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# Fuzzy $\boldsymbol{\rho}$-filter and fuzzy c-$\boldsymbol{\rho}$-filter in $\boldsymbol{\rho}$-algebra

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***Abstract***: In this paper we introduce the notions of fuzzy $\boldsymbol{\rho}$-filter and fuzzy complete$\boldsymbol{\rho}$-filter, in $\boldsymbol{\rho}$-algebra. Also, we give some theorems and relationships between them.

**Keywords:** fuzzy $\boldsymbol{\rho}$-filter , fuzzy complete-$\boldsymbol{\rho}$- filter,

**1.Introduction** In 1980, E. Y. Deeba introduced the notation of filters and in the setting of bounded implicative BCK-algebra constructed quotient algebra via a filter [2] It is known that the class of BCK-algebra is proper subclass of the class of BCI-algebra. J. Naggers and H. S. Kim introduced the notion of d-algebra in1999, which is another useful generalization of BCK-algebra [5]. The ideal theory plays an important rule in d-algebra . In 1999 [5] J. Naggers Y. B. Jun and H. S. Kim introduce the notion of d-ideal in d-algebra. in 2017 [4] M. Alradha ,and S. M. Khalil introduced the notion of characterizations of ρ-algebra and generation permutation topological ρ-algebra using permutation in symmetric group. The aim of this paper is to introduce some kind of fuzzy $\boldsymbol{\rho}$-filter and fuzzy complete-$\boldsymbol{\rho}$-filter ,also we study the relationships between them .

## 2. Preliminaries of $\boldsymbol{\rho}$-algebra

This portion we're introducing definition $\boldsymbol{\rho}$-*filte*r and c-$\boldsymbol{\rho}$-*filter* in $\boldsymbol{\rho}$-algebra, and some of their properties are presented.

***Definition* (2.1)[4]**

A $\boldsymbol{\rho}$-algebra is a set$\mathcal{K}$ with a binary operation " $*$ " and constant "1" which satisfies the following axioms :

- $\mathcal{K} * \mathcal{K} = 1$
- $1 * \mathcal{K} = 1$
- $\mathcal{K} * \mathcal{Z} = 1$ and $\mathcal{Z} * \mathcal{K} = 1$ imply $\mathcal{K} = \mathcal{Z}$, For all $\kappa, \mathcal{Z}, \in \mathcal{K}$
- For all $\mathcal{K} \neq \mathcal{Z}, \mathcal{K}, \mathcal{Z} \in \mathcal{K} - \{1\}$ , imply $\mathcal{K} * \mathcal{Z} = \mathcal{Z} * \mathcal{K} \neq 1$

Remark : (2.2)

In $\boldsymbol{\rho}$-algebra, $\mathcal{K}$ we denoted $\mathcal{K} * 1$ by $\mathcal{K}^*$ for every $\kappa \in \mathcal{K}$.

***Definition (2.3)***[3]

A nonempty subset $\mathcal{F}$ of a $\boldsymbol{\rho}$-algebra $\mathcal{K}$ is said to be $\boldsymbol{\rho}$-*filter if*

- $1 \in \mathcal{F}$
- $(\mathcal{K}^* * \mathcal{Z}^*)^* \in \mathcal{F}$ , $\mathcal{Z} \in F$ implies $\mathcal{K} \in \mathcal{F}$.

***Definition (2.4)[3]***

A subset of a $\boldsymbol{\rho}$ -algebra $X$ is side to be complete $\boldsymbol{\rho}$-*filte*r (c-$\boldsymbol{\rho}$-*filte*r ) if,

- $1 \in \mathcal{F}$
- $(\mathcal{K}^* * \mathcal{Z}^*)^* \in \mathcal{F}$ ,$\forall\, \mathcal{Z} \in \mathcal{F}$ , implies $\mathcal{K} \in \mathcal{F}$

***Proposition(2.5):***[3]

In $\boldsymbol{\rho}$-algebra $\mathcal{K}$ every $\boldsymbol{\rho}$-*filter* is a c-$\boldsymbol{\rho}$-*filter*.

***Definition (2.6):[6]***

Let $X$ be a non-empty set. A fuzzy set in $X$ is a function $\mu : X \to [0,1]$. If $\mu$ and $\eta$ be two fuzzy subsets of $X$, then by $\mu \subseteq \eta$ we mean $\mu(x) \le \eta(x)$ for all $x \in X$.

***Definition (2.9):[1]***

Let $\mu$ and $\eta$ be two fuzzy sets in $X$. Then :

1- $(\mu \cap \eta)(x) = min\{\mu(x), \eta(x)\}$, for all $x \in X$.

2- $(\mu \cup \eta)(x) = max\{\mu(x), \eta(x)\}$, for all $x \in X$.

$\mu \cap \eta$ and $\mu \cup \eta$ are fuzzy sets in $X$.

In general, if $\{\mu_i , i \in \Delta\}$ is a family of fuzzy sets in $X$, then :

$\bigcap_{i\in\lambda} \mu_i (x) = \inf\{ \mu_i(x) , i \in \lambda \}$, for all $x \in X$ and

$\bigcup_{i\in\lambda} \mu_i (x) = \sup\{\mu_i(x) , i \in \lambda \}$, for all $x \in X$.

which are also fuzzy sets in $X$.

***3.1 Fuzzy ρ-filter***

In this portion, we introduce concept fuzzy $\boldsymbol{\rho}$–*filter*, and give some its examples and properties .

***Definition (3.1)***

In $\boldsymbol{\rho}$-algebra $\mathcal{K}$ A fuzzy set $\mu$ is said to be fuzzy $\boldsymbol{\rho}$-*filter* of , if

(1) $\mu(1) \ge \mu(\mathcal{K})$ ,$\forall\ \kappa \in \mathcal{K}$

(2) $\mu(\mathcal{K}) \geq \min\{\mu((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu(\mathcal{Z})\}$ , $\forall\ \kappa, \mathcal{Z} \in \mathcal{K}$

***Example (3. 2):***

(I) every constant fuzzy set in $\boldsymbol{\rho}$-algebra is fuzzy $\boldsymbol{\rho}$–*filter*

(II) Let $\mathcal{K} = \{1, b, a, c, d\}$ and a binary operation $*$ is defined by

| $*$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $c$ | c |
| $b$ | $b$ | $c$ | 1 | $a$ |
| $c$ | $c$ | $c$ | $a$ | 1 |

Then $(\mathcal{K}, *, 1)$ is a $\boldsymbol{\rho}$ -algebra and. Let $\mu$ be the fuzzy set defined by

$$\mu(\mathcal{K}) = \begin{cases} 0.9 \ if\ \mathcal{K} = 1, a \\ 0.5\ if\ \mathcal{K} = b, c \end{cases}$$

Then $\mu$ is fuzzy $\boldsymbol{\rho}$-filter in $\mathcal{K}$. Since

$\mu(1) = 0.9 \geq \mu(\mathcal{K})$, $\forall$ k $\in \mathcal{K}$

$\mu(b) = 0.5 \geq \min\{\mu((b^* * 1^*)^*), \mu(1)\} = 0.5$

$\mu(b) = 0.5 \geq \min\{\mu((b^* * a^*)^*), \mu(c)\} = 0.5$

$\mu(c) = 0.5 \geq \min\{\mu((c^* * 1^*)^*), \mu(1)\} = 0.5$

$\mu(c) = 0.5 \geq \min\{\mu((c^* * a^*)^*), \mu(c)\} = 0.5$

***Example (3.3):***

Let$\mathcal{K} = \{1, b, a, c\}$ and a binary operation $*$ is defined by

| $*$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $a$ | $c$ |
| $b$ | $b$ | $a$ | 1 | $b$ |
| $c$ | $c$ | $c$ | $b$ | 1 |

$\mu(\mathcal{K}) = \begin{cases} 0.7 & if \quad \mathcal{K} = 1, b \\ 0.6 & if \quad \mathcal{K} = c, a \end{cases}$ , Then $\mu$ is not fuzzy $\boldsymbol{\rho}$-filter in $X$ since

$\mu(c) = 0.6 \ngeq \min\{\mu((c^* * b^*)^*), \mu(b)\} = 0.7$

***Proposition (3.4):***

Let $\{\mu_i : i \in \Delta\}$ be a family of fuzzy $\boldsymbol{\rho}$-filters in $\boldsymbol{\rho}$-algebra $\mathcal{K}$. the intersection of a family of fuzzy $\boldsymbol{\rho}$-filters is fuzzy $\boldsymbol{\rho}$-filters

*Proof :*

- $\mu_i(1) \geq \mu_i(1), \ \forall\, i \in \Delta$

$\inf\{\mu_i(1)\} \geq \inf\{\mu_i(\mathcal{K})\}$

So $\bigcap_{i\in\Delta} \mu_i\,(1) \geq \bigcap_{i\in\Delta} \mu_i\,(\mathcal{K})$

- let $\mathcal{K}, \mathcal{Z} \in \mathcal{K}$, then $\mu_i(\mathcal{K}) \geq \min\{\mu_i((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu_i(\mathcal{Z})\}$

Thus $\inf\{\mu_i\,(\mathcal{K})\} \geq \inf\{\min\{\mu_i((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu_i(\mathcal{Z})\}\}$

$$\geq \{\min\{inf\, \mu_i((\mathcal{K}^* * \mathcal{Z}^*)^*), inf\, \mu_i(\mathcal{Z})\}\}$$

So $\bigcap_{i\in\Delta} \mu_i\,(\mathcal{K}) \geq \min\{\bigcap_{i\in\Delta} \mu_i\,((\mathcal{K}^* * \mathcal{Z}^*)^*), \bigcap_{i\in\Delta} \mu_i\,(\mathcal{Z})\}$

***Remark (3. 5):***

The union of two fuzzy $\boldsymbol{\rho}$-filters it is not fuzzy $\boldsymbol{\rho}$-*filter* in general , as it is shown in the next example.

## ***Example (3. 6):***

Let $\mathcal{K} = \{1, b, a, c\}$ and a binary operation $*$ is defined by

| $*$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $c$ | $c$ |
| $b$ | $b$ | $c$ | 1 | $a$ |
| $c$ | $c$ | $c$ | $a$ | 1 |

Then $(\mathcal{K}, *, 1)$ is a $\boldsymbol{\rho}$-algebra. Let $\mu$ and $\eta$ be the fuzzy sets defined as the following

$$\mu(\mathcal{K}) = \begin{cases} 0.9 & if\ \mathcal{K} = 1, a \\ 0.3 & if\ \mathcal{K} = b, c \end{cases}, \quad \eta(\mathcal{K}) = \begin{cases} 0.6 & if\ \mathcal{K} = 1, b \\ 0.5 & if\ \mathcal{K} = a, c \end{cases}$$

Then $\mu$, is fuzzy $\boldsymbol{\rho}$-filters , by example (3.2)( II) and $\eta$ is fuzzy $\boldsymbol{\rho}$-filters since

$$\eta(a) = 0.5 \geq \min\{\eta((a * 1)^*), \eta(1)\} = 0.5$$

$$\eta(a) = 0.5 \geq \min\{\eta((a * b)^*), \eta(c)\} = 0.5$$

$$\eta(c) = 0.5 \geq \min\{\eta((c * 1)^*), \eta(1)\} = 0.5$$

$$\eta(c) = 0.5 \geq \min\{\eta((c * b)^*), \eta(b)\} = 0.5$$

$$\text{But}(\mu \cup \eta)(\mathcal{K}) = \begin{cases} 0.9 & if & \mathcal{K} = 1, a \\ 0.6 & if & \mathcal{K} = b \\ 0.5 & if & \mathcal{K} = c \end{cases}$$

is not fuzzy $\boldsymbol{\rho}$-filter since $(\mu \cup \eta)(c) = 0.5 \ngeq$

$\min\{\mu \cup \eta)((c * b)^*), \mu \cup \eta)(b)\} = 0.6$

## ***4- Fuzzy complete ρ-filter***

In this section, we describe fuzzy c-$\boldsymbol{\rho}$-filter of $\boldsymbol{\rho}$-algebra and study their relationship with fuzzy $\boldsymbol{\rho}$-filter.

***definition (4.1)***

Let $\mathcal{F}$ be c-$\boldsymbol{\rho}$-filter of $\boldsymbol{\rho}$-algebra $\mathcal{K}$. A fuzzy subset $\mu_F$ of $X$ is said to be fuzzy complete $\boldsymbol{\rho}$-*filter* ( fuzzy c-$\boldsymbol{\rho}$-filter) , at $\mathcal{F}$,if

- $\mu_{\mathcal{F}}(1) \geq \mu_{\mathcal{F}}(\mathcal{K})$ ,$\forall \mathcal{K} \in \mathcal{K}$
- $\mu_{\mathcal{F}}(\mathcal{K}) \geq \min\{\mu_{\mathcal{F}}((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu_{\mathcal{F}}(y)\}$ ,$\forall \mathcal{Z} \in \mathcal{F}$

***Example (4. 2):***

Let $\mathcal{K} = \{1, b, a, c, d\}$ and a binary operation $*$ is defined by

| $*$ | 1 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $a$ | $b$ | $d$ |
| $b$ | $b$ | $a$ | 1 | $b$ | $d$ |
| $c$ | $c$ | $b$ | $b$ | 1 | $d$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | 1 |

Then $(\mathcal{K}, *, 1)$ is a $\boldsymbol{\rho}$-algebra and $\mathcal{F} = \{1, c\}$ is c-$\boldsymbol{\rho}$-filter in $\mathcal{K}$. Let $\mu_F$ be the fuzzy set defined as the following

$$\mu_{\mathcal{F}}(x) = \begin{cases} 0.8 & if \ \ x = 1, c, d \\ 0.5 & if \ x = a, b \end{cases}$$

Then $\mu_{\mathcal{F}}$ is fuzzy c-$\boldsymbol{\rho}$-filter at $\mathcal{F}$ . Since

$\mu_{\mathcal{F}}(1) = 0.8 \geq \mu_{\mathcal{F}}(\mathcal{K})$ ,$\forall x \in \mathcal{K}$

$\mu_{\mathcal{F}}(a) = 0.5 \geq \min\{\mu_{\mathcal{F}}((a^* * 1^*)^*), \mu_{\mathcal{F}}(1)\} = 0.5$

$\mu_{\mathcal{F}}(a) = 0.5 \geq \min\{\mu_{\mathcal{F}}((a^* * c^*)^*), \mu_{\mathcal{F}}(c)\} = 0.5$

$\mu_{\mathcal{F}}(b) = 0.5 \geq \min\{\mu_F((b^* * 1^*)^*), \mu_{\mathcal{F}}(1)\} = 0.5$

$\mu_{\mathcal{F}}(b) = 0.5 \geq \min\{\mu_F((b^* * c^*)^*), \mu_{\mathcal{F}}(c)\} = 0.5$

***Example (4.3):***

Let $X = \{1, b, a\ c\}$ and a binary operation " $*$ " is defined by

| $*$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $a$ | c |
| $b$ | $a$ | $a$ | 1 | $a$ |
| $c$ | $c$ | $c$ | $a$ | 1 |

Then $(\mathcal{K}, *, 1)$ is a $\boldsymbol{\rho}$-algebra and $\mathcal{F} = \{1, a\}$ is c-$\boldsymbol{\rho}$-filter in $X$ but

$$\mu(\mathcal{K}) = \begin{cases} 0.8 & if \quad \mathcal{K} = 1, a \\ 0.4 & if \quad \mathcal{K} = b, c \end{cases}$$

$\mu$ is not fuzzy c-$\boldsymbol{\rho}$-filter at *since*

$\mu_{\mathcal{F}}(b) = 0.4 \ngeq \min\{\mu_{\mathcal{F}}((b^* * a^*)^*), \mu_{\mathcal{F}}(a)\} = 0.8$

$\mu_{\mathcal{F}}(b) = 0.4 \ngeq \min\{\mu_{\mathcal{F}}((b^* * 1^*)^*), \mu_{\mathcal{F}}(1)\} = 0.8$

***Proposition (4.4)***

Every fuzzy $\boldsymbol{\rho}$-filter is fuzzy c-$\boldsymbol{\rho}$-filter,.

*Proof:*

Let $\mathcal{F}$ be c-$\boldsymbol{\rho}$-filter and let $\mu$ be a fuzzy $\boldsymbol{\rho}$-filter, since

- $\mu(1) \geq \mu(\mathcal{K}), \ \forall \ \mathcal{K} \in \mathcal{K}.$
- $\mu(\mathcal{K}) \geq \min\{\mu((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu(\mathcal{Z})\}, \ \forall \mathcal{K}, \mathcal{Z} \in \mathcal{K}$ ,

since $\mathcal{F} \subseteq \mathcal{K}$ then $\mu(\mathcal{K}) \geq \min\{\mu((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu(\mathcal{Z})\}$ ,$\forall \mathcal{Z} \in \mathcal{F}$ Thus μ is fuzzy c-$\boldsymbol{\rho}$-filter, at $\mathcal{F}$ .

***Remark(4. 5):***

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The conversely of Proposition (4.4) is not true in general as in the example (4. 2). $\mu_F$ is fuzzy c-$\boldsymbol{\rho}$-filter at $\mathcal{F} = \{1, c\}$, but $\mu_F$ is not fuzzy $\boldsymbol{\rho}$-filter, since,

$$\mu_F(b) = 0.5 \ngeq \min\{\mu_F((b^* * \ d^*)^*), \mu_F(d)\} = 0.8$$

***proposition (4.6):***

Let $F$ be c-$\boldsymbol{\rho}$-filter in $\boldsymbol{\rho}$-algebra $\mathcal{K}$. If $\{\mu^i{}_F : i \in \Delta\}$ is a family of fuzzy c-$\boldsymbol{\rho}$-filters at $\mathcal{F}$, then $\bigcap_{i\in\Delta} \mu^i{}_F$ is fuzzy c-$\boldsymbol{\rho}$-filter. at $\mathcal{F}$

*Proof :* Since

- $\mu^i{}_{\mathcal{F}}(1) \geq \mu^i{}_{\mathcal{F}}(\mathcal{K})$, $\forall \mathcal{K} \in \mathcal{K}$. and , $\forall\ i \in \Delta$ , then

  $\inf\{\mu^i{}_{\mathcal{F}}(1)\} \geq \inf\{\mu^i{}_{\mathcal{F}}(\mathcal{K})\}$, so $\bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}(1) \geq \bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}(\mathcal{K})$, $\forall \mathcal{K} \in$
- $\mu^i{}_{\mathcal{F}}(\mathcal{K}) \geq \min\{\mu^i{}_{\mathcal{F}}((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu^i{}_{\mathcal{F}}(\mathcal{Z})\}$, $\forall \mathcal{Z} \in \mathcal{F}$

Thus, $\inf\{\mu^i{}_{\mathcal{F}}(\mathcal{K})\} \geq \inf\{\min\{\mu^i{}_{\mathcal{F}}((\mathcal{K}^* * \mathcal{Z}^*)^*), \mu^i{}_{\mathcal{F}}(\mathcal{Z})\}\}$, $\forall \mathcal{Z} \in \mathcal{F}$

$$\geq \{\min\{\inf \mu^i{}_{\mathcal{F}}((\mathcal{K}^* * \mathcal{Z}^*)^*), \inf \mu^i{}_{\mathcal{F}}(\mathcal{Z})\}\}, \forall \mathcal{Z} \in \mathcal{F}$$

So $\bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}(\mathcal{K}) \geq \min\{\bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}((\mathcal{K}^* * \mathcal{Z}^*)^*), \bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}(\mathcal{Z})\}$, $\forall \mathcal{Z} \in \mathcal{F}$

Then $\bigcap_{i\in\Delta} \mu^i{}_{\mathcal{F}}$ is fuzzy c-$\boldsymbol{\rho}$-filter at $\mathcal{F}$

***Remark (4.9):***

ιn general , the union of two fuzzy c-$\boldsymbol{\rho}$-filters it is not necessarily fuzzy c-$\boldsymbol{\rho}$-filter, as it is shown in the following example.

***example (4.10):***

Let $\mathcal{K} = \{1, a, b, c\}$ and a binary operation $*$ is defined by

| $*$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | $a$ | $a$ |

| | | | | |
|---|---|---|---|---|
| $b$ | $b$ | $a$ | 1 | $b$ |
| $c$ | c | $a$ | $b$ | 1 |

It is clear that ( $\mathcal{K}$,$*$ , 1 ) is a $\boldsymbol{\rho}$-algebra and $\mathcal{F} = \{1, c\}$ is c-$\boldsymbol{\rho}$-filter in $X$. Let $\mu_{\mathcal{F}}$ and $\eta_{\mathcal{F}}$ be two fuzzy sets defined as the following

define $\mu_{\mathcal{F}}(\mathcal{K}) = \begin{cases} 0.8 & if \;\; \mathcal{K} = \;1, c \\ 0.5 & if \;\; \mathcal{K} = a, b, \end{cases}$ $\eta_{\mathcal{F}}(\mathcal{K}) = \begin{cases} 0.6 & if \;\; \mathcal{K} = 1, b \\ 0.3 & if \;\; \mathcal{K} = a, c \end{cases}$

Then $\mu_{\mathcal{F}}$, $\eta_{\mathcal{F}}$ are fuzzy c-$\boldsymbol{\rho}$-filters at $\mathcal{F}$ since

$\mu_{\mathcal{F}}(1) = 0.8 \geq \mu_{\mathcal{F}}(\mathcal{K})$ , $\forall \mathcal{K} \in \mathcal{K}$, and

$\mu_{\mathcal{F}}(a) = 0.5 \geq \min\{\mu_{\mathcal{F}}((a * 1)^*), \mu_{\mathcal{F}}(1)\} = 0.5$

$\mu_{\mathcal{F}}(a) = 0.5 \geq \min\{\mu_{\mathcal{F}}((a * c)^*), \mu_{\mathcal{F}}(c)\} = 0.5$

$\mu_{\mathcal{F}}(b) = 0.5 \geq \min\{\mu_{\mathcal{F}}((b * 1)^*), \mu_{\mathcal{F}}(1)\} = 0.5$

$\mu_{\mathcal{F}}(b) = 0.5 \geq \min\{\mu_{\mathcal{F}}((b * c)^*), \mu_{\mathcal{F}}(c)\} = 0.5$

Also

$\eta_{\mathcal{F}}(1) = 0.6 \geq \eta_{\mathcal{F}}(\mathcal{K})$ , $\forall \mathcal{K} \in \mathcal{K}$, and

$\eta_{\mathcal{F}}(a) = 0.3 \geq \min\{\eta_{\mathcal{F}}((a * 1)^*), \eta_{\mathcal{F}}(1)\} = 0.3$

$\eta_{\mathcal{F}}(a) = 0.3 \geq \min\{\eta_{\mathcal{F}}((a * c)^*), \eta_{\mathcal{F}}(c)\} = 0.3$

$\eta_{\mathcal{F}}(c) = 0.3 \geq \min\{\eta_{\mathcal{F}}((c * 1)^*), \eta_{\mathcal{F}}(1)\} = 0.3$

$\eta_{\mathcal{F}}(c) = 0.6 \geq \min\{\eta_{\mathcal{F}}((c * c)^*), \eta_{\mathcal{F}}(c)\} = 0.6$

But $(\mu_{\mathcal{F}} \cup \eta_{\mathcal{F}})(\mathcal{K}) = \begin{cases} 0.8 & if \;\; \mathcal{K} = 1, c \\ 0.6 & if \;\; \mathcal{K} = b \\ 0.5 & if \; \mathcal{K} = a \end{cases}$

is not fuzzy c-$\boldsymbol{\rho}$-filter at $\mathcal{F}$ since $(\mu_{\mathcal{F}} \cup \eta_{\mathcal{F}})(a) = 0.5 \ngeq \min\{(\mu_{\mathcal{F}} \cup \eta_{\mathcal{F}})((a * c)^*), (\mu_{\mathcal{F}} \cup \eta_{\mathcal{F}})(c)\} = 0.6$

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