International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4 Issue 2, February – 2020, Pages: 1-4

On the Local Irregularity Vertex Coloring of Related Grid Graph

Nadia Azahra[1], Arika Indah Kristiana[2], Dafik[3], Ridho Alfarisi[4]

[1,2,3]Department of Mathematics Education, University of Jember , Jember, Indonesia
[4]Department of Primary Education, University of Jember, Jember, Indonesia
[1]nadiaazahraa29@gmail.com, [2]arika.fkip@unej.ac.id, [3]d.dafik@unej.ac.id, [4]alfarisi.fkip@unej.ac.id

Abstract: *All graph in this paper is connected and simple graph. Let graph d(u,v) be a distance between any vertex u and v in graph* $\boldsymbol{G}=(\boldsymbol{V},\boldsymbol{E})$. *A functio* $\boldsymbol{l}:\boldsymbol{V}(\boldsymbol{G})\to\{\mathbf{1},\mathbf{2},\dots,\boldsymbol{k}\}$*n is called vertex irregular k-labelling and* $\boldsymbol{w}:\boldsymbol{V}(\boldsymbol{G})\to\boldsymbol{N}$ *where* $\boldsymbol{w}(\boldsymbol{u})=\sum\boldsymbol{v}\in\boldsymbol{N}(\boldsymbol{u})\boldsymbol{l}(\boldsymbol{v})$. *If for every* $\boldsymbol{uv}\in\boldsymbol{E}(\boldsymbol{G}),\boldsymbol{w}(\boldsymbol{u})\neq\boldsymbol{w}(\boldsymbol{v})$ *and maks(l) = min{maks{l_i}; l_i , vertex irregular labelling} is called a local irregularity vertex coloring. χlis(G) or chromatic number of local irregularity vertex coloring of graph* (*G*) *is the minimum cardinality of the largest label over all such local irregularity vertex coloring. In this paper, we will study about local irregularity vertex coloring of related grid graphs, and we have found the exact value of their chromatic number local irregularity, namely ladder graph, triangular ladder graph, and H-graph.*

Keywords : local irregularity; vertex coloring; grid graph**.**

1. INTRODUCTION

Let $G(V,E)$ be a connected and simple graphs with vertex set *V* and edge set *E*. In this paper, we combine the two concepts, namely combining the local antimagic vertex coloring and the distance irregular labelling. So, we study a new notion of coloring type of graph, namely a local irregularity vertex coloring. This concept firstly was introduced by Kristiana, et.al [3]. By the local irregularity vertex coloring, we recall a bijection $l:V(G)\to\{1,2,\dots,k\}$ is called vertex irregular k-labelling and $w:V(G)\to N$ where $w(u)=\sum v\in N(u)l(v)$, l is called local irregularity vertex coloring. A condition for *w* to be a local irregularity vertex coloring, if *maks(l) = min{maks{l_i}; l_i , vertex irregular labelling}* and for every $uv\in E(G),w(u)\neq w(v)$. The chromatic number of local irregular denoted by $\chi lis(G)$, is minimum of cardinality local irregularity vertex coloring.

For ilustration local irregulairty vertex coloring and chromatic number local irregularity is provided in Figure 1.

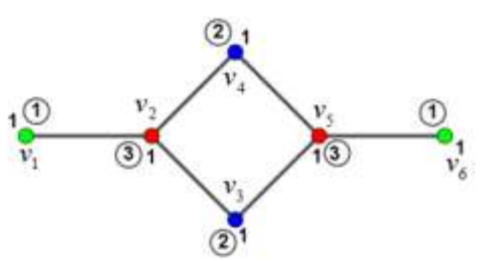

Fig. 1. *Chromatic number local irregular, χlis(G) = 4*

Lemma. Let *G* simple and connected graph,$\chi lis(G)\geq\chi(G)$.

The last, Kristiana, et.al [3] have discovered the exact value of the chromatic number local irregular of some graph, namely path, cycle, complete, star, friendship, and related wheel graphs.

2. RESULT

Theorem 2.1 Let *G* be a ladder graph (L_n). For $n\geq 2$, the chromatic number local irregular is χlis(L_n) = 4.

Proof. Let L_n for $n\geq 2$ be a ladder. Then L_n has the vertex set $V(L_{n,})=\{x_i,y_i:1\leq i\leq n\}$, and the edge set $E(L_{n,})=\{x_ix_{i+1},y_iy_{i+1}:1\leq i\leq n-1\}\cup\{x_iy_i:1\leq i\leq n\}$.If every vertex is labelled by 1, so we have $w(x_i)=l(x_{i-1})+l(x_{i+1})+l(y_i)=1+1+1=3$ and $w(x_{i+1})=l(x_i)+l(x_{i+2})+l(y_{i+1})=1+1+1=3$. It means a contradiction with definition. Since, $x_ix_{i+1}\in E(L_n),w(x_i)\neq w(x_{i+1})$. Hence *max(l)* = 2.

Based on Lemma, the lower bound for the chromatic number $\chi lis(L_{n,})\geq\chi(L_{n,})=2$, so that $\chi lis(L_{n,})\geq 2$.

Furthermore, the upper bound for the chromatic number local irregular, we define $l:V(L_n)\to\{1,2\}$ with vertex irregular 2-labelling as follows :

$$l(x_i)=\begin{cases}1, & \text{for } i \text{ odd}\\ 2, & \text{for } i \text{ even}\end{cases}$$

$$l(y_i)=\begin{cases}1, & \text{for } i \text{ even}\\ 2, & \text{for } i \text{ odd}\end{cases}$$

Hence, *max(l)* = 2 and the labelling provides vertex-weight as follows:

for *n* odd

$$w(x_i)=\begin{cases}3, & for\ i=even,\ \ 2\leq i\leq n-1\\ 6, & for\ i=odd,\ \ 2\leq i\leq n-1\\ 4, & for\ i=1\ and\ i=n\end{cases}$$

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4 Issue 2, February – 2020, Pages: 1-4

$$w(y_i)=\begin{cases}3, & for\ i=odd,\ \ 2\le i\le n-1\\ 6, & for\ i=even,\ \ 2\le i\le n-1\\ 2, & for\ i=1\ and\ i=n\end{cases}$$

for n even

$$w(x_i)=\begin{cases}4, & for\ i=1\\ 3, & for\ i=even,\ \ 2\le i\le n-1\\ 6, & for\ i=odd,\ \ 2\le i\le n-1\\ 2, & for\ i=n\end{cases}$$

$$w(y_i)=\begin{cases}2, & for\ i=1\\ 3, & for\ i=even,\ \ 2\le i\le n-1\\ 6, & for\ i=odd,\ \ 2\le i\le n-1\\ 4, & for\ i=n\end{cases}$$

For every $uv\ \in E(L_n)$, take any $u=x_i$ and $v=x_{i+1}$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(x_{i+1})$, then $u=y_i$ and $v=y_{i+1}$, $i=1,2,3,\dots n-1$, $w(y_i)\neq w(y_{i+1})$ and $u=x_i$ and $v=y_i$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(y_i)$.
Based on that, we have the set of vertices weight of the ladder graph L_n is $W(L_n)=\{2,3,4,6\}$, so $|w(V(L_n))|=4$. Hence, $\chi lis(L_n)=4$.

Theorem 2.2 Let G be a triangular ladder graph (TL_n). For $n\geq 5$, the chromatic number local irregular is

$$\chi lis(TL_n)=\begin{cases}5, for\ n=5\\ 6, for\ n\geq 6\end{cases}$$

Proof. Let TL_n for $n\geq 2$ be a triangular ladder. Then TL_n has the vertex set $V(TL_{n,})=\{x_i,y_i\!:1\le i\le n\}$, and the edge set $E(TL_{n,})=\{x_ix_{i+1},y_iy_{i+1}\!:1\le i\le n-1\}\cup\{x_iy_i\!:1\le i\le n\}\cup\{x_iy_{i+1}\!:1\le i\le n\}$. If every $x\in V(TL_n)$ is labelled by 1 and $y\in V(TL_n)$ is labelled by 2, for $1\le i\le n$, so we have $w(x_i)=w(x_{i+1})$. It means a contradiction with definition. Since, $x_ix_{i+1}\ \in\ E(TL_n), w(x_i)\neq w(x_{i+1})$. Hence *max(l)* = 3.
Based on Lemma, the lower bound for the chromatic number $\chi lis(TL_{n,})\geq\chi(TL_{n,})=3$, so that $\chi lis(TL_{n,})\geq 3$.

Case 1: for $n=5$
Furthermore, the upper bound for the chromatic number local irregular, we define $l\!:V(TL_n)\rightarrow\{1,2,3\}$ with vertex irregular 3-labelling as follows :

$$l(x_i)=\begin{cases}1, & for\ i\equiv 1\ (mod\ 3)\\ 2, & for\ i\equiv 2\ (mod\ 3)\\ 3, & for\ i\equiv 0\ (mod\ 3)\end{cases}$$

$$l(y_i)=\begin{cases}1, & for\ i\equiv 2\ (mod\ 3)\\ 2, & for\ i\equiv 0\ (mod\ 3)\\ 3, & for\ i\equiv 1\ (mod\ 3)\end{cases}$$

Hence, *max(l)* = 3 and the labelling provides vertex-weight as follows:

$$w(x_i)=\begin{cases}2, for\ i=n\\ 6, for\ i=1\\ 7, for\ i=2\\ 8, for\ i=3\\ 9, for\ i=4\end{cases}$$

$$w(y_i)=\begin{cases}2, for\ i=1\\ 6, for\ i=n\\ 7, for\ i=4\\ 8, for\ i=2\\ 9, for\ i=3\end{cases}$$

For every $uv\ \in E(TL_n)$, take any $u=x_i$ and $v=x_{i+1}$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(x_{i+1})$, then $u=y_i$ and $v=y_{i+1}$, $i=1,2,3,\dots n-1$, $w(y_i)\neq w(y_{i+1})$ and $u=x_i$ and $v=y_i$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(y_i)$, and the last, take $u=x_i$ and $v=y_{i+1}$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(y_{i+1})$. Based on that, we have the set of vertices weight of the triangular ladder graph TL_n is $W(TL_n)=\{2,6,7,8,9\}$, so $|w(V(TL_n))|=5$. Hence, $\chi lis(TL_n)=5$.

Case 2: for $n\geq 6, n\equiv 1(mod\ 3)$
Furthermore, the upper bound for the chromatic number local irregular, we define $l\!:V(TL_n)\rightarrow\{1,2,3\}$ with vertex irregular 3-labelling as follows :

$$l(x_i)=\begin{cases}1, & for\ i\equiv 1\ (mod\ 3)\\ 2, & for\ i\equiv 2\ (mod\ 3)\\ 3, & for\ i\equiv 0\ (mod\ 3)\end{cases}$$

$$l(y_i)=2,\ \ for\ 1\le i\le n$$

Hence, *max(l)* = 3 and the labelling provides vertex-weight as follows:

$$w(x_i)=\begin{cases}5, for\ i=n\\ 6, for\ i=1\\ 7, for\ i\equiv 0\ (mod\ 3), n\geq 3\\ 8, for\ i\equiv 2\ (mod\ 3), n\geq 2\\ 9, for\ i\equiv 1\ (mod\ 3), n\geq 4\end{cases}$$

$$w(y_i)=\begin{cases}3, for\ i=1\\ 6, for\ i=n\\ 7, for\ i\equiv 2\ (mod\ 3), n\geq 2\\ 8, for\ i\equiv 1\ (mod\ 3), n\geq 4\\ 9, for\ i\equiv 0\ (mod\ 3), n\geq 3\end{cases}$$

For every $uv\ \in E(TL_n)$, take any $u=x_i$ and $v=x_{i+1}$, $i=1,2,3,\dots n-1$, $w(x_i)\neq w(x_{i+1})$, then $u=y_i$ and

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4 Issue 2, February – 2020, Pages: 1-4

$v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(y_i) \neq w(y_{i+1})$ and $u = x_i$ and $v = y_i$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_i)$, and the last, take $u = x_i$ and $v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_{i+1})$. Based on that, we have the set of vertices weight of the triangular ladder graph TL_n is $W(TL_n) = \{3,5,6,7,8,9\}$, so $|w(V(TL_n))| = 6$ Hence, $\chi lis(TL_n) = 6$.

Case 3: for $n \geq 6, n \equiv 0 (mod\ 3)$

Furthermore, the upper bound for the chromatic number local irregular, we define $l: V(TL_n) \to \{1,2,3\}$ with vertex irregular 3-labelling as follows :

$$l(x_i) = \begin{cases} 1, & for\ i \equiv 1\ (mod\ 3) \\ 2, & for\ i \equiv 2\ (mod\ 3) \\ 3, & for\ i \equiv 0\ (mod\ 3) \end{cases}$$

$$l(y_i) = 3, \quad for\ 1 \leq i \leq n$$

Hence, *max(l)* = 3 and the labelling provides vertex-weight as follows:

$$w(x_i) = \begin{cases} 5, for\ i = n \\ 8, for\ i = 1 \\ 9, for\ i \equiv 0\ (mod\ 3), n \geq 3 \\ 10, for\ i \equiv 2\ (mod\ 3), n \geq 2 \\ 11, for\ i \equiv 1\ (mod\ 3), n \geq 4 \end{cases}$$

$$w(y_i) = \begin{cases} 4, for\ i = 1 \\ 8, for\ i = n \\ 9, for\ i \equiv 2\ (mod\ 3), n \geq 2 \\ 10, for\ i \equiv 1\ (mod\ 3), n \geq 4 \\ 11, for\ i \equiv 0\ (mod\ 3), n \geq 3 \end{cases}$$

For every $uv \in E(TL_n)$, take any $u = x_i$ and $v = x_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(x_{i+1})$, then $u = y_i$ and $v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(y_i) \neq w(y_{i+1})$ and $u = x_i$ and $v = y_i$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_i)$, and the last, take $u = x_i$ and $v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_{i+1})$. Based on that, we have the set of vertices weight of the triangular ladder graph TL_n is $W(TL_n) = \{4,5,8,9,10,11\}$, so $|w(V(TL_n))| = 6$. Hence, $\chi lis(TL_n) = 6$.

Case 4: for $n \geq 6, n \equiv 2 (mod\ 3)$

Furthermore, the upper bound for the chromatic number local irregular, we define $l: V(TL_n) \to \{1,2,3\}$ with vertex irregular 3-labelling as follows :

$$l(x_i) = \begin{cases} 1, & for\ i \equiv 1\ (mod\ 3) \\ 2, & for\ i \equiv 2\ (mod\ 3) \\ 3, & for\ i \equiv 0\ (mod\ 3) \end{cases}$$

$$l(y_i) = 3, \quad for\ 1 \leq i \leq n$$

Hence, *max(l)* = 3 and the labelling provides vertex-weight as follows:

$$w(x_i) = \begin{cases} 4, for\ i = n \\ 8, for\ i = 1 \\ 9, for\ i \equiv 0\ (mod\ 3), n \geq 3 \\ 10, for\ i \equiv 2\ (mod\ 3), n \geq 2 \\ 11, for\ i \equiv 1\ (mod\ 3), n \geq 4 \end{cases}$$

$$w(y_i) = \begin{cases} 4, for\ i = 1 \\ 6, for\ i = n \\ 9, for\ i \equiv 2\ (mod\ 3), n \geq 2 \\ 10, for\ i \equiv 1\ (mod\ 3), n \geq 4 \\ 11, for\ i \equiv 0\ (mod\ 3), n \geq 3 \end{cases}$$

For every $uv \in E(TL_n)$, take any $u = x_i$ and $v = x_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(x_{i+1})$, then $u = y_i$ and $v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(y_i) \neq w(y_{i+1})$ and $u = x_i$ and $v = y_i$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_i)$, and the last, take $u = x_i$ and $v = y_{i+1}$, $i = 1,2,3,\ldots n-1$, $w(x_i) \neq w(y_{i+1})$. Based on that, we have the set of vertices weight of the triangular ladder graph TL_n is $W(TL_n) = \{4,6,8,9,10,11\}$, so $|w(V(TL_n))| = 6$. Hence, $\chi lis(TL_n) = 6$.

Theorem 2.3 Let G be a H graph (H_n). For $n \geq 2$, the chromatic number local irregular is $\chi lis(H_n) = 4$.

Proof. $V(H_{n,}) = \{x_i, y_i : 1 \leq i \leq 2n\} \cup \{z_j : 1 \leq j \leq 2n\}$, and the edge set $E(H_{n,}) = \{x_i x_{i+1}, y_i y_{i+1} : 1 \leq i \leq 2n-1\} \cup \{z_j z_{j+1} : 1 \leq j \leq n\} \cup \{x_i z_j : 1 \leq i = j \leq 2n\} \cup \{y_i z_j : 1 \leq i = j \leq 2n\}$. If every $x \in V(H_n), y \in V(H_n), z \in V(H_n)$ is labelled by 1, so we have $w(x_i) = l(x_{i-1}) + l(x_{i+1}) + l(z_j) = 1 + 1 + 1 = 3$ and $w(x_{i+1}) = l(x_i) + l(x_{i+2}) + l(z_{j+1}) = 1 + 1 + 1 = 3$. It means a contradiction with definition. Since, $x_i x_{i+1} \in E(H_n), w(x_i) \neq w(x_{i+1})$. Hence *max(l)* = 2.

Based on Lemma, the lower bound for the chromatic number $\chi lis(H_{n,}) \geq \chi(H_{n,}) = 2$, so that $\chi lis(H_{n,}) \geq 2$.

Furthermore, the upper bound for the chromatic number local irregular, we define $l: V(H_n) \to \{1,2\}$ with vertex irregular 2-labelling as follows :

$$l(x_i) = \begin{cases} 1, & \text{for } i \text{ odd} \\ 2, & \text{for } i \text{ even} \end{cases}$$

$$l(y_i) = \begin{cases} 1, & \text{for } i \text{ odd} \\ 2, & \text{for } i \text{ even} \end{cases}$$

$$l(z_j) = \begin{cases} 1, & \text{for } j \text{ even} \\ 2, & \text{for } j \text{ odd} \end{cases}$$

Hence, *max(l)* = 2 and the labelling provides vertex-weight as follows:

$$w(x_i) = \begin{cases} 4, & for\ i = 1 \\ 3, & for\ i = even,\ \ 2 \le i \le 2n-1 \\ 6, & for\ i = odd,\ \ 2 \le i \le 2n-1 \\ 2, & for\ i = n \end{cases}$$

$$w(y_i) = \begin{cases} 4, & for\ i = 1 \\ 3, & for\ i = even,\ \ 2 \le i \le n-1 \\ 6, & for\ i = odd,\ \ 2 \le i \le n-1 \\ 2, & for\ i = n \end{cases}$$

$$w(z_j) = \begin{cases} 3, for\ j\ odd, j = 1,2,3, \dots ,2n \\ 6, for\ j\ even, j = 1,2,3, \dots ,2n \end{cases}$$

For every $uv \in E(H_n)$, take any $u = x_i$ and $v = x_{i+1}$, $i = 1,2,3, \dots ,2n$, $w(x_i) \neq w(x_{i+1})$, then $u = y_i$ and $v = y_{i+1}$, $i = 1,2,3, \dots ,2n$, $w(y_i) \neq w(y_{i+1})$ also take $u = z_j$ and $v = z_{j+1}$, $i = 1,2,3, \dots ,2n$, $w(z_j) \neq w(z_{j+1})$. Then $u = x_i$ and $v = z_j$, $1 \le i = j \le 2n$, $w(x_i) \neq w(z_j)$, and the last, take $u = y_i$ and $v = z_j$, $1 \le i = j \le 2n$, $w(y_i) \neq w(z_j)$. Based on that, we have the set of vertices weight of the H graph is $W(H_n) = \{2,3,4,6\}$, so $|w(V(H_n))| = 4$. Hence, $\chi lis(H_n) = 4$.

3. CONCLUSION

In this paper, we have studied local irregularity vertex coloring of ladder graph, triangular ladder graph, and H graph. We have concluded the exact value of the chromatic number local irregular of ladder graph, namely $\chi lis(L_n) = 4$, the chromatic number local irregular of triangular ladder graph, namely for $n = 5$ is $\chi lis(TL_n) = 5$ and for $n \ge 6$ is is $\chi lis(TL_n) = 6$ and the chromatic number local irregular of H graph, namely $\chi lis(H_n) = 4$. Hence the following problem arises naturally.

4. ACKNOWLEDGEMENT

We gratefully acknowledge the support from CGANT University of Jember Indonesian of year 2019.

5. REFERENCES

[1] Gross, J.L., and Zhang, P. (2014). Handbook of graph theory second edition., CRC Press Taylor and Francis Group.

[2] Chartrand G and Lesniak L. (2000). Graphs and digraphs 3rd ed., London: Chapman and Hall.

[3] Kristiana I. A., Dafik, Utoyo M. I., Slamin, Alfarisi R, Agustin I.H., and Venkatachalam M. (2019). Local irregularity vertex coloring of graphs, International Journal of civil Engineering and Technology (IJCIET), vol.10, pp.1606-1616.

[4] Kristiana I. A., Utoyo M. I., Dafik, Agustin I.H., Alfarisi R, and Waluyo E. (2019). On the chromatic number local irregularity of related wheel graph, IOP Conf. Series : Journal of Physics: Conf. Series. 1211. 0120003.