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Quotient Tritopology, Quotient Maps and Quotient Spaces in Tritopological Spaces

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Abstract— *In this work we shall investigate the reciprocal situation. If* $\boldsymbol{f}$ *is a function from tritopological space* $(\boldsymbol{X},\boldsymbol{\mathcal{T}},\boldsymbol{\mathcal{P}},\boldsymbol{\mathcal{Q}})$ *onto a set* $\boldsymbol{Y}$*, how may* $\boldsymbol{Y}$ *be tritopologized so that* $\boldsymbol{f}$ *is* $\boldsymbol{\delta^*}$*-continuous?. And we present theorem shows that there exists the* $\boldsymbol{\delta^*}$*-largest tritopology for* $\boldsymbol{Y}$ *relative to which* $\boldsymbol{f}$ *is* $\boldsymbol{\delta^*}$*-continuous.*

A main motivation for studying this subject is that, there is no published article about the Quotient Tritopological Spaces, we have defined it w.r.t. $\boldsymbol{\delta^*}$*-open set in tritopological spaces which we named it "* $\boldsymbol{\delta^*}$*- Quotient Spaces". Furthermore, we provide necessary conditions to some theorems for these spaces to be preserved.*

Keywords— Tritopological spaces, equivalence relation, δ^*-quotient tritopology, δ^*-quotient map, δ^*-quotient spaces.

1. INTRODUCTION

In mathematics, the study of any concept one may attempt to generalize the concept so that for a special case of the generalization the original concept is obtained. This is done with the hope that the generalization will lead to some interesting results. It is the object here to generalize the procedure for defining the δ^*-quotient tritopology and δ^*-quotient space.

The theory of quotient spaces is one of the most important and constitutes a very interesting and complex part of set-theoretic topology. Then the quotient space is the family (quotient set) X/R with the quotient topology (relative to the quotient map π) [13], [15].

Yet no concept of tritopologization in quotient spaces has been given till now. The purpose of this work is to give an introduction to the quotient tritopology. The quotient tritopology is one of the most ubiquitous constructions in set-theoretic tritopology. It is also among the most difficult concepts in set-theoretic tritopology. Hopefully these notes will assist you on your journey.

Up to this point in the text, when introducing important concepts for tritopological spaces such as δ^*-connectedness [6], δ^*-countability and δ^*-separability [7], δ^*-compactness [9] and δ^*-lindelofness [12] etc.

In this article we introduce a rather simple yet powerful method for constructing new tritopological spaces from given ones by means of taking "δ^*-quotients". This operation is well known in set-theoretic topology and we extend it here to tritopology by endowing the quotient set with a δ^*-quotient tritopology (relative to the δ^*-quotient map).

A tritopological space is simply a set X which is associated with three arbitrary topologies, was initiated by Kovar [14]. In 2004, Asmhan (the author) was introduced the definition of δ^*-open set in tritopological spaces as follows, a subset A of X is said to be δ^*-open set iff $A \subseteq \mathcal{T}\,\mathrm{int}(\mathcal{P}\,\mathrm{cl}(\mathcal{Q}\,\mathrm{int}(A)))$ [1]. And Asmhan et al.[5] defined the δ^*-base in tritopological spaces. In [2-4] the reader can find a relationships among separation axioms, and a relationships among some types of continuous and open functions in topological, bitopological and tritopological spaces. In 2017, the author presented the concept of the soft tritopological spaces [8]. In 2019, the author presented the concept of the fuzzy soft tritopological spaces [10]. Also, in the same year she presented the concept of the δ^*-product spaces [11].

In section 2, some preliminary and basic concepts about tritopological spaces and general quotient spaces in topological spaces are given. The main section of the manuscript is the third, which introduces the relevant concepts and explores some of the properties and theorems of the definitions δ^*-quotient tritopology, δ^*-quotient space, Upper Semi-δ^*-continuous Decomposition. Finally in section 4 the conclusions and some recent ideas of future work is suggested.

2. PRELIMINARIES

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In the following we will mention some basic definitions and notations in tritopological space and general quotient spaces in topological spaces which we need in this work.

Definition 2.1.[1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, a subset A of X is said to be δ^*-open set iff $A \subseteq \mathcal{T}\ \mathrm{int}(\mathcal{P}\ \mathrm{cl}(\mathcal{Q}\ \mathrm{int}(A)))$, and the family of all δ^*-open sets is denoted by $\delta^*.O(X)$. ($\delta^*.O(X)$ not always represent a topology). The complement of δ^*-open set is called a δ^*-closed set.

Definition 2.2. [1] $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called a discrete tritopological space with respect to δ^*-open if $\delta^*.O(X)$ contains all subsets on X. And $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called an indiscrete tritopological space with respect to δ^*-open if $\delta^*.O(X) = \{X,\emptyset\}$.

Definition 2.3. [1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let $x \in X$, a subset $\mathcal{N}$ of X is said to be a δ^*-nhd of a point x iff there exists a δ^*-open set U such that $x \in U \subset \mathcal{N}$. The set of all δ^*-nhds of a point x is denoted by $\delta^* - \mathcal{N}(x)$.

Definition 2.4. [5] A collection δ^*-β of a subsets of X is said to form a δ^*-base for the tritopology $(\mathcal{T},\mathcal{P},\mathcal{Q})$ iff:

1. δ^*-β $\subset \delta^*.O(X)$. for each point $x \in X$.
2. each δ^*-neighbourhood $\mathcal{N}$ of x there exists some $\mathcal{B} \in \delta^*$-β such that $x \in \mathcal{B} \subset \mathcal{N}$.

Definition 2.5. [1] The function $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ is said to be δ^*-continuous at $x \in X$ iff for every δ^*-open set V in Y containing $f(x)$ there exists δ^*-open set U in X containing x such that $f(U) \subset V$. We say f is δ^*-continuous on X iff f is δ^*-continuous at each $x \in X$.

Definition 2.6. [1] The function $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ is said to be δ^*-open (δ^*-closed) iff $f(G)$ is δ^*-open (δ^*-closed) in Y for every δ^*-open (δ^*-closed) set G in X.

Definition 2.7. [1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ and $(Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ are two tritopological spaces and $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ be a function, then f is δ^*-homeomorphism if and only if :

1. f is bijective (one to one , on to). 2. f and f^{-1} are δ^*-continuous.

Definition 2.8. [1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, a point x is called δ^*-limit point of a subset A of X iff for each δ^*-open set G containing another point different from x in A ; that is $(G/\{x\}) \cap A \neq \emptyset$, and the set of all δ^*-limit points of A is denoted by $\delta^* - lm(A)$.

Definition 2.9. [1] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called δ^*-T_2-space (δ^*-Hausdorff) if and only if for each pair of distinct points x , y of X, there exists two δ^*-open sets G , H such that $x \in G$, $y \in H$, $G \cap H = \emptyset$.

Definition 2.10. [1] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be a δ^*-regular iff for each δ^*-closed set F in X , and each $x \notin F$, there exist δ^*-open sets U, V such that $x \in X$, $F \subset V$, $U \cap V = \emptyset$.

Definition 2.11. [1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let A be any subset of X , then the collection $C = \{G_\lambda : \lambda \in \Lambda\}$ is called δ^*-open cover to A if C is a cover to A and $C \subset \delta^*.O(X)$.

Definition 2.12. [1] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let A be any subset of X , then A is called δ^*-compact set iff every δ^*-open cover of A has a finite sub-cover, i.e. for each $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $A \subset \cup\{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \dots, G_{\lambda n}$ among the G_λ's such that $A \subset G_{\lambda 1} \cup \dots \cup G_{\lambda n}$.
In particular, the space X is called δ^*-compact iff for each collection $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $X = \cup\{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \dots, G_{\lambda n}$ among the G_λ's such that $X = G_{\lambda 1} \cup \dots \cup G_{\lambda n}$.

Definition 2.13. [9] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be locally δ^*-compact iff every point in X has at least one δ^*-neighbourhood whose δ^*-closure is δ^*-compact.

Definition 2.14. [7] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space. The space is said to be a δ^*-second countable (or to satisfy the second axiom of δ^*-countability in tritopology) iff there exists a δ^*-countable base for a tritopology.

Definition 2.15. [7] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be a δ^*-separable iff X contains a countable δ^*-dense subset in $\delta^*.O(X)$, that is, iff there exists a countable subset say A of X such that δ^*-cl(A) = X.

Definition 2.16.[6] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said δ^*-connected iff the only non-empty subset of X which is both δ^*-open and δ^*-closed in X is X itself .

Definition 2.17.[6] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is δ^*-locally connected at a point $x \in X$ iff every δ^*-open neighbourhood of x, i.e. iff the collection of all connected δ^*-open neighbourhood of x forms a δ^*-local Base at x. The space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be δ^*-locally connected iff it is δ^*-locally connected at each of its points.

Definition 2.18.[12] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be a δ^*-Lindelof space if for any collection $\mathcal{C}$ of δ^*-open sets such that $X = \bigcup_{\mathcal{A}\in\mathcal{C}} \mathcal{A}$, there exists a countable sub-collection $\mathfrak{B} \subseteq \mathcal{C}$ such that $X = \bigcup_{B\in\mathfrak{B}} B$, That is, a tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be a δ^*- Lindelof space iff every δ^*-open cover of X in $\delta^*.O(X)$ has a δ^*-countable sub-cover.

Definition 2.19.[9] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is said to be locally δ^*-compact iff every point in X has at least one δ^*-neighbourhood whose δ^*-closure is δ^*-compact.

Definition 2.20.[11] A tritopology $(\mathcal{T},\mathcal{P},\mathcal{Q})$ on a set X is said to be δ^*-weaker (or δ^*-coarser or δ^*-smaller) than another Tritopology $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ on X . Or we can say that $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ is said to be δ^*-stronger (or δ^*-finer or δ^*-larger) than $(\mathcal{T},\mathcal{P},\mathcal{Q})$) iff

$\delta^*.O(X) \subset \delta^*.\acute{O}(X)$, (where $\delta^*.O(X)$ is the family of all δ^*-open sets in$(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ and $\delta^*.\acute{O}(X)$ is the family of all δ^*-open sets in $(X,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$).

According to this definition, indiscrete tritopology on any set X with respect to δ^*-open set is the δ^*-weakest whereas the discrete tritopology on any set X with respect to δ^*-open set is the δ^*-strongest. It is easy to see that the collection C off all tritopologies on a set X is a δ^*-partially ordered set with respect to the relation $\leq$ defined by setting $(\mathcal{T},\mathcal{P},\mathcal{Q}) \leq (\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ iff $(\mathcal{T},\mathcal{P},\mathcal{Q})$ is δ^*-weaker than $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$, where $(\mathcal{T},\mathcal{P},\mathcal{Q})$ *and* $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ are members of C. The indiscrete tritopology on X w.r.t. δ^*-open set is the δ^*-infimum and the discrete tritopology on X w.r.t. δ^*-open set is the δ^*-supremum of $(C,\leq)$.

Definition 2.21. [11] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ and $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ be two tritopological space. Then the tritopology (U,V,W) whose δ^*-base is $E = \{G \times H: G \in \delta^*.O(X) \text{ and } H \in \delta^*.O(Y)\}$ is called the δ^*-product tritopology for $X \times Y$ and $(X \times Y, U, V, W)$ is called the δ^*-product space of X and Y.

Definition 2.22. [15] Let X be non-empty set, and D a decomposition of X. Then the mapping π from X onto D such that $\pi(x)$ is the unique member of D to which x belongs is called the quotient map (or the projection map or canonical map).

Definition 2.23. [13] Let X be a topological space and R an equivalence relation on X. Let π be the quotient map of X onto the quotient set X/R of X over R so that $\pi(x) = [x]$ is the equivalence class to which x belongs. Then the quotient space is the family X/R with the quotient topology (relative to π).

3. δ^*-Quotient Spaces in Tritopological Spaces .

In this section we shall describe the technique for constructing a quotient tritopology, and concepts of a quotient space will be defined in tritopological spaces and those definitions and theorems which will be needed in the following subject will be given.
Because the families of all δ^*-open sets $\delta^*.O(X)$ and $\delta^*.O(Y)$ does not always represent a topology [1]. We provide some necessary conditions for these theorems to be valid under a δ^*-quotient.

3.1.Theorem. Let f be a mapping of a tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ onto a set Y , then the collection $\delta^*.O(Y)$ of all subsets G in Y such that $f^{-1}[G]$ is δ^*-open in X, then $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ is the δ^*-largest tritopology for Y such that f is δ^*-continuous. Further a subset F of Y is δ^*-closed if and only if $f^{-1}[F]$ is δ^*-closed in X.
[This theorem is valid when the collection $\delta^*.O(X)$ represent a topology on X]
Proof. We first show that $\delta^*.O(Y)$ is a topology for Y.
[T1]: Since $f^{-1}[\emptyset]= \emptyset$ is δ^*-open in X, we have $\emptyset \in \delta^*.O(Y)$.
Since $f^{-1}[Y] = X$ is δ^*-open in X, we have $Y \in \delta^*.O(Y)$.
[T2]: Let $G_1, G_2 \in \delta^*.O(Y)$. Then $f^{-1}[G_1]$ and $f^{-1}[G_2]$ are δ^*-open in X by hypothesis and so $f^{-1}[G_1] \cap f^{-1}[G_2] = f^{-1}[G_1 \cap G_2]$ is also δ^*-open in X (since $\delta^*.O(X)$ represent a topology). Therefor $G_1 \cap G_2 \in \delta^*.O(Y)$.
[T3]: Let $G_\lambda \in \delta^*.O(Y)$for every $\lambda \in \Lambda$ where Λ is an arbitrary index set. Then each $f^{-1}[G_\lambda]$ is δ^*-open in X by hypothesis and so (since $\delta^*.O(X)$ represent a topology) their union $\cup\{f^{-1}[G_\lambda]: \lambda \in \Lambda\} = f^{-1}[\cup\{G_\lambda: \lambda \in \Lambda\}]$ is also δ^*-open in X and therefore $\cup\{G_\lambda: \lambda \in \Lambda\} \in \delta^*.O(Y)$. Hence $\delta^*.O(Y)$ is represent a topology for Y. It then follows from theorem [1] that f is δ^*-continuous from $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ to $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$.

Now let (U',V',W') be another tritopology for Y such that f is δ^*-continuous from $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ to (Y,U',V',W'). Then $f^{-1}[H]$ is δ^*-open in $\delta^*.O(X)$ for every $H \in \delta^*.\acute{O}(Y)$ (where $\delta^*.\acute{O}(Y)$ is the family of all δ^*-open sets of (Y,U',V',W')) by theorem in [1], and therefore $H \in \delta^*.O(Y)$ by hypothesis. Hence $\delta^*.\acute{O}(Y) \subset \delta^*.O(Y)$. In other words, $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ is the δ^*-largest tritopology for Y such that f is δ^*-continuous from $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ to $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$. Finally let F be subset of Y. Then $(F \in \delta^*.C(Y))$ i.e. F is δ^*-closed in $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}}) \Leftrightarrow (Y - F \in \delta^*.O(Y))$ i.e. $Y - F$ is δ^*-open in $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}}) \Leftrightarrow f^{-1}[Y - F] = X - f^{-1}[F]$ is δ^*-open in X [8] $\Leftrightarrow f^{-1}[F]$ is δ^*-closed in X.

δ^*-Quotient Tritopology.

3.2.Definition. Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, Y a set, and f a mapping of $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ onto $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$. Then the δ^*-largest tritopology $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ for Y such tha f is δ^*-continuous is called the δ^*-quotient tritopology for Y (relative to f) denoted by $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})_f$ and $\delta^*.O(Y)$ shall be denoted by $\delta^*.O(Y)_f$ and the map f is called the δ^*-quotient map.

3.3.Theorem. A subset G of Y is δ^*-open in the δ^*-quotient tritopology (i.e. in $\delta^*.O(Y)_f$) (relative to $f:(X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$)) if and only if $f^{-1}[G]$ is a δ^*-open subset of X.

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Proof. Let G be a δ^*-open in the δ^*-quotient tritopology relative to $f: X \to Y$. Since f is δ^*-continuous by definition (3.2) , $f^{-1}[G]$ is δ^*-open in X by theorem [1].

Conversely, the collection of all sets G such that $f^{-1}[G]$ is δ^*-open in X is the δ^*-largest tritopology for Y such that f is δ^*-continuous. [See theorem 3.1].

3.4.Theorem. If f is δ^*-continuous mapping of a tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ onto another space $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ such that is either δ^*-open or δ^*-closed, then $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ must be the δ^*-quotient tritopology for Y.

Proof. First let f be δ^*-continuous and δ^*-open mapping. We want to show that $\delta^*.O(Y)_f = \delta^*.O(Y)$ where $\delta^*.O(Y)_f$ is the family of all δ^*-open sets of the δ^*-quotient tritopology $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ for Y.

By definition (3.2) , $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})_f$ is the δ^*-largest tritopology for Y making f δ^*-continuous and $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is any tritopology for Y for which f is δ^*-continuous. It follows that $\delta^*.O(Y) \subset \delta^*.O(Y)_f$.

Conversely, let $G \in \delta^*.O(Y)_f$. Then $f^{-1}[G]$ is δ^*-open in $\delta^*.O(X)$ by theorem (3.3). Since f is a δ^*-open mapping and onto, $f[f^{-1}[G]] = G$ is δ^*-open in $\delta^*.O(Y)$ i.e. $G \in \delta^*.O(Y)$ and therefore $\delta^*.O(Y)_f \subset \delta^*.O(Y)$. Hence $\delta^*.O(Y) = \delta^*.O(Y)_f$, $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is the δ^*-quotient tritopology for Y.

Now, let f be δ^*-continuous and δ^*-closed. Then $\delta^*.O(Y) \subset \delta^*.O(Y)_f$ as before. Conversely, if $G \in \delta^*.O(Y)_f$, then $f^{-1}[G]$ is δ^*-open in X (i.e. $f^{-1}[G] \in \delta^*.O(X)$) by theorem (3.3) and so $X - f^{-1}[G]$ is δ^*-closed in $\delta^*.C(X)$. But by theorem in [8] Since f is a δ^*-closed and onto map, $f[f^{-1}[Y - G]] = Y - G$ is δ^*-closed in $\delta^*.C(Y)$ and so $G \in \delta^*.O(Y)$ Therefore $\delta^*.O(Y)_f \subset \delta^*.O(Y)$. Thus $\delta^*.O(Y)_f = \delta^*.O(Y)$ in this case also.

3.5.Theorem. If f is a δ^*-continuous map of a δ^*-compact space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ onto a δ^*-Hausdorff space $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$, then $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ must be the δ^*-quotient tritopology for Y relative to f.

Proof. Let F be any δ^*-closed subset of X. Then F is δ^*-compact by theorem in [9] and so its δ^*-continuous image f[F] is also a δ^*-compact subset of Y by theorem in [9] Since Y is δ^*-Hausdorff, $f[F]$ is a δ^*-closed subset of Y by [9] and therefore f is a δ^*-closed mapping.

Hence by theorem (3.4), $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ must be the δ^*-quotient tritopology for Y.

3.6.Theorem. Let f be a δ^*-continuous map of a tritopological space X onto a tritopological space Y, and let Y have the δ^*-quotient tritopology (relative to f). Then a mapping g on Y onto a tritopological space Z is δ^*-continuous if and only if the composition map $g \circ f$ is δ^*-continuous.

Proof. Suppose $g \circ f$ is δ^*-continuous and let G be any δ^*-open set in Z (i.e. $G \in \delta^*.O(Z)$). Then $(g \circ f)^{-1}[G] = f^{-1}[g^{-1}[G]]$ is δ^*-open in X. Theorem in [1] and so $g^{-1}[G]$ is δ^*-open in Y by theorem (3.3) . Hence g is a δ^*-continuous map by theorem in [1].

Conversely, let $g: Y \to Z$ be δ^*-continuous.

Since $f: X \to Y$ is given to be δ^*-continuous, it follows from [1] that $g \circ f$ is a δ^*-continuous map.

3.7.Remark. It can be shown that the δ^*-quotient tritopology and the properties of δ^*-open and δ^*-closed maps have little to do with the range space. In fact, if f is a δ^*-continuous map of a space X onto a space Y with δ^*-quotient tritopology, then a tritopological copy of Y may be reconstructed from X, its tritopology and the family of all sets of the from $f^{-1}(y)$ with y in Y. The δ^*-continuation can be described as follows. Let D be the collection of all subsets X of the form $f^{-1}(y)$ with y in Y and let $\mathcal{F}$ be the function from X into D defined by $\mathcal{F}(x) = f^{-1}[f(x)]$ for all $x \in X$. Define $g: Y \to D$ by $g(y) = f^{-1}(y)$. Then it is easy to see that is a one-one map of Y onto D. Also $g \circ f = \mathcal{F}$ and $f = g^{-1} \circ \mathcal{F}$. If now we assign the δ^*-quotient tritopology to D (relative to $\mathcal{F}$), then theorem (3.6) shows that g and g^{-1} are δ^*-continuous (since $g \circ f = \mathcal{F}$ and $g^{-1} \circ \mathcal{F} = f$). Hence g is a δ^*-homeomorphism.

δ^*-quotient space.

3.8.Definition. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$be a tritopological space and R an equivalence relation on X. Let π be the δ^*-quotient map of X onto the quotient set X/R of X over R so that $\pi(x) = [x]$ is the equivalence class to which x belongs. Then the δ^*-quotient space is the family X/R with the δ^*-quotient tritopology (relative to π).

3.9.Remark.

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(i) Recall (3.2) that the δ^*-quotient tritopology for X/R is the δ^*-strongest tritopology for X/R rendering the δ^*-quotient map π δ^*-continuous on X. Then theorem (3.3) shows that the δ^*-quotient tritopology for X/R consists of all subsets G of X/R such that $\pi^{-1}[G]$ is δ^*-open in X.

(ii) If A is a subset of X , then the set of all points which are R-relatives of points A shall be denoted by [A] or R[A]. Thus $R[A] = \{y: (x, y) \in R$ for some $x \in A\}$. In other words, $R[A] = \cup \{D: D \in X/R$ and $D \cap A \neq \emptyset\}$. If x is a point of X, then we abbreviate $R[\{x\}]$ as $R[x]$ or $[\{x\}]$ as $[x]$. The set $R[x]$ or $[x]$ is the equivalence class to which x belongs and if π is the projection of X onto the decomposition, then $\pi(x) = R[x]$.

(iii) If $\mathcal{A} \subset X/R$, then $\pi^{-1}[\mathcal{A}] = \cup \{A: A \in \mathcal{A}\}$ and therefore $\mathcal{A}$ is δ^*-open (δ^*-closed) relative to the δ^*-quotient tritopology iff $\cup \{A: A \in \mathcal{A}\}$ is δ^*-open (respectively δ^*-closed) in X.

3.10.Theorem. Let π be the δ^*-quotient map of the tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ onto the δ^*-quotient space $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$, Then the following statements are equivalent.
(a) π is an δ^*-open (δ^*-closed) mapping.
(b) If A is δ^*-open (δ^*-closed) in X, then $R[A]$ is δ^*-open (respectively δ^*-closed).
(c) If B is a δ^*-closed (δ^*-open) subset of X, then the union of all members of X/R which are subsets of B is δ^*-closed (respectively δ^*-open).

Proof. (a) ⇔ (b). First observe that if A is a subset of X, then the set

$$R[A] = \pi^{-1}[\pi[A]]. \qquad \ldots (1)$$

Now let π be an δ^*-open map and let A be any δ^*-open subset of X. Then by definition of an δ^*-open map $\pi[A]$ is an δ^*-open subset of X/R. Since π is δ^*-continuous, $\pi^{-1}[\pi[A]]$ is δ^*-open in X and so by (1), $R[A]$ is an δ^*-open subset of X.

Conversely, let $R[A]$ be δ^*-open for each δ^*-open subset A of X so that by (1), $\pi^{-1}[\pi[A]]$ is δ^*-open in X. Then $\pi[A]$ is δ^*-open in X/R [See the remark (3.9) (i)] , and consequently π is an δ^*-open mapping.

(b) ⇔ (c): We first notice that the union of all those members of X/R which are subsets of B is $X - R[X - B]$. Now assume that $R[A]$ is δ^*-open for each δ^*-open subset A of X and let B be any δ^*-closed subset of X so that $X - B$ is δ^*-open is X Hence by hypothesis $R[X - B]$ is δ^*-open in X and consequently $X - R[X - B]$ is δ^*-closed in X, that is, the union of all those members of X/R which are subsets of B is δ^*-closed. This proves (b) ⇒ (c). To prove (c) ⇒ (b), assume that for each δ^*-closed subset B of X, the union of all those members of X/R which are subsets of B is δ^*-closed, that is, assume that $X - R[X - B]$ is δ^*-closed. Let A be any δ^*-open subset of X so that $X - A$ is δ^*-closed in X. Then by hypothesis $X - R[X - (X - A)]$ is δ^*-closed. That is, $X - R[A]$ is δ^*-closed and consequently $R[A]$ is δ^*-open.

If we interchange the words "δ^*-open" and "δ^*-closed" throughout, we get the proof of the dual proposition.

3.11.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space. If X is δ^*-compact, δ^*-connected, δ^*-locally connected, δ^*-separable or δ^*-Lindelof, then so is X/R.

Proof. Since δ^*-compactness, δ^*-connectedness, δ^*-separability and δ^*-Lindelofness are preserved under δ^*-continuous mappings and since X/R is a δ^*-continuous image of X under the quotient map π it follows that X/R also has these properties. To prove δ^*-local connectedness of X/R, we shall use the fact that a tritopological space X is δ^*-locally connected iff every component of a δ^*-open set is δ^*-open in X [6]. So let A be an δ^*-open subset of X/R and let B be a component of A. Then, the set $G = \pi^{-1}[A]$ is an δ^*-open subset of X containing the set $C = \pi^{-1}[B]$. [See the remark (3.9) (i)] . Let x be any point of C and let C_x be the component of C containing x.

Now $\quad x \in C \Rightarrow \pi(x) \in \pi[C] \Rightarrow \pi(x) \in \pi[\pi^{-1}(B)]$
$\qquad\qquad\qquad \Rightarrow \pi(x) \in B$
and $\quad x \in C_x \Rightarrow \pi(x) \in \pi[C_x]$.

definition of component, C_x is δ^*-connected and so its δ^*-continuous image $\pi[C_x]$ is also δ^*-connected. Also B, being a component, is Since $\pi(x) \in B$ and $\pi(x) \in \pi[C_x]$, we have $\pi[C_x] \neq \emptyset$. It follows from theorem in [6] that their $B \cup \pi[C_x]$ is δ^*-connected. Since $\pi[C_x]$ is a δ^*-connected subset it follows by the maximality of components that $\pi[C_x] \subset B$, hence $\pi^{-1}[\pi[C_x]] \subset \pi^{-1}[B]$, is, $C_x \subset C$. Since X is δ^*-locally connected, each component C_x be δ^*-open [6], and hence C is a δ^*-neighbor of x . Since x was an arbitrary point of C, we conclude that an δ^*-open set in X . Finally, since $C = \pi^{-1}[B]$, B is a δ^*-open of X/R [See the remark (3.9) (i)] and consequently X/R is δ^*-locally connected.

3.12.Remark. The δ^*-countability and separation axioms in tritopological spaces are not preserved. For example the δ^*-quotient space of a δ^*-Hausdorff space need not be a δ^*-Hausdorff space as the following example.

3.13.Example. Give an example of a δ^*-Hausdorff space X which a non-δ^*-Hausdorff quotient space.

Solution. Consider the space $(\mathbb{R}, U, U, U)$ where $\mathbb{R}$ denotes the set real numbers and U is the usual topology for $\mathbb{R}$. We know this tritopological space is δ^*-Hausdorff. Now consider the relation R on $\mathbb{R}$ of all pairs (x, y) such that $x - y$ is rational. Then it to see

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that R is an equivalence relation on $\mathbb{R}$. Also it is that the δ^*-quotient space $\mathbb{R}/R$ in this case is indiscrete tritopological space (w.r.t. δ^*-open set) and non-δ^*-Hausdorff.

3.14.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space and let the δ^*-quotient space $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be δ^*-Hausdorff. Then R is a δ^*-closed subset of the δ^*-product $X \times X$.

Proof. We shall show that no point of $X \times X - R$ can be a point R. Let $(x, y) \in X \times X - R$. Then $(x, y) \notin R$ and $\pi(x) \neq \pi(y)$. Since $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is δ^*-Hausdorff, there exist δ^*-open G and H in X/R such that $\pi(x) \in G, \pi(y) \in H$ and $G \cap H = \emptyset$. [remark (3.9) (i)], $\pi^{-1}[G]$ and $\pi^{-1}[H]$ are δ^*-open subsets of X and their images $\pi[\pi^{-1}[G]] = G$ and $\pi[\pi^{-1}(H)] = H$ disjoint, no member of $\pi^{-1}[G]$ can be R–related to a member of Hence $\pi^{-1}[G] \times \pi^{-1}[H]$ is a δ^*-neighbourhood of (x, y) which does not contain any point of R and consequently (x, y) is not a δ^*-limit point of R. It follows that R is δ^*-closed.

3.15.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space and let $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be the δ^*-quotient space. If the δ^*-quotient map π is an δ^*-open map and R is δ^*-closed in $X \times X$, then $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is a δ^*-Hausdorff space.

Proof. Let $\pi(x)$ and $\pi(y)$ be any two members of $(X/R, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ such that $\pi(x) \neq \pi(y)$. Then x is not R–related to y so that $(x, y) \notin R$.

Since R is δ^*-closed, (x, y) cannot be a δ^*-limit point of R. Hence there exists a basic δ^*-open subset $G \times H$ of the δ^*-product space $X \times X$ containing (x, y) and disjoint from R. This implies that Gand H are δ^*-open neighbourhoods of x and y respectively such that no point of G is R–related to a point of H. Hence $\pi[G]$ and $\pi[H]$ are disjoint. Since π is given to be an δ^*-open map, it follows that $\pi[G]$ and $\pi[H]$ are δ^*-open neighbourhoods of $\pi(x)$ and $\pi(y)$ respectively and consequently X/R is a δ^*-Hausdorff space.

Upper Semi-δ^*-continuous Decomposition.

3.16.Definition. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space. Then a decomposition D of X is said to be upper semi-δ^*-continuous iff for every $\mathcal{D} \in D$ and every δ^*-open set G containing $\mathcal{D}$ there exists an δ^*-open set H such that $\mathcal{D} \subset H \subset G$ and H is the union of members of D.

3.17.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space and let D be an upper semi-δ^*-continuous decomposition of X and let D have the δ^*-quotient tritopology $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$. If G is a δ^*-open set in $\delta^*.O(X)$ containing a member A of D, then $\pi[G]$ is a δ^*-neighbourhood of A in $\delta^*.O(D)$ where π is the δ^*-projection or (δ^*-quotient) map.

Proof. Since G is a δ^*-open set in $\delta^*.O(X)$ containing $A \in D$, by the definition of upper semi-δ^*-continuity above, there exists a δ^*-open set H in $\delta^*.O(X)$ which is a union of members of D such that $A \subset H \subset G$.

Hence $\pi[A] \subset \pi[H] \subset \pi[G]$. ... (1)

Since $A \in D$, by the definition of π we have $\pi[A] = \{A\}$,

and consequently $A \in \pi[A]$. ... (2)

It will now be shown that $\pi[H]$ is δ^*-open in $\delta^*.O(D)$. By definition of H, we have

$$H = \cup \{A_1 : A_1 \in D' \subset D\}$$

Then

$$\pi[H] = \pi[\cup \{A_1 : A_1 \in D' \subset D\}] = \cup \{\pi[A_1] : A_1 \in D' \subset D\} = \cup \{\{A_1\} : A_1 \in D' \subset D\}$$

Therefore

$$\pi^{-1}[\pi[H]] = \pi^{-1}[\cup \{A_1 : A_1 \in D' \subset D\}] = \cup \{\pi^{-1}[\{A_1\}] : A_1 \in D' \subset D\} = \cup \{A_1 : A_1 \in D' \subset D\} = H.$$

Since H is δ^*-open in $\delta^*.O(X)$, it follows from the definition of δ^*-quotient topology that $\pi[H]$ is a δ^*-open set in $\delta^*.O(D)$. Hence it follows from (1) and (2) that $\pi[G]$ is a δ^*- neighbourhood of A in $\delta^*.O(D)$.

3.18.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space and let D be a decomposition of X. Then D is upper semi-δ^*-continuous iff the δ^*-projection π of X onto D is δ^*-closed.

Proof. By theorem (3.10) , π is a δ^*-closed map iff for each δ^*-open subset G of X, the union H of all members of D which are subsets of G is an δ^*-open set. Now let π be a δ^*-closed map. Let $\mathcal{D} \in D$ and G be a δ^*-open set containing $\mathcal{D}$. If H is the union of all those members of D which are subsets of G, then by theorem (3.10), H is a δ^*-open set such that $\mathcal{D} \subset H \subset G$. Hence D is upper semi-δ^*-continuous.

To prove the converse, let D be upper semi-δ^*-continuous. Let G be a δ^*-open subset of X and let H be the union of all members of D which are subsets of G. We shall show that H is δ^*-open. Let $x \in H$. Then by definition of union, $x \in \mathcal{D} \subset G$. for some $\mathcal{D} \in D$.

Hence by upper semi-δ^*-continuity, there exists an δ^*-open set K which is the union of members of D such that $\mathcal{D} \subset K \subset G$. It follows that $K \subset H$. Thus we have shown that to each $x \in H$, there exists a δ^*-open set K such that $x \in K \subset H$. Hence H is a δ^*-open subset of X since it is a δ^*-neighbourhood of each of its points. It follows from theorem (3.10) , that π is a δ^*-closed map.

3.19.Theorem. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space and let D be an upper semi-δ^*-continuous decomposition of X such that every member of D is a δ^*-compact subset of X and let D have the δ^*-quotient tritopology $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$. If X is (i) δ^*-Hausdorff, (ii) δ^*-regular, (iii) δ^*-locally compact or (iv) satisfies the δ^*-second axiom of countability, then D also has the corresponding properties.

Proof. (i) Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a δ^*-Hausdorff space and let A and B be distinct members of D. Since A and B are δ^*-compact subsets of the δ^*-Hausdorff space X, there exist disjoint δ^*-nhds N and M of A and B in $\delta^*.O(X)$ respectively. Then by theorem (3.17) , $\pi[N]$ and $\pi[M]$ are disjoint δ^*-nhds of A and B in $\delta^*.O(D)$ respectively and consequently $(D, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is a δ^*-Hausdorff space.

(ii) Let X be δ^*-regular. Let A be any point of D and let N be a δ^*-nhd of A in $\delta^*.O(D)$. Then the union G of members of N is a δ^*-nhd of A in $\delta^*.O(X)$. Since$(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is δ^*-regular, for each $x \in A$, there exists a δ^*-open nhd U_x of x such that $\overline{U_x} \subset G$ theorem in [1]. Now the collection $\{U_x : x \in A\}$ is δ^*-open cover of A. Since A is δ^*-compact there exists a finite subcover $U_1, U_2, \ldots, U_n$ of A such that $\overline{U}_\iota \subset G$ for each i , so that $U_i \subset \cup\{A_i : i = 1,2, \ldots, n\}$. Let $V = \cup\{\overline{U}_\iota : i = 1,2, \ldots, n\}$. Then V is a δ^*-closed neighbourhood of A in $\delta^*.O(X)$ such that $V \subset G$. It follows by theorem (3.17) and (3.18) that $\pi[V]$ is a δ^*-closed neighbourhood in $\delta^*.O(D)$ of A such that $\pi[A] \subset \pi[V] = N$. Hence $(D, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is δ^*-regular theorem [1].

(iii) Let X locally δ^*-compact and A any member of D .

Then by definition of local δ^*-compactness, to each $x \in A$, there exists a δ^*-open compact nhd U_x of x in $\delta^*.O(X)$. Now the collection $\{U_x : x \in A\}$ is δ^*-open cover of A. Since A is δ^*-compact, there exists a finite subcover $U_1, U_2, \ldots, U_n$ so that $A \subset \cup\{U_i : i = 1,2, \ldots, n\}$. Let $V = \cup\{U_i : i = 1,2, \ldots, n\}$.

Also since V is the union of a finite number of δ^*-compact sets, it is easy to see that V is also δ^*-compact. Thus V is a δ^*-compact neighbourhood of A. Since π is δ^*-continuous, $\pi[V]$ is δ^*-compact. Also by theorem (3.17), $\pi[V]$ is a δ^*-neighbourhood A in $\delta^*.O(D)$. Thus we have shown that each point of D has a δ^*-compact neighbourhood, and hence by definition of local δ^*-compactness, $(D, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is locally δ^*-compact [9].

(iv) Let X be δ^*-second countable and let B be a countable δ^*-base for $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$. The family H of unions of finite subfamilies of B is a countable. For every member H of H, let H^* denote the union of all those members of D which are subsets of H and M be the family of all sets H^* corresponding to every $H \in$ H. Then it is easy to see that $\pi[H^*]$ is δ^*-open in D for every $H^* \in$ M. We will show that the countable collection $\{\pi[H^*] : H^* \in M\}$ is a δ^*-base for the δ^*-qountient tritopology $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$. For this purpose, it is sufficicnt to prove that for each $A \in D$ and each δ^*-neighbourhood N of A, there exists some $H^* \in$ M such that $A \subset H^* \subset N$.

Since A is δ^*-compact, it may be covered by a finite number of members of B such that the union H of these members, which is a member of H, is contained in N. Then $H^* \in$ M is such that $A \subset H^* \subset N$. This proves the theorem.

3.20.Note: In proving (iii) of (3.19) , we have adopted the following definition of local δ^*-compactness instead of the definition in [9] : X is locally δ^*-compact iff every point of X has a δ^*-compact neighbourhood. However (iii) can also be proved by using the same definition in [9].

Conclusions:

We define and study in this article the quotient spaces in tritopological spaces (which we named it δ^*- quotient space). And to motivate our definition several properties of δ^*-quotient spaces concept are established. Moreover, we obtain a characterization and preserving theorems with the help of some necessary conditions and interesting examples. And we are generalize some theorems in quotient spaces with δ^*-compact, locally δ^*-compact, δ^*-connected, δ^*-locally connected, δ^*-separable, δ^*-second countable and δ^*-Lindelof spaces in tritopology. Furthermore, Uses of tritopological results in this paper and some other papers is worthy for some possible applications in various areas of science and social science for future .

I want to show ideas and be clear enough that you can later explore them. The main emphasis will be towards combinatorics and the connection of tritopological spaces with topological and bitopological spaces by using some kinds of continuous functions which defined in past from tritopological to topological spaces, bitopological to topological spaces or vise verse. Then the researcher can define and study many quotient spaces in this manner.

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