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Intuitionistic Fuzzy Prime Ideal on Q-Algebra

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Abstract: *The aim of this paper is to study the fuzzy prime ideal in Q-algebra and we attempt to generalize the definition of fuzzy prime ideal in bounded Q-algebra and give some useful proposition about fuzzy prime ideal in bounded Q-algebra .Also we study intuitionistic fuzzy prime ideal in Q-algebra and bounded Q-algebra and give some proposition of it . As well as we will study the relationship between them.*

.**Keywords**: Fuzzy, Prime, Ideal, Q-Algebra

1 Introduction

Q-algebra is introduced by H.S.Kim ([1]) in 2001 .The concept of fuzzy set was first initiated by Zadeh ([8]) in 1965 . Since then these ideas have been applied to other algebraic structures such as group , semi group , ring , vector spaces etc. Intuitionistic fuzzy sets have been defined by G.Takeuti and S.Titanti in ([9]) . H.K.Abdullah , M.T.Shadhan ([10]) introduced the concept of prime ideal in Q-algebra. In this paper we presented the definition of the fuzzy prime ideal on Q-algebra and bounded Q-algebra also it is relationship with the level set μ_t ,and studied intersection on the fuzzy prime ideal. We also showed some of its properties of it . Then we expanded after that to study the intuitionistic fuzzy set prime ideal in Q-algebra also bounded Q-algebra , and we presented its definition and some properties that are equivalent to it and its relationship with upper level set $U(\mu,t)$ and lower level set $L(\mu,t)$, as well as its relationship with the $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$.

2 Background

In this section ,we recalled the definition of Q-algebra, bounded Q-algebra, homomorphism , epimorphism, commutative , fuzzy set , fuzzy ideal, Intuitionistic fuzzy set , $\circ A = (\mu_A, \overline{\mu_A})$, $\bullet A = (\overline{\nu_A}, \nu_A)$, , intuitionistic fuzzy ideal, upper -level cut set, lower -level cut set in Q-algebra , and some of the features we need in the paper. Moreover, we have presented and demonstrated some good properties of intersection and union on the fuzzy prime ideal and study intuitionistic fuzzy prime ideal on Q-algebra.

Definition (2.1) [2]
A Q- algebra is a set X with a binary operation $*$ and constant 0 that fulfilled the following axioms:
1. $x * x = 0, \quad \forall x \in X.$
2. $x * 0 = x, \quad \forall x \in X.$
3. $(x * y) * z = (x * z) * y, \quad \forall x, y, z \in X.$

Remark (2.2) [3]
In a Q-algebra X , we can define a binary relation $\leq$ by putting x^* represented relation $\leq$ by which $x \leq y$ if and only if $x * y = 0, \ \forall x, y \in X.$

Definition (2.3) [1]
A Q-algebra $(X, *, 0)$ is called bounded if there is an element $e \in X$ that satisfies $x \leq e$, $\forall x \in X$. Then e is said to be an unit , we denotted e $*$ x by $x^*, for\ each\ x \in X$.

Proposition(2.4)
Let X be a bounded Q-algebra then $x \wedge y \leq x$.

Proof
Let $x, y \in X$,
since $x \wedge y = (x * y) * y$

then $(x \wedge y) * x = \big((x * y) * y\big) * x$
$= \big((x * y) * x\big) * y$
$= \big((x * x) * y\big) * y$
$= (0 * y) * y$
$= 0$

Hence $x \wedge y \leq x$.

Definition(2.5)[7]
A Q-algebra $(X,*,0)$is said to be commutative if it satisfies
$(x * y) * y = (y * x) * x$, $\forall\, x, y \in X$, such that $x \neq 0, y \neq 0$ ($That\ is\ x \wedge y = y \wedge x$).

Proposition (2.6)
Let X be a commutative bounded Q-algebra then $x \wedge y \leq y$.
Proof
it's clear by **(Proposition (2.4))**

Definition (2.7)[3]
Let $(X ,* , 0)$ and $(Y ,\circ , 0')$ be two Q-algebra , A mapping $g : X \rightarrow Y$ is called
1. $Homomorphism\ if\ \ g(x * y) = g(x) \circ g(y)$.
2. $Epimorphism\ if\ \ g\ \ is\ homomorphism\ \ and\ onto$.

Definition (2.8) [8]
Let X be a none empty set . A fuzzy set μ in X is a function : $X \rightarrow [0 , 1]$.

Definition(2.9)[3]
For any $t \in [0 ,1]$ and a fuzzy set μ in a nonempty set of X , the sets :
$U(\mu , t) = \{x : \mu(x) \geq t \}$, it's said to be an upper t-level cut of X .
$L(\mu , t) = \{x , \mu(x) \leq t \}$, it's said to be a lower t-level cut of X .

Definition(2.10)[3]
Let X be a Q-algebra . A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies
1. $\mu(0) \geq \mu(x)$, $\quad \forall x \in X$.
2. $\mu(x) \geq \min\{\mu(x * y) , \mu(y)\}$, $\quad \forall\, x , y , z \in X$.

Proposition(2.11)[3]
A fuzzy subset μ of a Q-algebra X is a fuzzy ideal of X if and only if , for every $t \in [0 ,1]$
$U(\mu , t)$ is either empty or an ideal of X .

Theorem(2.12)[3]
The intersection of any set of fuzzy ideal in Q-algebra X is also a fuzzy ideal .

Proposition (2.13)
Let μ be a fuzzy ideal of Q-algebra $(X ,* , 0)$. If $x \leq y$ then μ (x) $\geq$ μ (y) $\quad \forall\, x \in X$.
Proof:
Since μ is fuzzy ideal then
$\mu(x) \geq \min\{ \mu(x * y), \mu(y)\}, \quad \forall\, x, y \in X.$
$= \min\{ \mu(0), \mu(y)\}$
$= \mu(y)$.

Definition(2.14) [6]
An intuitionistic fuzzy set ($I - F - S$ for short) A in a set X is object having the form
A $= \{< x,\ \mu_A(x),\ \nu_A(x)\ > : x \in X\}$, such that $\mu_A: X \rightarrow [0;\ 1]$ and $\nu_A: X \rightarrow [0;\ 1]$ denoted
the degree of membership (namely $\mu_A(x)$) , and the dagree of non membership (namely $\nu_A(x)$) for any element
$x \in X$ to the set A , and $0 \leq \mu_A(x) + \nu_A(x) \leq 1,\ \forall x \in X$ for the sake
of simplicity, we shall use the notation A $= \{< x,\ \mu_A(x),\ \nu_A(x)\ >\}$

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instead of A $= \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$.

Definition(2.15) [6]

An $I - F - S$ $A = (\mu_A, \nu_A)$ of a none –empty set X . Then

1.∘ $A = \{ (x , \mu_A(x), 1 - \mu_A(x)) \mid x \in X \}$
$= \{ (x , \mu_A(x) , \overline{\mu_A}(x))\}$
$= (\mu_A(x) , \overline{\mu_A}(x)))$

2.• $A = \{ (x , 1 - \nu_A(x) , \nu_A(x)) \mid x \in X \}$
$= \{ (x , \overline{\nu_A}(x) , \nu_A(x))\} =$
$= (\overline{\nu_A}(x), \nu_A(x))$

Definition(2.16)[4]

A intuitionistic fuzzy set $A = \{< x , \mu_A(x) , \nu_A(x) >\}$ in a Q − algebra X is called a intuitionistic fuzzy ideal (in short I − F − ID) if

1. $\mu_A(0) \geq \mu_A(x)$, $\forall x \in X$.
2. $\nu_A(0) \leq \nu_A(x)$, $\forall x \in X$.
3. $\mu_A(x) \geq Min\{ \mu_A(x * y), \mu_A(y)\}$, $\forall x, y \in X$.
4. $\nu_A(x) \leq Max\{\nu_A(x * y), \nu_A(y)\}$, $\forall x, y \in X$.

Proposition(2.17) [4]

Let A be a $I - F - ID$ of a Q-algebra $(X ,* , 0)$. If $x \leq y$ then $\mu_A(x) \geq \mu_A(y)$ *and* $\nu_A(x) \leq \nu_A(y)$ $\forall x ,\in X$.

Proposition(2.18) [4]

An I-F-S $A = (\mu_A, \nu_A)$ is an $I - F - ID$ of X if and only if $\forall s , t \in [0 ,1]$, the set $U(\mu_A , t)$ and $L(\mu_A , s)$ are either empty or ideal of X .

Proposition(2.19)

An $I - F - S$ $A = (\mu_A , \nu_A)$ of a bounded Q-algebra $(X ,* , 0)$ is $I - F - ID$ if and only if μ_A and $\overline{\nu_A}$ are fuzzy ideals of X .

Proof:

⇒

Let A be an $I - F - ID$ of X
then it's clear μ_A is fuzzy ideal of X .
Since $\overline{\nu_A}(x) = 1 - \nu_A(x)$ $\forall x \in X$
then $\overline{\nu_A}(0) = 1 - \nu_A(0) \leq 1 - \nu_A(x) = \overline{\nu_A}(x)$
since $\nu_A(0) \leq \nu_A(x)$,
then $\overline{\nu_A}(x) = 1 - \nu_A(x) \geq \max\{ \nu_A(x * y) , \nu_A(y)\}$
$= \min\{1 - \nu_A(x * y) , 1 - \nu_A(y)\}$
$= \min\{\overline{\nu_A}(x * y) \;\; \overline{\nu_A}(y)\}$

Hence μ_A and $\overline{\nu_A}$ are fuzzy ideals of X .

⇐

Let $\overline{\nu_A}$ and μ_A be fuzzy ideal of X then

1. $\mu_A(0) \geq \mu_A(x)$, *and*
$1 - \nu_A(0) = \overline{\nu_A}(0) \geq 1 - \nu_A(x) = \overline{\nu_A}(x)$
so $\nu_A(0) \leq \nu_A(x)$, $\forall x \in X$.
2. $\mu_A(x) \geq Min\{ \mu_A(x * y), \mu_A(y)\}$, and
$1 - \nu_A(x) = \overline{\nu_A}(x) \geq \min\{ \overline{\nu_A}(x * y) \;\; \overline{\nu_A}(y) \}$
$= \min\{ 1 - \nu_A(x * y) , 1 - \nu_A(y)\}$

$= 1 - \max\{ \nu_A(x * y) , \nu_A(y)\}$

so $\nu_A(x) \leq \max\{ \nu_A(x * y) , \nu_A(y)\}$

Then $A = (\mu_A , \nu_A)$ is an I-F-ID.

Proposition(2.20)

If $A = (\mu_A , \nu_A)$ is an $I - F - S$ a in bounded Q-algebra $(X ,* , 0)$, then $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$ $are\ I - F - ID$ if and only if A is $I - F - ID$.

proof :

⇒ suppose that $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$ $are\ I - F - ID$.

then the fuzzy set μ_A and $\overline{\nu_A}$ are fuzzy ideal of X **(by Proposition (2.19))**

⇐ suppose that $A = (\mu_A , \nu_A)$ is an I-F-ID of X

then $\mu_A = \overline{\overline{\mu_A}}$ and $\overline{\nu_A}$ are fuzzy ideal of X . **(by Proposition (2.19)**

Hence $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$ $are\ I - F - ID$.

Proposition(2.21)

The intersection of any $I - F - ID$ in Q-algebra X is also an $-F - ID$.

Proof :

Let A_i be a family of $I - F - ID$ in Q-algebra X .

then for any $x , y \in X$

1. $\cap\left(\mu_{A_i}(0)\right) = \inf\left(\mu_{A_i}(0)\right) \geq \inf\left(\mu_{A_i}(x)\right) = \cap\left(\mu_{A_i}(x)\right)$
   and
   $\cup\left(\nu_{A_i}(0)\right) = \sup\left(\nu_{A_i}(0)\right) \leq \sup\left(\nu_{A_i}(x)\right) = \cup\left(\nu_{A_i}(x)\right)$
2. $\cap\left(\mu_{A_i}(x)\right) = \inf\left(\mu_{A_i}(x)\right) \geq \inf(\min\{\mu_{A_i}(x * y), \mu_{A_i}(x)\})$
   $= \min\{\inf\left(\mu_{A_i}(x * y)\right) , \inf\left(\mu_{A_i}(x)\right) \}$
   and
   $\cup\left(\nu_{A_i}(x)\right) = \sup\left(\nu_{A_i}(x)\right) \leq \sup(\max\{\nu_{A_i}(x * y), \nu_{A_i}(x)\})$
   $= \max\{\sup\left(\nu_{A_i}(x * y)\right) , \sup\left(\nu_{A_i}(x)\right) \}$

Proposition(2.22)

Let $(X ,* , 0)$ and $(Y ,*' , 0)$ be two bounded Q-algebra . If g is an epimorphosim mapping from $(X ,* , 0)$ into $(Y ,*' , 0)$, then $g^{-1}(A)$ is an $I - F - ID$ of X if $A = (\mu_A , \nu_A)$ is an $I - F - ID$ of Y.

Proof :

Let $x , y \in X$ and $A = (\mu_A , \nu_A)$ be an $I - F - ID$ of Y . Now

1. $\mu_{g^{-1}(A)}(0) = \mu_A\big(g(0)\big) \geq \mu_A\big(g(x)\big) = \mu_{g^{-1}(A)}(x)$ and
   $\nu_{g^{-1}(A)}(0) = \nu_A\big(g(0)\big) \leq \nu_A\big(g(x)\big) = \nu_{g^{-1}(A)}(x).$ $[Since\ A\ is\ an\ I - F - ID\ of\ X]$
2. $\mu_{g^{-1}(A)}(x) = \mu_A\big(g(x)\big) \geq \min\{\mu_A\big(g(x) * g(y)\big) , \mu_A\big(g(y)\big) \}$
   $= \min\{\mu_A\big(g(x * y)\big) , \mu_A\big(g(y)\big) \}$
   $= \min\{\mu_{g^{-1}(A)}(x * y) , \mu_{g^{-1}(A)}(y) \}$
3. $\nu_{g^{-1}(A)}(x) = \nu_A\big(g(x)\big) \leq \max\{\nu_A\big(g(x) * g(y)\big) , \nu_A\big(g(y)\big) \}$
   $= \max\{\nu_A\big(g(x * y)\big) , \nu_A\big(g(y)\big) \}$
   $= \max\{\nu_{g^{-1}(A)}(x * y) , \nu_{g^{-1}(A)}(y) \}$.

Hence $g^{-1}(A)$ is an $I - F - ID$ of X

3 $fuzzy\ prime\ ideal$

In this section we gave the definition of the fuzzy prime ideal with an example that achieves the definition, and we also showed that the intersection is achieved on the fuzzy prime ideal , and we show that μ is a fuzzy prime ideal if and only if μ_t is prime ideal.

Definition(3.1)
A fuzzy ideal μ in a Q-algebra X is said to be fuzzy prime ideal of X (in short $F-P-ID$) if it satisfies
$\mu(x \wedge y) \leq \max\{\mu(x)\ ,\mu(y)\}$, $\quad \forall\, x , y \in X.$

Example(3.2)
Let $X = \{0 , a , b , c\}$ and a binary operation $*$ is defined by

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

then $x \wedge y\ \ is$

∧	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c

and $\mu(x) = \begin{cases} 0.8 & if \quad x = 0\,,\ a \\ 0.3 & if \quad x = \ b, c \end{cases}$

it's clear μ is $F-P-ID$.

Proposition (3.3)
Every $F-P-ID$ in bounded Q-algebra is constant .

Proof :
Let $x , y \in X$ then
$\mu(x \wedge x) \leq \max\{\mu(x)\ \ ,\mu(x)\}$,
$\mu(0) \leq\ \mu(x)$
and since $\mu(0) \geq\ \mu(x)$ (**by Definition(2.10)**)

hence $\mu(0) =\ \mu(x)$.
In general if $x \wedge y\ = 0$
then $\mu(0) = \mu(x) = \mu(y) \quad \forall\, x , y \in X$
thus the fuzzy prime is constant

We will exclude from the definition of the fuzzy prime ideal the case of $x \wedge y = 0$, so that we do not have a problem, as shown in the above Proposition and the following example illustrates that.

Example (3.4)
Let $X = \{0 , a , b\}$ and let $*$ be a binary operation on X which is defined by :

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$*$	**0**	$\boldsymbol{a}$	$\boldsymbol{b}$
0	**0**	**0**	**0**
$\boldsymbol{a}$	$\boldsymbol{a}$	**0**	**0**
$\boldsymbol{b}$	$\boldsymbol{b}$	$\boldsymbol{a}$	**0**

$$\mu(x) = \begin{cases} 0.8 & : \quad if \ x = 0 \\ 0.4 & : \quad if \ x = \text{a , b} \end{cases}$$

Then μ is $F-P-ID$ sense
$\mu(a \wedge 0) = \mu(a) = 0.4 \leq \max\{\mu(a), \mu(0)\} = 0.8$
$\mu(b \wedge 0) = \mu(b) = 0.4 \leq \max\{\mu(b), \mu(0)\} = 0.8$

Proposition(3.5)
Let $(X, *, 0)$ be a bounded commutative Q-algebra . Then μ is a $F-P-ID$ if and only if
$\mu(x \wedge y) = \mu(x)$ or $\mu(x \wedge y) = \mu(y)$, $\forall x, y \in X$.
Proof :

$\Rightarrow$ Let μ be a $F-P-ID$ and let $x, y, \in X$
since $x \wedge y \leq x$, $x \wedge y \leq y$ (by **Proposition (2.4)** and **Proposition (2.6))**
then $\mu(x \wedge y) \geq \mu(x)$ and $\mu(x \wedge y) \geq \mu(y)$ (**by Proposition(2.13)**)
since μ is a $F-P-ID$ then
$\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$ means
$\mu(x \wedge y) \leq \mu(x)$ or $\mu(x \wedge y) \leq \mu(y)$.
Hence $\mu(x \wedge y) = \mu(x)$ or $\mu(x \wedge y) = \mu(y)$.

$\Leftarrow$ Let $\mu(x \wedge y) = \mu(x)$ or $\mu(x \wedge y) = \mu(y)$
we must prove that μ is a $F-P-ID$
Let $x, y \in X$ $s.t$ $x \wedge y \neq 0$
since $\mu(x \wedge y) = \mu(x)$ or $\mu(x \wedge y) = \mu(y)$
then it is clear $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$.

Proposition(3.6)
Let μ be a fuzzy subset of bounded Q-algebra $(X, *, 0)$. Then μ is a $F-P-ID$ in X if and only if $U(\mu, t)$ is a prime ideal in X , $\forall t \in [0,1]$ s.t $U(\mu, t) \neq \phi$.

Proof :
$\Rightarrow$ Let μ be a $F-P-ID$ in X and $t \in [0,1]$ such that $U(\mu, t) \neq \phi$
Then $U(\mu, t) \neq$ is an ideal (***by Proposition*(2.11)**)
$Let\ x \wedge y \in U(\mu, t)\ s.t\ x \wedge y \neq 0 \quad then\ \ \mu(x \wedge y) \geq t$,
then $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$, $then$
$t \leq \max\{\mu(x), \mu(y)\}$, so
$t \leq \mu(x)$ or $t \leq \mu(y)$
thus $x \in U(\mu, t)$ or $y \in U(\mu, t)$
hence $U(\mu, t)$ is prime ideal of X

$\Leftarrow$ Let $U(\mu, t)$ be a prime ideal , $\forall t \in [0,1]$ s.t $U(\mu, t) \neq \phi$.
since μ is a fuzzy ideal **(by Proposition(2.11))**
Let $x, y \in X, s.t\ x \wedge y \neq 0$, $and\ \mu(x \wedge y) = t$
thus $x \wedge y \in U(\mu, t)$. Since $U(\mu, t)$ is a prime ideal of X .
then either $x \in U(\mu, t)$ or $y \in U(\mu, t)$
So $\max\{\mu(x), \mu(y)\} \geq t = \mu(x \wedge y)$
Hence μ is a $-P-ID$.

Proposition(3.7)
Let P be a non-empty subset of bounded Q-algebra and $n \in [0,1]$. If μ is a fuzzy set in X defined by **:**

$$\mu(x) = \begin{cases} 1 & : \quad x \in P \\ n & : \quad \text{otherwise} \end{cases}$$

Then P is a prime ideal if and only if μ is a F-P-ID of X .

Proof :

$\Rightarrow$ Let P is a prime ideal. Then
1. $0 \in P$, so $\mu(0) = 1 \geq \mu(x)$, $\forall x \in X$.

2. $Let\ x, y \in X, then\ we\ have\ two\ cases$
 a) $If\ x \in P$, $then\ \mu(x) = 1 \geq \min\{\mu(x * y), \mu(y)\}$
 b) $If\ x \notin P$, $then$
 $x * y \notin P$, $\forall y \in P$ $(since\ P\ is\ an\ ideal)$,
so $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$.
 $Thus\ \ \mu\ is\ a\ fuzzy\ ideal\ of\ X$.
3. $Let\ x, y \in X, s.t\ x \wedge y \neq 0 . If\ x \wedge y \notin P$
 $then\ \mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$.
 $And\ if\ x \wedge y \in P\ \ then$
 $x \in P\ \ or\ \ y \in P$ $(since\ P\ is\ an\ prime\ ideal\ of\ X)$
 so $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$

Hence μ be a $F - P - ID$ of X .

$\Leftarrow$ Let μ is a $F - P - ID$ then

1. $since\ \mu(0) \geq \mu(x)\ \ \forall x \in X$
 $then\ \mu(0) = 1$, $thus\ 0 \in P$

2. $Let\ x * y, y \in P\ then$
 $\mu(x * y) = 1\ \ and\ \ \mu(y) = 1$
 $since\ \mu(x) \geq \min\{\mu(x * y), \mu(y)\} = 1$.
 $Thus\ x \in P$
 $hence\ P\ is\ an\ ideal\ of\ X$.
3. $Let\ x, y \in X\ \ such\ that\ x \wedge y \in P$, $x \wedge y \neq 0$.
 $Then\ \mu(x \wedge y) = 1 \leq \max\{\mu(x), \mu(y)\} = \mu(x)\ \ or\ \ \mu(y)$
 (since μ is a F-P-ID of X)
 so $x \in P$ or $y \in P$.
 hence P is a prime ideal of X .

Proposition(3.8)
In a bounded Q-algebra $(X, *, 0)$, if $\{\mu_i , i \in \Delta\}$ is an arbitrary family of $F - P - ID$, then $\cap \mu_i$ is a $F - P - ID$ of X .

Proof :
Since μ_i , $i \in \Delta$ is a F-P-ID ,
then μ_i is a fuzzy ideal of X .
So $\cap \mu_i$ is a fuzzy ideal of X $(\boldsymbol{by\ Theorem(2.12)})$.
Now $\mu_i(x \wedge y) \geq \max\{\mu_i(x), \mu_i(y)\}$,
$\forall x, y \in X\ \ s.t$, $x \wedge y \neq 0$, $then$
$\inf \mu_i(x \wedge y) \leq \inf\{\max\{\mu_i(x), \mu_i(y)\}\}$

Then $\cap \mu_i$ is a $F-P-ID$ of X .

4 intuitionistic fuzzy prime ideal

In this section we gave the definition of the an intuitionistic fuzzy prime ideal with an example that achieves the definition, and we also showed that the intersection is achieved on the an intuitionistic fuzzy prime ideal , and above all , we show that $A=(\mu_A, \nu_A)$ is an intuitionistic fuzzy prime ideal if and only if $U(\mu_A, t)$ and $L(\mu_A, s)$ is prime ideal. And also A is intuitionistic fuzzy set if and only if the $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$, are intuitionistic fuzzy prime ideal.

Definition(4.1)
An intuitionistic fuzzy ideal $A=(\mu_A, \nu_A)$ in Q-algebra is said to be intuitionistic fuzzy prime ideal of X (in short $I-F-P-ID$) if it satisfies

1. $\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\} \quad \forall x, y \in X.$
2. $\nu_A(x \wedge y) \geq \min\{\nu_A(x), \nu_A(y)\} \quad \forall x, y \in X.$

Example(4.2)
In example (3.2) we define $A=(\mu_A, \nu_A)$ by

$$\mu_A(x) = \begin{cases} 0.8 & if \quad x = 0, a \\ 0.3 & if \quad x = b, c \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0.2 & if \quad x = 0, a \\ 0.7 & if \quad x = b, c \end{cases}$$

then it's clear $A=(\mu_A, \nu_A)$ is $I-F-P-ID$.

Proposition (4.3)
Every $I-F-P-ID$ in bounded Q-algebra is constant .

Proof :
Let $x, y \in X$ then
$\mu_A(x \wedge x) \leq \max\{\mu_A(x), \mu_A(x)\}$,
$\mu_A(0) \leq \mu_A(x)$
and since $\mu_A(0) \geq \mu_A(x)$ (**by Definition(2.16)**)
hence $\mu_A(0) = \mu_A(x)$
$\nu_A(x \wedge x) \geq \min\{\nu_A(x), \nu_A(x)\}$,
$\nu_A(0) \geq \nu_A(x)$
and since $\nu_A(0) \leq \nu_A(x)$ (**by Definition(2.16)**)
. hence $\nu_A(0) = \nu_A(x)$
In general if $x \wedge y = 0$
then $\mu(0) = \mu(x) = \mu(y) \quad \forall x, y \in X$
thus the fuzzy prime is constant.

We will exclude from the definition of the intuitionistic fuzzy prime ideal the case of $x \wedge y = 0$, so that we do not have a problem, as shown in the above Proposition and the following example illustrates that.

Example(4.4)
In example (3.4) we define $A=(\mu_A, \nu_A)$ by :

$$\mu_A(x) = \begin{cases} 0.8 : & if \ x = 0 \\ 0.4 : & if \ x = a, b \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0.2 : & if \ x = 0 \\ 0.6 : & if \ x = a, b \end{cases}$$

Then A is $I-F-P-ID$.

Proposition(4.5)

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Let $(X, *, 0)$ be a bounded commutative Q-algebra . Then $A = (\mu_A, \nu_A)$ is a $I - F - P - ID$ if and only if
$\mu_A(x \wedge y) = \mu_A(x)$ or $\mu_A(x \wedge y) = \mu_A(y)$,
$and\ \nu_A(x \wedge y) = \nu_A(x)$ or $\nu_A(x \wedge y) = \nu_A(y)$ $\forall x, y \in X$.

Proof :

$\Rightarrow$ Let A be a $I - F - P - ID$ and let $x, y, \in X$
since $x \wedge y \leq x$, $x \wedge y \leq y$ (by **Proposition (2.4)** and **Proposition (2.6))**
then $\mu_A(x \wedge y) \geq \mu_A(x)$ and $\mu_A(x \wedge y) \geq \mu_A(y)$ (**by Proposition(2.17)**)
also $\nu_A(x \wedge y) \leq \nu_A(x)$ and $\nu_A(x \wedge y) \leq \nu_A(y)$
since A is a I- F-P-ID then
$\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$ means
$\mu_A(x \wedge y) \leq \mu_A(x)$ or $\mu_A(x \wedge y) \leq \mu_A(y)$.
Hence $\mu_A(x \wedge y) = \mu_A(x)$ or $\mu_A(x \wedge y) = \mu_A(y)$
And
$\nu_A(x \wedge y) \geq \min\{\nu_A(x), \nu_A(y)\}$ means
$\nu_A(x \wedge y) \geq \nu_A(x)$ or $\nu_A(x \wedge y) \geq \nu_A(y)$.
Hence $\nu_A(x \wedge y) = \nu_A(x)$ or $\nu_A(x \wedge y) = \nu_A(y)$

$\Leftarrow$ suppose that $\mu_A(x \wedge y) = \mu_A(x)$ or $\mu_A(x \wedge y) = \mu_A(y)$,
$and\ \nu_A(x \wedge y) = \nu_A(x)$ or $\nu_A(x \wedge y) = \nu_A(y)$
we must prove that A is a $I - F - P - ID$
Let $x, y \in X\ s.t\ x \wedge y \neq 0$
since $\mu_A(x \wedge y) = \mu_A(x)$ or $\mu_A(x \wedge y) = \mu_A(y)$
and and $\nu_A(x \wedge y) = \nu_A(x)$ or $\nu_A(x \wedge y) = \nu_A(y)$
then it is clear $\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$
and
$\nu_A(x \wedge y) \leq \min\{\nu_A(x), \nu_A(y)\}$.
Hence A is a $I - F - P - ID$.

Proposition(4.6)
Let P be a non-empty subset of bounded Q-algebra and $n \in [0,1]$. If $A = (\mu_A, \nu_A)$ is an $I - F - S$ in X defined by **:**

$$\mu_A(x) = \begin{cases} 1 & : \quad x \in P \\ n & : \quad \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0 & : \quad x \in P \\ n & : \quad \text{otherwise} \end{cases}$$

Then P is a prime ideal if and only if A is an $I - F - P - ID$ of X .

Proof :

$\Rightarrow$ Let P be a prime ideal then
1. $\mu_A(0) = 1 \geq \mu_A(x)$ and
$\nu_A(0) = 0 \leq \nu_A(x)$ $\forall x \in X$ $(since\ 0 \in P)$.

2. $Let\ x, y \in X, then\ we\ have\ two\ cases$
   a) $If\ x \in P$ $,then$ $\mu_A(x) = 1 \geq \min\{\mu_A(x * y), \mu_A(y)\}$
   $and\ \nu_A(x) = 0 \leq \max\{\nu_A(x * y), \nu_A(y)\}$.
   b) $If\ x \notin P$, $then$
   $x * y \notin P$, $\forall y \in P$ $(since\ P\ is\ an\ ideal)$.
   $Thus\ \ A\ is\ an\ I - F - ID\ of\ X$.
3. $Let\ x, y \in X\ s.t\ x \wedge y \neq 0$ $,if\ x \wedge y \notin P$
   $then\ \mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$.
$and\ \nu_A(x \wedge y) \geq \min\{\nu_A(x), \nu_A(y)\}$.

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$And\ if\ x \wedge y\ \in P\quad then$
$x \in P\ \ or\ \ y \in\ P\quad (since\ P\ is\ a\ prime\ \ ideal\ of\ X\ )$
then $\mu_A(x \wedge y\ ) \leq \max\{\mu_A(x)\ \ ,\mu_A(y)\ \}$
$and\ \nu_A(x \wedge y\ ) \geq \min\{\nu_A(x)\ \ ,\nu_A(y)$

Hence A is an $I - \ F - P - ID$ of X .

$\Leftarrow$ Now we prove that P is prime ideal
Let A be a $I - F - P - ID$

1. $since\ \ \mu_A(0\ )\ \geq \mu_A\ (x)$
$and\ \ \ \nu_A(0\ ) \leq \nu_A\ (x)\qquad \forall\ x \in X$
$then\ \mu_A(0) = 1$
$and\ \nu_A(0) = 0\ \ ,thus\ \ 0 \in P$

2. $Let\ x * y\ ,y\ \in\ \ P\ \ then$
$\mu_A(x * y) = 1\ \ and\ \ \mu_A(y) = 1$
$since\ \mu_A(x) \geq \min\{\mu_A(x * y\ ),\mu_A(y)\ \} = 1$.
so $\mu_A(x) = 1$
$Thus\ x \in P$
$hence\ \ P\ \ is\ \ an\ \ ideal\ of\ \ X$.
3. $Let\ \ \ x\ ,y\ \in X\quad such\ that\ x \wedge y \neq 0\ ,\ \ x \wedge y\ \in P$.
$Then\ \ \mu_A(x \wedge y\ ) = 1\ \ \leq \max\{\mu_A(x)\ \ ,\mu_A(y)\}\ \ \ = \mu_A(x)\quad or\ \ \mu_A(y)$
(since A is an $I - \ F - P - ID$ of X)
so $x \in P\quad or\quad y \in\ \ P$.
hence P is a prime ideal of X .

Proposition(4.7)
In a bounded Q-algebra $(X\ ,*\ ,0)$, if $\{\ A_{\ i}\ \ ,i \in\ \Delta\ \}$ is an arbitrary family of $I - \ F - P - ID$, then $\cap\ A_{\ i}$ is a $I - \ F - P - ID$ of X .

Proof :
Since $A_{\ i}\ \ ,i \in\ \Delta$ is a $- F - P - ID$,
then $A_{\ i}$ is an $I - F - ID\ \ of\ X$.
So $\cap\ A_{\ i}$ is an $I - F - ID\ \ of\ X$ of X (**by Proposition(2.21)**).
Now $\mu_{\ A_i}\ (x \wedge y\ ) \geq \max\{\mu_{\ A_i}(x)\ \ ,\ \mu_{\ A_i}\ (y)\ \}\qquad \forall x\ ,y\ \in X\ \ s.t\ \ x \wedge y \neq 0$,
thus $\inf\ \mu_{\ A_i}\ (x \wedge y\ ) \geq \inf\{\max\{\mu_{\ A_i}(x)\ \ ,\mu_{\ A_i}(y)\ \}$,
so $\cap\ \mu_{\ A_i}\ (x \wedge y\ )\ \geq \max\{\inf\left(\mu_{\ A_i}(x)\right)\ ,\inf\left(\mu_{\ A_i}(y)\right)\}$
$= \max\{\ \cap\ \mu_{\ A_i}(x)\ \ ,\ \ \cap\ \mu_{\ A_i}(y)\}$
And
$\nu_{\ A_i}\ (x \wedge y\ ) \leq \min\{\nu_{\ A_i}(x)\ \ ,\ \nu_{\ A_i}\ (y)\ \}\qquad \forall x\ ,y\ \in X\ \ s.t\ \ x \wedge y \neq 0$, ,
thus $\sup\ \nu_{\ A_i}\ (x \wedge y\ ) \leq \sup\{\min\{\nu_{\ A_i}(x)\ \ ,\nu_{\ A_i}(y)\ \}$,
so $\cup\ \nu_{\ A_i}\ (x \wedge y\ )\ \leq \min\{\sup\left(\nu_{\ A_i}(x)\right)\ ,\sup\left(\nu_{\ A_i}(y)\right)\}$
$= \min\{\cup\ \nu_{\ A_i}(x)\ \ ,\cup\ \nu_{\ A_i}(y)\}$.

Thus $\cap\ A_{\ i}$ is a $I - \ F - P - ID$ of X .

Proposition(4.8)
An I-F-S $A = (\mu_A\ ,\nu_A)$ of a bounded Q-algebra $(X\ ,*\ ,0\ )$ is $I - F - P - ID$ if and only if $\mu_A\quad and\quad \overline{\nu_{\ A}}$ are $F - P - ID$.

Proof :
$\Rightarrow$ Let $A\ = (\mu_A\ ,\nu_A)$ be an $I - F - P - ID$ of X .
Then A is $I - F - ID$, so by (**by Propostion (2.19)**)
$\mu_A\quad and\quad \overline{\nu_A}$ are fuzzy ideal of X .

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clearly μ_A is a $F-P-ID$ of X .
Now , let $x, y \in X \quad s.t \; x \wedge y \neq 0$
we have $\overline{\nu_A}(x \wedge y) = 1-\nu_A(x \wedge y) \leq 1-\min\{\nu_A(x), \nu_A(y)\}$
$= \max\{1-\nu_A(x), 1-\nu_A(y)\}$
$= \max\{\overline{\nu_A}(x), \overline{\nu_A}(y)\}$
Hence $\overline{\nu_A}$ is a F-P-ID of X .

$\Leftarrow$ suppose that μ_A *and* $\overline{\nu_A}$ be a fuzzy prime ideal of X ,
so μ_A *and* $\overline{\nu_A}$ are fuzzy ideal of X .
then $A=(\mu_A, \nu_A)$ is an I-F-ID ,
$\mu_A(x \wedge y) \geq \max\{\mu_A(x), \mu_A(y)\}$ and
$1-\nu_A(x \wedge y) = \overline{\nu_A}(x \wedge y) \leq \max\{\overline{\nu_A}(x), \overline{\nu_A}(y)\}$
$= \max\{1-\nu_A(x), 1-\nu_A(y)\}$
$= 1-\min\{\nu_A(x), \nu_A(y)\}$.

So $\nu_A(x \wedge y) \geq \min\{\nu_A(x), \nu_A(y)\}$
Thus $A=(\mu_A, \nu_A)$ is an $I-F-P-ID$ of X .

Proposition(4.9)
Let $A=(\mu_A, \nu_A)$ be an I-F-S in a bounded Q-algebra $(X, *, 0)$ then $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$, are $I-F-P-ID$ if and only if A is a $I-F-P-ID$.

Proof :
$\Rightarrow$ clear by Proposition (4.7)
$\Leftarrow$ Since $A=(\mu_A, \nu_A)$ is an $I-F-P-ID$

we get $\circ A = (\mu_A, \overline{\mu_A})$ and $\bullet A = (\overline{\nu_A}, \nu_A)$ are $I-F-ID$ by $\mathbf{Proposition(2.20)}$,
Now , since $1-\mu_A(x \wedge y) \geq 1-\max\{\mu_A(x), \mu_A(y)\}$
$\geq \min\{1-\mu_A(x), 1-\mu_A(y)\}$
$= \min\{\overline{\mu_A}(x), \overline{\mu_A}(y)\} \quad \forall x, y \in X$.

Hence $\circ A = (\mu_A, \overline{\mu_A})$ is an $I-F-P-ID$ of X .
Also
$\overline{\nu_A}(x \wedge y) = 1-\nu_A(x \wedge y) \leq 1-\min\{\nu_A(x), \nu_A(y)\}$
$= \max\{1-\nu_A(x), 1-\nu_A(y)\}$
$= \max\{\overline{\nu_A}(x), \overline{\nu_A}(y)\}$

Hence $\bullet A = (\overline{\nu_A}, \nu_A)$ is an $I-F-P-ID$ of X .

Proposition(4.10)
An $I-F-S \;\; A=(\mu_A, \nu_A)$ of bonded Q-algebra $(X, *, 0)$ is an $I-F-P-ID$ if and only if the sets $U(\mu_A, t)$ *and* $L(\mu_A, s)$, $\forall t, s \in [0,1]$ are empty sets or prime ideal of X .

Proof :
$\Rightarrow$ suppose that $A=(\mu_A, \nu_A)$ be an $I-F-P-ID$ of X ,
and let $s, t \in [0,1] \quad s.t \quad U(\mu_A, t) \neq \phi \;, L(\mu_A, s) \neq \phi$
then $U(\mu_A, t), L(\mu_A, s)$ *are ideal* $\forall t, s \in [0,1]$ $\left(by\, \boldsymbol{Theorem}\ (\mathbf{2.18})\right)$
Let $x, y \in X$ *such that* $x \wedge y \in U(\mu_A, t)$.
So $\mu_A(x \wedge y) \geq t$.
Since $\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$,
then $\mu_A(x) \geq t$ *and* $\mu_A(y) \geq t$.
So $x \in U(\mu_A, t)$ *or* $y \in U(\mu_A, t)$.
Hence $U(\mu_A, t)$ is a prime ideal of X .

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Similarly , if $x , y \in X$, $such\ that\ x \wedge y \in L(\nu_A , s)$,
$so\ \nu_A(x \wedge y) \leq s$
Since $\nu_A(x \wedge y) \geq \min\{\nu_A(x) , \nu_A(y) \}$
so $\nu_A(x) \leq s$ and $\nu_A(y) \leq s$ then
$then\ x \in L(\nu_A , s)$ or $y \in L(\nu_A , s)$.
Hence $L(\nu_A , s)$ is a prime ideal .

$\Leftarrow$ suppose that $L(\nu_A , s)$ and $U(\mu_A , t)$ are prime ideal of X , $\forall\ t , s \in [0,1]$,
such that $U(\mu_A , t) \neq \phi \neq L(\nu_A , s)$,
then $U(\mu_A , t)$ and $L(\nu_A , s)$ are ideals so by $\textbf{Proposition}(\mathbf{2.18})$
$A = (\mu_A , \nu_A)$ is an $I - F - ID$ of X .
Let $A = (\mu_A , \nu_A)$ not be an $-F - P - ID$.
Then there exist $x , y \in X$

such that $\mu_A (x \wedge y) > \max\{\mu_A(x) , \mu_A(y)\}$
$\nu_A(x \wedge y) < \min\{\nu_A(x) , \nu_A(y)\}$.
If we put $= \mu_A(x \wedge y)$, so $x \wedge y \in L(\nu_A , s)$,
but , $y \neq L(\nu_A , s)$, Which is contradiction .
Hence $A = \mu_A , \nu_A)$ is an $I - F - P - ID$ of .

Proposition(4.11)
Let g be an epimorphism mapping from $(X ,* , 0)$ $into$ $(Y ,*' , 0')$, and $A = (\mu_A , \nu_A)$ be an $I - F - S$ of Y such that $g^{-1}(A) = < \mu_{g^{-1}(A)} , \nu_{g^{-1}(A)} >$ $is\ an\ I - F - P - ID$ $of\ X$. Then A is $an\ I - F - P - ID$ of Y .

Proof :
$\forall c , d \in Y , \exists x , y \in X$, $such\ that\ g(x) = c .\ \ g(y) = d$, $then$

$$\begin{aligned}\mu_A(c \wedge d) &= \mu_A\big(g(x) \wedge g(y)\big)\\ &= \mu_A\big(g(x \wedge y)\big)\\ &= \mu_{g^{-1}(A)}(x \wedge y)\\ &\leq \max\{\mu_{g^{-1}(A)}(x) , \mu_{g^{-1}(A)}(y) \}\\ &= \max\{ \mu_A(c) , \mu_A(d)\}\end{aligned}$$

And

$$\begin{aligned}\nu_A(c \wedge d) &= \nu_A\big(g(x) \wedge g(y)\big)\\ &= \nu_A\big(g(x \wedge y)\big)\\ &= \nu_{g^{-1}(A)}(x \wedge y)\\ &\geq \min\{\nu_{g^{-1}(A)}(x) , \nu_{g^{-1}(A)}(y) \}\\ &= \min\{ \nu_A(c) , \nu_A(d)\}\end{aligned}$$

Also A is an $I - F - ID\ of\ Y\ (by$ $\textbf{Proposition}(\mathbf{2.22})$
Hence A is $an\ I - F - P - ID$ of Y

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