

# Modeling Cybercafé Internet Service for Revenue Optimization under Stochastic Demand

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**Abstract:** We consider a set of cybercafés faced with an optimal choice of bandwidth for internet service under stochastic stationary demand. The choice is made over uniformly time horizons with a goal of optimizing revenue for internet service provision. Considering customer demand and price for internet service at cybercafés, we formulate a finite state markov decision process model where states of a markov chain represent possible states of demand for internet service. A revenue matrix is generated; representing the long run measure of performance for the Markov decision process problem. The problem is to determine an optimal bandwidth adjustment policy at cybercafés so that the long run revenue generated is maximized for a given state of demand. Using dynamic programming, the optimal bandwidth adjustment policies are determined over a finite period planning horizon. Results from the case study demonstrate the existence of an optimal state-dependent option for bandwidth adjustment policy and revenue generated from internet service provision at the cybercafés.

**Keywords**— Bandwidth adjustment; internet service; modeling; revenue optimization; stochastic demand

## 1. INTRODUCTION

Communities all over the world use the internet to access information for academic, business and social welfare. In effect, the internet becomes a vital tool in bridging gap between the ICT disadvantaged people; according to Doezi [1]. The e-business era today has changed the mode of business operations and communication among millions of people all over the world. The internet is a vital means for job seekers; acting as a platform where employers and employees interact for possible recruitment. In the health sector, the internet allows healthy living to access medication and drugs using the e-pharmacy platform. Citizens from several countries have benefited in education; since quality degrees can be earned from reputable institutions of leaning without one leaving his/her town or village [2]. The use of internet in Uganda is picking at a steadily increasing rate; Uganda being one of the first countries in sub-Saharan Africa to gain full internet connectivity [3]. Although a substantial number of cybercafés are scattered in urban areas, major initiatives require bringing internet services to rural areas as well. Stewart J [4] defines a cyber café as a café or shop open to the public; where a computer can be hired for specified periods to access the internet, write a CV or play a game. In African countries, power outages pose a major problem against information/service provision as noted by Jensen [5]. Nyomoko, Richard and Makori [6] vividly point out how internet service as a venture for business investment has essential areas to evaluate with respect to the cybercafé to invest in, how decisions made improve cybercafé business performance and the performance of cybercafé after implementation. However, Farbey [7] shows how the use of

such areas evaluated provide varying responses to different organizations. Situations vary from organization to organization and the range of circumstance the technique would be applicable is extremely wide. As a business investment venture, Berghout [8] concludes that both qualitative and quantitative methods are desirable in evaluation cybercafé investment strategies among potential users.

## 2. OBJECTIVES OF THE STUDY

### 2.1 General Objective

The general objective of the study was to develop a mathematical model that optimizes bandwidth adjustment decisions and revenue generated from internet service at cybercafés under stochastic demand.

### 2.2 Specific Objectives

Specifically, the study sought to attain the following objectives:

1. To select and define model variables and parameters
2. To determine demand transition and revenue(reward) matrices of internet service in cybercafés
3. To develop a finite-period dynamic programming model that optimize bandwidth adjustment decisions and revenue in cybercafés
4. To solve the model using a real life case study

### 3. LITERATURE REVIEW

According to Clark [9] cost allocation and pricing of internet is vital to give the relationship between the range of service offered to users and the cost of providing these services. The author provides a new scheme for resource allocation, pricing and the expected capacity allocation. This is compared with a number of resource allocation schemes under consideration. The literature behind internet resource pricing models, mechanisms and methods benefits from the works of Huan H, Xu K and Li Y [10]; who guide us to understand how to effectively use internet resources by examining pricing strategies, mechanisms and methods. Therefore, with the evolution of service types, several corresponding mechanisms which can ensure price implementation and resource allocation with special reference to utility optimization economics. As a mode of improving internet service provision, Turan, Nihatkasap and Hüseyn [11] provide a heuristic algorithm from firm's perspective at managerial level to solve the bandwidth sourcing and task allocation problem. The authors illustrate how bandwidth provides selection and task allocation with stochastic constraints; where delay and jitter are considered as random variables in order to capture the stochastic nature of telecom network environment. Serafeimidis and Smithson [12] however argue the difficulty task associated with measuring and identifying the potential benefits and costs of an IT investment. It is also true that IT evaluation is complex and elusive Dillon [13]; since a substantial amount of money is lost because of inability of organizations to realize benefits.

While studies have tried to examine the dynamics of price and cost in internet service provision, a stochastic approach is sought to handle demand uncertainty among users in cybercafés with special reference to bandwidth provision as a revenue maximization strategy.

### 4. MODEL DESCRIPTION

We consider a set of cybercafés that periodically adjust the bandwidth to meet customer requirements at maximum revenue. The demand for internet service during each time period over a fixed planning horizon is described as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities at cybercafés over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action namely; to adjust bandwidth for faster internet speed (a decision denoted by  $Z=1$ ) or not to adjust bandwidth (a decision denoted by  $Z=0$ ) during each time period over a fixed planning horizon where  $Z$  is a binary decision variable. Optimality is defined such that the maximum revenue generated at cybercafés is accumulated at the end of  $N$  consecutive time periods spanning the planning horizon. In this paper, a two-period

( $N=2$ ) planning period is considered for two ( $b=2$ ) cybercafés

#### 4.1 Notation

##### Sets

- $i, j$  Set of states of demand
- $Z$  Set of bandwidth adjustment policies
- $b$  Set of cybercafés'

##### Parameters

- $D$  Demand matrix
- $Q$  Demand transition matrix
- $p_r$  Unit price
- $R$  Revenue matrix
- $e$  Expected revenue
- $a$  Accumulated revenue
- $Q_{ij}^Z$  Demand transition matrix
- $S$  Customer matrix

##### Others

- $n, N$  Stages
- $F$  Favorable demand
- $U$  Unfavorable demand

$$i, j \in \{F, U\} \quad Z \in \{0, 1\} \quad b \in \{1, 2\}$$

#### 4.2 Finite-Period Dynamic Programming Model

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal bandwidth adjustment policy can be expressed as a finite period dynamic programming model. Assuming  $g_n(i)$  denotes the optimal expected revenue accumulated at the end of periods  $n, n+1, \dots, N$  given that the state of the system at the beginning of period  $n$  is  $i \in \{F, U\}$ . The recursive equation relating  $g_n$  and  $g_{n+1}$  is

$$g_n(i) = \max_Z [Q_{iF}^Z(b) R_{iF}^Z(b) + g_{n+1}(F, b)] \\ + \max_Z [Q_{iU}^Z(b) R_{iU}^Z(b) + g_{n+1}(U, b)]$$

together with the conditions

$$h_{N+1}(b, F) = h_{N+1}(b, U) = 0$$

$$i \in \{F, U\}, \quad Z \in \{0, 1\}, \quad b = \{1, 2\} \quad n = 1, 2, \dots, N$$

This recursive relationship may be justified by noting that the cumulative revenue  $R_{ij}^Z(b) + h_{N+1}(b, j)$  at cybercafé  $b$  resulting from reaching state  $j \in \{F, U\}$  at the beginning of period  $n+1$  from state  $i \in \{F, U\}$  at the beginning of period  $n$  occurs with probability  $Q_{ij}^Z(b)$

Clearly,

$$e^Z(b) = [Q^Z(b)][R^Z(b)]^T \\ Z \in \{0, 1\}, \quad b = \{1, 2\} \quad (2)$$

where “T” denotes matrix transposition. Hence, the dynamic programming recursive equations

$$g_N(i, b) = \max_Z [e_i^Z(b) + \max_Z [Q_{iF}^Z(b)g_{N+1}(F, b)] + \max_Z [Q_{iU}^Z(b)g_{N+1}(U, b)]] \quad (3)$$

$$g_N(i, b) = \max_Z [e_i^Z(b)] \quad (4)$$

#### 4.3 Computing $Q^Z(b)$ and $C^Z(b)$

The demand transition probability from state  $i \in \{F, U\}$  to state  $j \in \{F, U\}$  at cybercafé  $b$ , given adjustment policy  $Z$  may be taken as the number of customers observed with demand is initially in state  $i$  and later with demand changing to state  $j$  divided by the number of customers over all states

That is

$$Q_{ij}^Z(b) = S_{ij}^Z(b) / [S_{iF}^Z(b) + S_{iU}^Z(b)]$$

$i \in \{F, U\}, b = \{1, 2\}, Z \in \{0, 1\}$

The revenue generated for providing internet service can be expressed as the product of unit price and demand

That is

$$R^Z(b) = p_r(b)D^Z(b)$$

$$R_{ij}^Z(b) = p_r(b)D_{ij}^Z(b)$$

## 5. Optimization

### 5.1 Optimization during Period 1

When demand is *favorable* (ie. in state F), the optimal bandwidth adjustment policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } e_F^1(b) > e_F^0(b) \\ 0 & \text{if } e_F^1(b) \leq e_F^0(b) \end{cases}$$

The associated generated revenue is then

$$h_1(b, F) = \begin{cases} e_F^1(b) & \text{if } Z = 1 \\ e_F^0(b) & \text{if } Z = 0 \end{cases}$$

Similarly, when demand is *unfavorable* (ie. in state U), the optimal bandwidth adjustment policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } e_U^1(b) > e_U^0(b) \\ 0 & \text{if } e_U^1(b) \leq e_U^0(b) \end{cases}$$

In this case, the generated revenue is

$$h_1(b, U) = \begin{cases} e_U^1(b) & \text{if } Z = 1 \\ e_U^0(b) & \text{if } Z = 0 \end{cases}$$

### 5.2 Optimization during Period 2

Using (3) and (4), and recalling that  $a_i^Z(b)$  denotes the already accumulated revenue at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^Z(b) = e_i^Z(b) + Q_{iF}^Z(b) \max [e_F^1(b), e_F^0(b)] + Q_{iU}^Z(b) \max [e_U^1(b), e_U^0(b)]$$

$$a_i^Z(b) = e_i^Z(b) + Q_{iF}^Z(b)h_2(b, F) + Q_{iU}^Z(b)h_2(b, U)$$

Therefore, when demand is *favorable* (ie. in state F), the optimal bandwidth adjustment policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_F^1(b) > a_F^0(b) \\ 0 & \text{if } a_F^1(b) \leq a_F^0(b) \end{cases}$$

while the generated revenue is

$$h_2(b, F) = \begin{cases} a_F^1(b) & \text{if } Z = 1 \\ a_F^0(b) & \text{if } Z = 0 \end{cases}$$

Similarly, when demand is *unfavorable* (ie. in state U), the optimal bandwidth adjustment policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_U^1(b) > a_U^0(b) \\ 0 & \text{if } a_U^1(b) \leq a_U^0(b) \end{cases}$$

In this case, the associated revenue generated is

$$h_2(b, U) = \begin{cases} a_U^1(b) & \text{if } Z = 1 \\ a_U^0(b) & \text{if } Z = 0 \end{cases}$$

## 5. A CASE STUDY ABOUT ZION AND COMPUTECH CYBERCAFÉS IN UGANDA

In order to demonstrate use of the model in §3-4, a case study from *Zion Cybercafe* and *Computech cybercafé* in Uganda is presented in this section. Customer demand for internet service fluctuates every week based on the bandwidth and speed realized at the two cybercafés. The goal of managers at both cafes is to maximize revenue generated when demand for internet service is favorable (state F) or unfavorable (state U) and hence, decision support is sought in terms of an optimal bandwidth adjustment policy and the associated revenue generated for offering internet service in a two-week planning horizon.

### 5.1 Data Collection

Samples of customers, demand (in minutes) of internet service were collected. The state transitions of demand and the respective bandwidth adjustment policies were examined

over twelve weeks. The data is presented in Table 1 and Table 2.

Table 1: Customers versus state-transitions at Cybercafés

Cybercafé (b)	States (F/U)	Adjustment Policy 1		Adjustment Policy 0	
		F	U	F	U
Zion (1)	F	18	3	16	2
	U	13	8	10	4
Computech (2)	F	15	6	11	7
	U	10	8	9	5

Table 2: Demand (in minutes) versus state-transitions at Cybercafés

Cybercafé (b)	States (F/U)	Adjustment Policy 1		Adjustment Policy 0	
		F	U	F	U
Zion (1)	F	5000	800	3500	1500
	U	750	1000	1350	1050
Computech (2)	F	4800	1500	1500	2500
	U	2000	3000	1200	1050

In either case, the unit price of using internet service = US\$ 0.0080 per minute.

### 5.2 Computation of Model Parameters

Using (5) and (6), the state-transition matrices and revenue matrices (in US\$) are as follows for the case when the bandwidth was adjusted (Z=1) during week 1 are:

$$Q^1(1) = \begin{bmatrix} 0.857 & 0.143 \\ 0.684 & 0.316 \end{bmatrix} \quad R^1(1) = \begin{bmatrix} 40 & 6.4 \\ 6 & 8 \end{bmatrix}$$

$$Q^1(2) = \begin{bmatrix} 0.714 & 0.286 \\ 0.556 & 0.444 \end{bmatrix} \quad R^1(2) = \begin{bmatrix} 38.4 & 12 \\ 16 & 24 \end{bmatrix}$$

while these matrices are given by

$$Q^0(1) = \begin{bmatrix} 0.889 & 0.111 \\ 0.714 & 0.286 \end{bmatrix} \quad R^0(1) = \begin{bmatrix} 28 & 12 \\ 10.8 & 8.4 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.611 & 0.389 \\ 0.643 & 0.357 \end{bmatrix} \quad R^0(2) = \begin{bmatrix} 12 & 20 \\ 9.6 & 8.4 \end{bmatrix}$$

for the case when the bandwidth is *not* adjusted (Z=0) during week 1.

When the bandwidth is adjusted (Z=1) during week 1, the matrices  $Q^1(1), Q^1(2)$  and  $R^1(1), R^1(2)$  yield the expected revenue

$$\begin{aligned} e_F^1(1) &= (0.857)(40) + (0.143)(6.4) = 35.20 \\ e_U^1(1) &= (0.684)(6) + (0.316)(8) = 6.63 \\ e_F^1(2) &= (0.714)(38.4) + (0.286)(12) = 30.85 \\ e_U^1(2) &= (0.556)(16) + (0.444)(24) = 19.55 \end{aligned}$$

However, when the bandwidth is *not* adjusted (Z=0) during week 1, the matrices  $Q^0(1), Q^0(2)$  and  $R^0(1), R^0(2)$  yield the expected revenue

$$\begin{aligned} e_F^0(1) &= (0.889)(28) + (0.111)(12) = 26.22 \\ e_U^0(1) &= (0.714)(10.8) + (0.286)(8.4) = 10.11 \\ e_F^0(2) &= (0.611)(12) + (0.389)(20) = 15.11 \\ e_U^0(2) &= (0.643)(9.6) + (0.357)(8.4) = 9.17 \end{aligned}$$

When the bandwidth is adjusted (Z=1)

the accumulated revenue during week 2 is

$$\begin{aligned} a_F^1(1) &= 35.20 + (0.857)(35.20) + (0.143)(10.11) = 66.81 \\ a_U^1(1) &= 6.63 + (0.684)(35.20) + (0.316)(10.11) = 33.90 \\ a_F^1(2) &= 30.85 + (0.714)(30.85) + (0.286)(19.55) = 58.47 \\ a_U^1(2) &= 19.55 + (0.556)(30.85) + (0.444)(19.55) = 45.38 \end{aligned}$$

However, when the bandwidth is *not* adjusted (Z=0), the accumulated revenue during week 2 is

$$\begin{aligned} a_F^0(1) &= 26.22 + (0.889)(35.20) + (0.111)(10.11) = 58.64 \\ a_U^0(1) &= 10.11 + (0.714)(35.20) + (0.286)(10.11) = 38.13 \\ a_F^0(2) &= 15.11 + (0.611)(30.85) + (0.389)(19.55) = 41.56 \\ a_U^0(2) &= 9.17 + (0.643)(30.85) + (0.357)(19.55) = 35.99 \end{aligned}$$

### 5.3 The Optimal bandwidth adjustment policy at cybercafés'

#### Week 1

##### Zion cybercafé

Since  $35.20 > 26.22$ , it follows that Z=1 is an optimal bandwidth adjustment policy for week 1 with associated expected revenue of \$35.20 when demand is favorable. Since  $10.11 > 6.63$ , it follows that Z=0 is an optimal bandwidth adjustment policy for week 1 with associated expected revenue of \$10.11 when demand is unfavorable.

##### Computech cybercafé

Since  $30.85 > 15.11$ , it follows that Z=1 is an optimal bandwidth adjustment policy for week 1 with associated expected revenue of \$30.85 when demand is favorable. . Since  $19.55 > 9.17$ , it follows that Z=1 is an optimal bandwidth adjustment policy for week 1 with associated expected revenue of \$19.55 when demand is unfavorable

#### Week 2

##### Zion cybercafé

Since  $66.81 > 58.64$ , it follows that Z=1 is an optimal bandwidth adjustment policy for week 2 with associated accumulated revenue of \$66.81 when demand is favorable. Since  $38.13 > 33.90$ , it follows that Z=0 is an optimal bandwidth adjustment policy for week 2 with associated accumulated revenue of \$38.13 when demand is unfavorable

##### Computech cybercafé

Since  $58.49 > 41.56$ , it follows that Z=1 is an optimal bandwidth adjustment policy for week 2 with

associated accumulated revenue of \$58.49 when demand is favorable. Since  $45.38 > 35.99$ , it follows that  $Z=1$  is an optimal bandwidth adjustment policy for week 2 with associated accumulated revenue of \$45.38 when demand is unfavorable

## 6. CONCLUSIONS AND DISCUSSION

The optimal bandwidth adjustment policy for internet service provision at cybercafés' and the corresponding revenue generated can be modelled using markov decision processes under stochastic stationary demand for internet service. Results of the proposed model indicate optimal state-dependent bandwidth adjustment policies and revenue at the two cybercafés' used as the case study. As a revenue maximization strategy for internet service at cybercafés', computational efforts of using markov decision processes provide promising results. However, further extensions of this study is sought in order to examine a considerable number of cybercafés' for critical analysis. The model developed also raises a number of salient issues to consider eg. Power disruptions during internet service provision and abrupt changes in the speed of internet service; a major determinant for customer retention. Special interest is also sought to analyze optimality of bandwidth adjustment policies under dynamic stochastic demand. By the same token, further extensions of the model are deemed vital to analyze the problem using continuous time markov chains.

## 7. ACKNOWLEDGEMENTS

The authors would like to thank the staff of Zion and Computech cybercafés whose data contributed to the stage of testing the developed model.

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