_h(x_i-x_0)\left\{-\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]\underset{\sim}{x}_i(x_0)+\sum_{k=0}^{s}\frac{(y_{1i}-k)\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]\underset{\sim}{x}_i(x_0)}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(1)}(x_0)\right]-\gamma}\right\}=0 \quad (32)$$

(34)

$$\sum_{i=1}^{n}K_h(x_1-x_0)\left\{-\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]\underset{\sim}{x}_i(x_0)+\sum_{k=0}^{s}\frac{(y_{1i}-k)(y_{2i}-k)\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]\underset{\sim}{x}_i(x_0)}{\exp\left[\underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(2)}(x_0)\right]-\gamma}\right\}=0 \quad (33)$$

The first partial derivative obtained in Equation (31) to Equation(33) are nonlinear in parameters so that an iteration process is required to obtain the solution. The commonly used numerical method is the Newton-Raphson method. In the Newton-Raphson method, there is a Hessian matrix whose element is a partial derivative of the two functions of ln likelihood to the γ, $(\underset{\sim}{\beta}^{(1)})^T$, and $(\underset{\sim}{\beta}^{(2)})^T$ parameters.

5. Conclusion

Estimator bi-response Poisson regression model using local linear approach can be obtained from the first partial derivative of the likelihood function. The first partial derivative are nonlinear in parameters so that we required an iteration process to obtain the solution, it is the Newton-Raphson method.

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