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Stochastic Inventory Model for the Global Supply Chain Problem

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***Abstract**: We consider a single-item three-echelon global supply chain problem; consisting of manufacturing plants in worldwide locations. The item marketers consist of major distributors, wholesalers, and retailers at the respective locations.. Associated with each echelon at the respective location is stochastic stationary demand of item; where inventory replenishment periods are uniformly fixed over the echelons. Considering on-hand inventory positions of item, we determine the total inventory cost matrix for echelons; representing the long run measure of performance or the markov decision process problem. We formulate a finite-state markov decision process model where states of a markov chain represent possible states of demand. The objective is to determine over each echelon of the selected location; an optimal inventory replenishment policy of item so that the long run inventory costs are minimized for the given state of demand. The decisions of replenishing versus not replenishing additional units of item are made using dynamic programming over a finite period planning horizon. We present a numerical example for illustrative purposes. The model demonstrates the existence of an optimal state-dependent inventory replenishment policy and costs of item over the echelons and locations of the global supply chain network.*
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Keywords— Global supply chain; inventory; optimization; stochastic demand

1. Introduction

The global supply chain framework is a dynamic worldwide network that involves people, information, processes and resources used in the production, handling and distribution of materials and finished products, thus providing a service to the customer. To achieve successful integration, flows of information, materials and finances thorough coordination of the supply chain management is paramount. Swaminathat and Tayur [1] illustrate how global supply chain management plays a critical role in the digital economy as the rapid growth and adoption of the internet has already had great impact in all aspects of business performance. It is estimated that during the next five years, collaboration by supply chain partners over the internet can potentially save $223 billion with reduction in transaction, production, and inventory costs as Keenan and Ante [2] explain. It is eminent

Supply chain management has played an important role in traditional businesses; whose role is still needed at the global level. Both plants and purchased inventories among major regional distributors, wholesalers and retail outlets play a vital role to support cost-reduction strategies of inventory along the product supply chains. For example, Lundegaard [3] vividly show how Autoliv reduced the plant inventories by 37% after coordinating orders online with suppliers. Inventory reduction has a large potential impact on product supply chains and the study of inventory optimization in a global supply chain framework makes this study timely and important since inventory is a powerful and compelling enabler of the entire supply chain to function or to operate. All companies that are key players of the global supply chain need to interact, and use of e-business is central to devise appropriate inventory policies in order to support members that form the chain. However, development of an appropriate inventory optimization model is not easy in such an effort; especially under stochastic demand.

2. Literature Review of Supply Chain Models

Supply chain management science traces its origin in 1960s.In one of the first works, Clark and Scarf [4] developed inventory management models at multiple locations. In an effort to foster inventory optimization in a global supply chain, inventory and allocation in a distribution network has been studied by researchers for several years; for instance Eppen and Shrage [5] examined the inventory allocation decisions in a distribution network. Past research related to inventory problems under stochastic environments; mostly from a centralized perspective is well captured in the research handbook by Graves, de Kok et al [6]. However, one way to effectively model inventory supply chains is to characterize the optimal inventory policies as Zipkin([7] , Federgruen[8] and Porteus[9] explain.Several other issues related to supply chain management have been tackled via modeling. For example, decentralized multi-agent models to analyze supply chain coordination models that integrate information availability across supply chain with logistics decisions, models for supply chain contracts and demand forecasting models that integrate product design with supply chain management.

The primary contributions of this paper to supply chain models under demand uncertainty are as follows:

1. We illustrate how inventory replenishment policies of an item in a global supply chain framework can be established using demand transition matrices and inventory cost (reward) matrices
2. We show how markov decision processes can be used to efficiently compute the expected and accumulated inventory costs of an item over the global supply chain network
3. We derive the optimal replenishment policies and costs under different states of demand on the side of distributors, wholesalers and retailers along the supply chain echelons of the respective location.

3. The Global Supply Chain Model Description

We consider a global supply chain management network consisting of manufacturing plants at designated locations worldwide, major distributors, wholesalers and retailers that form the product supply chain. The product demand during replenishment periods at a given location over a fixed planning horizon is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to replenish additional units of item (a decision denoted by Z=1) or *not* to replenish additional units (a decision denoted by Z=0) during each time period over the planning horizon, where Z is a binary decision variable. Optimality is defined such that the minimum inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon along echelon *h* of location *L*. In this paper, a three-echelon (*h*=3), two-location (L=2) and two-period (N=2) planning horizon are considered.

3.1 Notation

Sets

i,j Set of states of demand
L Set of locations
Z Set of replenishment policies
h Set of supply chain echelons

Parameters

M Demand matrix
Q Demand transition matrix
D Distributor matrix
W Wholesaler matrix
R Retailer matrix

Inventory

O On-hand inventory matrix

Costs

e Expected inventory costs
a Accumulated inventory costs
c_r Unit replenishment cost
c_h Unit holding cost
c_s Unit shortage cost
V Inventory cost matrix

Others

n,N Stages
F Favourable demand
U Unfavourable demand
Q^Z_{ij} Probability that demand changes from state i to state j given replenishment policy Z

i,j ε {F,U} Zε {0,1} h={1,2,3} L={1,2}
n=1,2,…..N

3.2 Finite-Period Dynamic Programming Formulation

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal replenishment policy can be expressed as a finite period dynamic programming model. Assuming $f_n(i,h,L)$ denotes the optimal expected inventory costs accumulated along echelon h of location L at the end of periods *n.n+1,………N* given that the state of the system at the beginning of period *n* is iε{F,U}.The recursive equation relating f_n and f_{n+1} is

$$f_n(i,h,L) = min_Z[Q^Z_{iF}(h,L)V^Z_{iF}(h,L)] + min_Z[f_{n+1}(F,h,L), Q^Z_{iU}(h,L)V^Z_{iU}(h,L)] + min_Z[f_{n+1}(U,h,L)] \quad (1)$$

together with the conditions

$$f_{N+1}(F,h,L) = f_{N+1}(U,h,L) = 0$$

This recursive relationship may be justified by noting that the cumulative inventory costs $V^Z_{ij}(h,L) + f_{N+1}(j)$ resulting from reaching state jε{F,U} at the start of period *n+1* from state iε {F,U} at the start of period *n* occurs with probability $Q^Z_{ij}(h,L)$

Clearly $e^Z(h,L)=[Q^R(h,L)][V^Z(h,L)]^T$

Zε{0,1}, h={1,2,3} , L={1,2} (2)

where "T" denotes matrix transposition.

Hence, the dynamic programming recursive equations

$$f_N(i,h,L) = min_Z[e^Z_i(h,L) + Q^Z_{iF}(h,L)f_{N+1}(F)] + min_Z[Q^Z_{iU}(h,L)f_{N+1}(U)] \quad (3)$$

$$f_N(i,h,L) = min_Z[e^Z_i(h,L)] \quad (4)$$

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result where (4) represents the markov chain stable state.

3.3 Computing Q^Z(h,L) and V^Z(h,L)

The demand transition probability from state iε{F,U} to state jε{F,U} given replenishment policy Zε{0,1}along echelon h of location L may be taken as the number of distributors, wholesalers and retailers observed whose demand is initially in state i and later with demand changing to state j divided by the sum of distributors, wholesalers and retailers over all states. That is,

$$Q_{ij}^{Z}(h,L) = \begin{cases} D_{ij}^{Z}(h,L)/D_{iF}^{Z}(h,L) + D_{iU}^{Z}(h,L) & if\ \ h=1 \\ W_{ij}^{Z}(h,L)/W_{iF}^{Z}(h,L) + W_{iU}^{Z}(h,L) & if\ \ h=2 \\ R_{ij}^{Z}(h,L)/R_{iF}^{Z}(h,L) + R_{iU}^{Z}(h,L) & if\ \ h=3 \end{cases} \quad (5)$$

iε{F,U} , Zε{0,1}, h={1,2,3} , L={1,2}

When demand outweighs on-hand inventory, the inventory cost matrix V^Z(h,L)may be computed by means of the relation

$$V^Z(h,L) = c_r(h,L)[D^Z(h,L) - O^Z(h,L)] + c_h(h,L)[D^Z(h,L) - O^Z(h,L)] + c_s(h,L)[D^Z(h,L) - O^Z(h,L)]$$

Therefore

The justification for expression (5) is that $[D^Z_{ij}(h,L) - O^Z_{ij}(h,L)]$ units must be replenished to meet excess demand\Otherwise replenishment is cancelled when demand is less than or equal to on hand inventory

The following conditions must, however hold:

1. Z=1 when $c_r(h,L) > 0$ and Z=0 when $c_r(h,L) = 0$
2. When shortages are allowed, $c_s(h,L)>0$ and $c_s(h,L)=0$ when shortages are not allowed

4. Deriving the Optimal Inventory Replenishment Policy and Costs

The optimal inventory replenishment policy and costs are found in this section along the respective echelons and locations during periods 1 and 2.

4.1 Optimization during period 1

When demand is favorable (ie. in state F), the optimal replenishment policy and associated expected inventory costs during period 1 are

$$Z = \begin{cases} 1 & if\ \ e_F^1(h,L) < e_F^0(h,L) \\ 0 & if\ \ e_F^1(h,L) \geq e_F^0(h,L) \end{cases}$$

$$f_1(F,h,L) = \begin{cases} e_F^1(h,L) & if\ \ Z=1 \\ e_F^0(h,L) & if\ \ Z=0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy and associated expected inventory costs during period 1 are

$$Z = \begin{cases} 1 & if\ \ e_U^1(h,L) < e_U^0(h,L) \\ 0 & if\ \ e_U^1(h,L) \geq e_U^0(h,L) \end{cases}$$

$$f_1(U,h,L) = \begin{cases} e_U^1(h,L) & if\ \ Z=1 \\ e_U^0(h,L) & if\ \ Z=0 \end{cases}$$

4.2 Optimization during period 2

Using (3) and (4) and recalling that $a^Z_i(h,L)$ denotes the already accumulated inventory costs of item along echelon h of location L during the end of period 1 as a result of decisions made during that period

$$a_i^Z(h,L) = e_i^Z(h,L) + Q_{iF}^Z(h,L)min[e_F^1(h,L), e_F^0(h,L)] + Q_{iU}^Z(h,L)min[e_U^1(h,L), e_U^0(h,L)]$$

$$a_i^Z(h,L) = e_i^Z(h,L) + Q_{iF}^Z(h,L)f_2(F,h,L) + Q_{iU}^Z(h,L)f_2(U,h,L)$$

When demand is favorable (state F), the optimal replenishment policy and associated inventory costs are during period 2 are

$$Z = \begin{cases} 1 & if\ \ a_F^1(h,L) < a_F^0(h,L) \\ 0 & if\ \ a_F^1(h,L) \geq a_F^0(h,L) \end{cases}$$

$$f_2(F,h,L) = \begin{cases} a_F^1(h,L) & if\ \ Z=1 \\ a_F^0(h,L) & if\ \ Z=0 \end{cases}$$

Similarly, when demand is unfavorable (state U), the optimal replenishment policy and associated inventory costs during period 2 are

$$Z = \begin{cases} 1 & if\ \ a_U^1(h,L) < a_U^0(h,L) \\ 0 & if\ \ a_U^1(h,L) \geq a_U^0(h,L) \end{cases}$$

$$f_2(U,h,L) = \begin{cases} a_U^1(h,L) & if\ \ Z=1 \\ e_U^0(h,L) & if\ \ Z=0 \end{cases}$$

5. Example Application

In order to demonstrate use of the model in §3-4, an illustrative example for plant manufacturers, major distributors, wholesalers and retailers of calculators are presented in this section. We consider two manufacturing plants with branches in different countries (locations) that produce and store calculators to sustain demand along the product supply chain.

The distributors replenish calculators in bulk based on the wholesaler demand and inventory positions of the product. The wholesalers replenish calculators from major distributors, whose stock is similarly influenced by the demand pattern of retailers. The demand of

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calculators fluctuates every week. Excess inventory must be avoided when demand is unfavorable (state U) and running out of stock when demand is favorable (state F) and hence, distributors, wholesalers and retailers seek decision support in terms of an optimal replenishment policy, and the associated inventory costs of calculators in a two-week planning period. The network topology of the global supply chain problem for calculators at the plants, major distributors, wholesalers and retailers is illustrated in Figure 1.

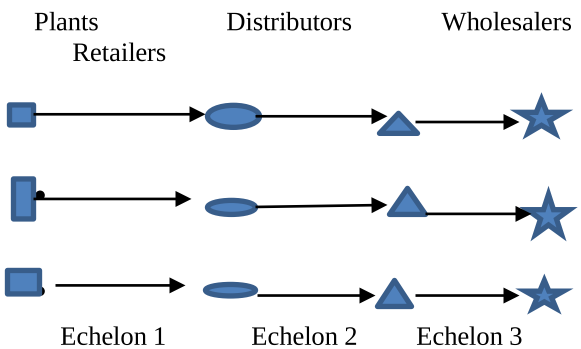

Fig 1: A Global Supply chain network for plants, distributors, wholesalers and retailers of calculators

Table 3: Customer category versus state transitions

Customer category	Echelon (h)	Location (L)	State of Demand (F/U)	Replenishment policy 1 F	Replenishment policy 1 U	Replenishment policy 0 F	Replenishment policy 0 U
Distributor (D)	1	1	F	300	350	450	280
			U	260	490	220	390
		2	F	650	640	425	400
			U	340	600	175	450
Wholesaler (W)	2	1	F	160	105	125	80
			U	80	155	65	135
		2	F	140	100	105	112
			U	80	155	65	135
Retailer (R)	3	1	F	85	50	55	90
			U	60	90	85	50
		2	F	95	60	85	50
			U	80	90	70	85

5.1 Data collection

Table 1: On-hand inventory of calculators versus state-transitions

Customer category	Echelon (h)	Location (L)	State of Demand (F/U)	Replenishment policy 1 F	Replenishment policy 1 U	Replenishment policy 0 F	Replenishment policy 0 U
Distributor (D)	1	1	F	12	30	42	38
			U	18	40	30	50
		2	F	50	48	85	25
			U	25	35	49	38
Wholesaler (W)	2	1	F	28	16	39	10
			U	10	10	25	18
		2	F	10	12	14	18
			U	14	10	11	12
Retailer (R)	3	1	F	6	5	4	9
			U	9	7	8	3
		2	F	4	2	5	3
			U	3	7	2	6

Table 2: Demand of calculators versus state-transitions

Customer category	Echelon (h)	Location (L)	State of Demand (F/U)	Replenishment policy 1 F	Replenishment policy 1 U	Replenishment policy 0 F	Replenishment policy 0 U
Distributor (D)	1	1	F	50	25	55	40
			U	25	38	26	65

		2	F U	45 28	30 40	60 30	35 22
Wholesaler (W)	2	1	F U	30 16	23 21	32 10	18 19
		2	F U	15 12	10 18	11 10	14 8
Retailer (R)	3	1	F U	8 5	3 9	6 10	4 7
		2	F U	7 5	3 6	1 3	4 1

Unit replenishment, holding, shortage) costs of calculators are similarly presented in Table 4.

Table 4: The unit replenishment, holding and shortage costs (in US$)

Echelon (h)	Location (L)	Replenishment cost (c_r)	Holding cost (c_h)	Shortage cost (c_s)
1	1	1.2	0.6	0.01
	2	1.2	0.6	0.01
2	1	1.5	0.6	0.03
	2	1.5	0.6	0.03
3	1	1.3	0.6	0.02
	2	1.3	0.6	0.02

5.2 Computation of Model Parameters

Using (5), the state-transition matrices can be calculated along the echelons and locations for the respective replenishment policies. When additional units are replenished (Z=1),

$$Q^1(1,1) = \begin{bmatrix} 0.588 & 0.412 \\ 0.347 & 0.653 \end{bmatrix} Q^1(1,2) = \begin{bmatrix} 0.596 & 0.404 \\ 0.362 & 0.638 \end{bmatrix}$$

$$Q^1(2,1) = \begin{bmatrix} 0.604 & 0.396 \\ 0.636 & 0.364 \end{bmatrix} Q^1(2,2) = \begin{bmatrix} 0.583 & 0.417 \\ 0.340 & 0.660 \end{bmatrix}$$

$$Q^1(3,1) = \begin{bmatrix} 0.630 & 0.370 \\ 0.400 & 0.600 \end{bmatrix} Q^1(3,2) = \begin{bmatrix} 0.613 & 0.387 \\ 0.467 & 0.533 \end{bmatrix}$$

When additional units are *not* replenished (Z=0),

$$Q^0(1,1) = \begin{bmatrix} 0.616 & 0.384 \\ 0.386 & 0.614 \end{bmatrix} Q^0(1,2) = \begin{bmatrix} 0.448 & 0.552 \\ 0.280 & 0.720 \end{bmatrix}$$

$$Q^0(2,1) = \begin{bmatrix} 0.610 & 0.390 \\ 0.354 & 0.646 \end{bmatrix} Q^0(2,2) = \begin{bmatrix} 0.484 & 0.516 \\ 0.325 & 0.675 \end{bmatrix}$$

$$Q^0(3,1) = \begin{bmatrix} 0.652 & 0.348 \\ 0.579 & 0.421 \end{bmatrix} Q^0(3,2) = \begin{bmatrix} 0.630 & 0.370 \\ 0.452 & 0.548 \end{bmatrix}$$

Using (6), the total inventory cost matrices are similarly calculated along the respective echelons and locations for the given replenishment policies. When additional units are replenished (Z=1)

$$V^1(1,1) = \begin{bmatrix} 32.6 & 3.0 \\ 12.7 & 1.2 \end{bmatrix} \qquad V^1(1,2) = \begin{bmatrix} 3 & 10.8 \\ 5.4 & 9.1 \end{bmatrix}$$

$$V^1(2,1) = \begin{bmatrix} 4.3 & 14.9 \\ 12.8 & 23.4 \end{bmatrix} \qquad V^1(2,2) = \begin{bmatrix} 10.7 & 1.2 \\ 1.2 & 4.3 \end{bmatrix}$$

$$V^1(3,1) = \begin{bmatrix} 3.8 & 1.2 \\ 2.4 & 3.8 \end{bmatrix} \qquad V^1(3,2) = \begin{bmatrix} 5.8 & 3.8 \\ 3.8 & 0.6 \end{bmatrix}$$

When additional units are *not* replenished (Z=0),

$$V^0(1,1) = \begin{bmatrix} 23.5 & 3.6 \\ 2.4 & 3.0 \end{bmatrix} \qquad V^0(1,2) = \begin{bmatrix} 3 & 1.8 \\ 10.2 & 3.6 \end{bmatrix}$$

$$V^0(2,1) = \begin{bmatrix} 17.4 & 19.2 \\ 7.2 & 2.4 \end{bmatrix} \quad V^0(2,2) = \begin{bmatrix} 1.8 & 2.4 \\ 0.6 & 2.4 \end{bmatrix}$$

$$V^0(3,1) = \begin{bmatrix} 2.4 & 1.92 \\ 1.92 & 3.0 \end{bmatrix} \quad V^0(3,2) = \begin{bmatrix} 2.4 & 1.92 \\ 1.92 & 3.0 \end{bmatrix}$$

Using (2) , the expected inventory costs are calculated along echelons at the respective locations, states of demand and replenishment policies(Table 5)

Table 5: Expected inventory costs (in thousand US$) for echelons, locations and states of demand

Echelon (h)	Location (L)	State of Demand (F/U)	Expected Inventory Costa Z=1	Z=0
1	1	F U	20.4 5.2	15.9 2.8
	2	F U	6.2 7.8	2.3 5.4
2	1	F U	8.5 16.5	18.1 4.1
	2	F U	6.7 .3.2	5.9 3.8
3	1	F U	2.8 3.2	2.2 2.4
	2	F U	5.0 2.1	2.2 2.7

Using (4), the accumulated inventory costs are calculated along the echelons at the respective locations, states of demand

and replenishment policies. Results are summarized in Tables 6.

Table 6: Accumulated inventory costs (in thousand US$) along echelons and locations given states of demand

Echelon (h)	Location (L)	State of Demand (F/U)	Accumulated Inventory Costa Z=1 Z=0	
1	1	F U	36.9 12.5	26.9 10.7
	2	F U	9.7 12.1	6.3 9.9
2	1	F U	15.3 22.3	24.9 9.8
	2	F U	11.4 7.3	10.3 7.8
3	1	F U	5.1 5.5	4.5 4.9
	2	F U	7.2 4.2	4.4 5.4

5.3 The optimal replenishment policy for distributors, wholesalers and retailers along echelons and locations

Week 1

Distributors

Echelon 1(Location 1)

Since 15.9 < 20.4, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 15.9US$ for the case of favorable demand. Since 2.8 < 5.2, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 2.8 US$ for the case when demand is unfavorable.

Echelon 1 (Location 2)

Since 2.3 < 6.2, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 2.3 US$ for the case of favorable demand. Since 5.4 < 7.8, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 5.4 US$ for the case when demand is unfavorable

Wholesalers

Echelon 2(Location 1)

Since 8.5 < 18.1, it follows that Z=1 is an optimal replenishment policy for week 1 with associated expected inventory costs of 8.5 US$ for the case of favorable demand. Since 4.1 < 16.5, it follows that Z=0 is an optimal replenishment policy for week 1 associated expected inventory costs of 4.1 US$ for the case when demand is unfavorable.

Echelon 2(Location 2)

Since 5.9 < 6.7, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 5.9 US$ for the case of favorable demand. Since 3.2 < 3.8, it follows that Z=1 is an optimal replenishment policy for week 1 with associated expected inventory costs of 3.2 US$ for the case when demand is unfavorable.

Retailers

Echelon 3(Location 1)

Since 2.2 < 2.8, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 2.2 US$ for the case of favorable demand

Echelon 3(Location 2)

Since 2.2 < 5.0, it follows that Z=0 is an optimal replenishment policy for week 1 with associated expected inventory costs of 2.2 US$ for the case of favorable demand. Since 2.1 < 2.7, it follows that Z=1 is an optimal replenishment policy for week 1 with associated inventory costs of 2.1 US$ for the case when demand is unfavorable.

week 2

Distributors

Echelon 1(Location 1)

Since 26.9 < 30.9, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of $26.9 for the case of favorable demand. Since 10.7<12.5, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of $10.7 for the case when demand is unfavorable.

Echelon 1 (Location 2)

Since 6.3 < 9.7, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 6.3 US$ for the case of favorable demand. Since 9.9 < 12.1, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 9.9 US$ for the case when demand is unfavorable.

Wholesalers

Echelon 2(Location 1)

Since 15.3 < 24.9, it follows that Z=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 15.3 US$ for the case of favorable demand. Since 9.8<22.3, it follows that Z=0 is an optimal replenishment policy for week 2 associated accumulated inventory costs of 9.8 US$ for the case when demand is unfavorable.

Echelon 2 (Location 2)

Since 10.3 < 11.4, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 10.3 US$ for the case of favorable demand. Since 7.3 < 7.8, it follows that Z=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 7.3 US$ for the case when demand is unfavorable.

Retailers

Echelon 3(Location 1)

Since 4.5 < 5.1, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 4.5 US$ for the case of favorable demand. Since 4.9<5.5, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 4.9 US$ for the case when demand is unfavorable.

Echelon 3(Location 2)

Since 4.4 < 7.2, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 4.4 US$ for the case of favorable demand. Since 4.2 < 5.4, it follows that Z=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 4.2 US$ for the case when demand is unfavorable. It is worth concluding this paper by identifying the limitations of the stochastic inventory model; which also indicate future research directions.

• Our model ignores factors that influence replenishment decisions in different regions of the global supply chain network.

• The developed model assumes uniform inventory cost structures (replenishment, holding and shortage) along the supply chain framework. This subject to critical analysis to support model validation

• It is an important challenge to extend the stochastic inventory model for optimal replenishment policies using continuous time markov chain modeling

• Finally, better and more robust models are required to analyze replenishment decisions under dynamic stochastic demand

6. Conclusion

The Global Supply Chain model with stochastic demand was presented in this paper. The model determines an optimal replenishment policy and inventory costs of an item under stochastic demand. The decision of whether or not to replenish additional units along supply chain echelons at a specific location was made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal replenishment policies and inventory costs on the side of distributors, wholesalers and retailers of calculators that form the supply chain network. .As a cost minimization strategy for inventory of the global supply chain problem, computational efforts of using Markov decision process approach provide promising results.

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