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Semi-Stability of Fixed Points

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Abstract: *In this paper, we introduced new types of stability:semi-stability, semi-c-stability, semi-ic-stability and we discussed the relationships among them and the among them and other types of stability which are stability, c-stability and ic-stability. We discussed the semi-stability of fixed point with respect to different topologies.*

Keywords: Semi-open ,Fixed point , Semi- stable fixed point , Orbit , and dynamical system.

1 Introduction

Dynamical systems theory studies mathematical structures that are abstractions of the most common scientific models of deterministic evolution. The two main elements in the theory are a space $\mathbb{X}$ that describes the possible states of the system (phase space or state space) and a rule that prescribes how the states evolve. The mathematical expression of this evolution rule is the action of a group or semi-group on the space. The group is often thought of as time, and dynamical systems theory is usually restricted to the cases where the semi- group is the non-negative integers, the integers, or the real numbers. In the first two cases one has discrete steps of evolution in time (discrete dynamical system), and in the latter, continuous evolution(continuous dynamical system)[11]. There are many different kind of stability depend on the phase space Lyapunov stability, Lagrange stability, Hurwitz stability, and structural stability or connective stability [12], c-stability, ic- stability [1]. In this paper, we introduced certain types of stability depend on the semi-open sets sets.

In this paper, $SO(\mathbb{X}), R, N, \overline{U}^{\circ}, \overline{U^{\circ}}$, $N_{\circ}$ and $D.D.S.$, will denote the family of semi-open sets, the set of real numbers, the set of natural numbers, the interior of the closure of U, the closure of the interior of U, $N_{\circ} = \{0,1,2,3,\dots\}$ and discrete dynamical system, respectively. For any non-empty set $\mathbb{X}$, we denote by $\tau_u, \tau_d, \tau_{ind}$ and τ_c, the usual topology on R, the discrete topology, the indiscrete topology and the cofinite topology respectively. Finally, we denote by A^c and $O(x)$, the complement of the set$A$ and the orbit of x.

2 Preliminaries

Definition 2.1 .*[7] A $D.D.S.$ consists of a phase space $\mathbb{X}$ and iterates f^s, where s belong to N of a map $f:\mathbb{X} \to \mathbb{X}$, the nth iterate of f is the s-fold composition $f^s = f \circ f \circ \dots \circ f$, we define f^0 to be the identity map. If f satisfy the invertible properties then $f^{-s} = f^{-1} \circ f^{-1} \circ \dots \circ f^{-1}$ (n-times). Since $f^{s+m} = f^s \circ f^m$, these iterates form a group if f is invertible, and semi group otherwise.*

Definition 2.2 .*[2] Let $(\mathbb{X},\tau)$ be a topological space, and $f:\mathbb{X} \longrightarrow \mathbb{X}$ be a continuous function, A point $x \in \mathbb{X}$ is said to be a fixed point of f if $f(x) = x$.*

Definition 2.3 *[2] let $(\mathbb{X},\tau)$ be a topological space, and $f:\mathbb{X} \longrightarrow \mathbb{X}$ be a continuous function. for all $x \in \mathbb{X}$, the orbit of x under f is the set $\{x, f(x), f^2(x), \dots, f^n(x), \dots\}$, and it is denoted by $O(x)$.*

Definition 2.4 .*[3] A subset A of a topological space $\mathbb{X}$ is called semi-open (S-O), if and only if there exists an open set U such that $U \subseteq A \subseteq \overline{U}$ and the set of all (S-O) sets in $\mathbb{X}$ denoted by $SO(\mathbb{X})$. The complement of (S-O) set in*

$\mathbb{X}$ *is called a semi-closed set. An equivalent definition of (S-O) sets is that a set A is (S-O) if and only if* $A \subseteq \overline{A^{\circ}}$, *[4].*

lemma 1 *[4] The concept of (S-O) sets and open sets in* τ_d, τ_{ind} *and* τ_c *are the same.*

Theorem 2.1 *[3] Let* $\mathbb{X}$ *be a topological space. If* A *is a (S-O) set in* $\mathbb{X}$, *Then* $A = U \cup B$, *where* U *is an open set in* $\mathbb{X}$, B *is an nowhere dense set in* $\mathbb{X}$, *and* $U \cap B = \phi$.

Theorem 2.2 *[4] The arbitrary union of (S-O) sets is also (S-O) set.*

Definition 2.5 *[2] Let* $(\mathbb{X}, \tau)$ *be a topological space,* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *be a continuous function, and* x_0 *be a fixed point of* f. x_0 *is called stable if for any open set* U *containing* x_0, *there exists an open set* $V \subseteq U$ *containing* x_0 *such that,* $O(x) \subseteq U, \forall x \in V$.

Otherwise, x_0 is called unstable fixed point.

Theorem 2.3 *[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* B_τ *is a basis for* τ, $f: \mathbb{X} \to \mathbb{X}$ *be a continuous function, and* x_0 *be a fixed point of* f. *If* x_0 *is stable point with respect to* B_τ, *then* x_0 *is stable point with respect to* τ.

Definition 2.6 .*[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *be a continuous function. A fixed point* x_0 *of* f *is called c-stable if for any open set* U *containing* x_0, *there exists an open set* $V \subseteq U$ *containing* x_0, *such that* $O(x) \subseteq \overline{U}, \forall x \in V$.

Otherwise, x_0 is called not c-stable fixed point.

Theorem 2.4 *[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* B_τ *is a basis for* τ, $f: \mathbb{X} \to \mathbb{X}$ *be a continuous function, and* x_0 *be a fixed point of* f. *If* x_0 *is c-stable point with respect to* B_τ, *then* x_0 *is c-stable point with respect to* τ.

Theorem 2.5 *[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *be a continuous function and* x_0 *be a fixed point of* f. *If* x_0 *is stable, then it is c-stable.*

Definition 2.7 *Let* $(\mathbb{X}, \tau)$ *be a topological space,*$f: \mathbb{X} \longrightarrow \mathbb{X}$ *be a continuous function. A fixed point* x_0 *of* f *is called ic-stable if for any open set* U *containing* x_0, *there exists an open set* $V \subseteq U$ *containing* x_0, *such that,* $O(x) \subseteq \overline{U}^{\circ}$, *for all* $x \in V$.

Otherwise, x_0 is called not ic-stable fixed point.

Theorem 2.6 *[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* B_τ *is a basis for* τ, $f: \mathbb{X} \to \mathbb{X}$ *be a continuous function, and* x_0 *be a fixed point of* f. *If* x_0 *is ic-stable point with respect to* B_τ, *then* x_0 *is ic-stable point with respect to* τ.

Theorem 2.7 *[1] Let* $(\mathbb{X}, \tau)$ *be a topological space,* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *be a continuous function and* x_0 *be a fixed point of* f. *If*

x_0 is stable, then it is ic-stable.
x_0 is ic-stable, then it is c-stable.

3 Main Results

Definition 3.1 : *Let* $(\mathbb{X}, \tau)$ *be a topological space, in a* $D.D.S.$ $\{f^n\}_{n \in N_\circ}$ *and let* x_0 *be a fixed point of* f. *We say that* x_0 *is semi-stable if for any (S-O) set* U *containing* x_0, *there exists a (S-O) set* $V \subseteq U$ *containing* x_0, *such that*

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$O(x) \subseteq U, \forall x \in V$.

Otherwise, x_0 is called not semi-stable fixed point.

Example 3.1: *Consider the topological space* (R, τ_u), *and* $f: R \longrightarrow R$ *is the function defined by* $f(x) = \frac{-1}{5}x$. *The fixed point of* f *is* 0, *and the* $D.D.S.$ *is* $\{(\frac{-1}{5})^n x\}_{n \in N_\circ}$.

Let U be any (S-O) set containing 0 of the form $U = (x_0, x_1), [x_0, x_1), (x_0, x_1] and [x_0, x_1]$ also a union of any of these forms. Then U contains (S-O) set $U^* = (x_0, x_1)$.

$V = (-a, a)$, $a = \min\{|x_0|, x_1\}$, $0 \in V \subseteq U^* \subseteq U$, $O(x) \subseteq U, \forall x \in V$

Then, 0 is semi-stable fixed point .

Example 3.2 *Consider the topological space* (R, τ_u),*and* $f: R \longrightarrow R$ *is the function defined by* $f(x) = x^2$. *The fixed points of* f *are* 0 *and* 1, *and the* $D.D.S.$ *is* $\{x^{2^n}\}_{n \in N_\circ}$. 0 *is semi-stable and* 1 *is not semi-stable. To show that* 0 *is semi-stable, let* U *be any (S-O) set containing* 0 *of the form* $U = (x_0, x_1), [x_2, x_3), (x_4, x_5] and [x_6, x_7]$ *also a union of any of these forms.*

i- If $U = (x_0, x_1)$ or it is a union of the above forms including (x_0, x_1) with $0 \in (x_0, x_1)$.

Let $a = \min\{|x_0|, x_1\}$. Take $V = \begin{pmatrix} (-\frac{1}{a}, \frac{1}{a}) & if \quad a > 1 \\ (-a, a) & o.w. \end{pmatrix}$

ii- If $U = [x_2, x_3)$ or it is a union of the above forms including $[x_2, x_3)$ with $0 \in [x_2, x_3)$ or $U = (x_4, x_5]$ or it is a union of the above forms including $(x_4, x_5]$ with $0 \in (x_4, x_5]$, we choose V in similar way as in (i).

iii- If U is as in (ii) with $x_2 = 0$, take $V = [0, \frac{1}{2}]$.

iv- If U is as in (ii) with $x_5 = 0$, take $V = [-\frac{1}{2}, 0]$.

v- If $U = [x_6, x_7]$ or it is a union of the above forms including $[x_6, x_7]$ with $0 \in [x_6, x_7]$,

take $V = \begin{pmatrix} [0, \frac{1}{2}] & if \quad x_6 = 0, \\ & if \quad x_7 = 0, \\ (-\frac{1}{a}, \frac{1}{a}) & if \quad x_6, x_7 \neq 0 \ and \ a > 1, \\ (-a, a) & if \quad x_6, x_7 \neq 0 \ "a \leq 1 (0 < a \leq 1)". \end{pmatrix}$

Where $a = \min\{|x_6|, x_7\}$.

V in each of $(i), (ii), (iii), (iv) and (v)$ is a (S-O) subset of U containing 0 and $O(x) \subseteq U, \forall x \in V$.

So, 0 is a semi-stable fixed point.

1 is not semi- stable :

$U = (0,2)$ (S-O) set; $1 \in U$. Let V be any (S-O) set containing 1 and $V \subseteq U$. There exists $x = 1 + \epsilon \in V$, $0 < \epsilon < 1$,.

$O(x) = \{1 + \epsilon, (1 + \epsilon)^2, (1 + \epsilon)^4, (1 + \epsilon)^{16}, \dots\} \nsubseteq U$.

Hence, 1 is not semi-stable fixed point.

Remark 3.1: *If* x_0 *is stable fixed point, then it needs not be semi-stable as we shall show in the following example:*

Example 3.3 *Let* $(\mathbb{X}, \tau)$ *be a topological space and* $\mathbb{X} = \{a, b, c, d\}$, $\tau = \{\mathbb{X}, \phi, \{a, b\}, \{a, b, c\}\}$ *and* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *is the function defined by* $f(a) = f(b) = f(c) = c$, $f(d) = d$. *The fixed point of* f *are* d *and* c, *the* $D.D.S.$ *generated by* f *is.*

x	$f(x)$	f^2	$f^3(x)$	…	$f^n(x)$	…
a	c	c	c	…	c	…
b	c	c	c	…	c	…

c	c	c	c	…	c	…
d	d	d	d	…	d	…

$SO(\mathbb{X}) = \tau \cup \{\{a,b,d\}\}$.
d is stable fixed point but not semi-stable:
$U = \{a,b,d\}$ is a (S-O) set containing d. The only (S-O) subset of U that containing d is U itself i.e. $V = U$.
$O(a) = \{a,c,c,c,\dots\} \nsubseteq U$.
d is not semi-stable fixed point.
So, stability $\nRightarrow$ semi-stability.

In the following theorem, we give a condition that makes stability implies semi-stability and semi-stability implies stability.

Theorem 3.1 *Let $(\mathbb{X},\tau)$ be a topological space and $\{f^n\}_{n\in N_\circ}$ be a $D.D.S.$ with a fixed point x_0 such that every open set contains x_0. Then x_0 is semi- stable if and only if it is stable.*

proof $\Rightarrow$: Let x_0 be a semi-stable fixed point and let U be any open set containing x_0. Then U is a (S-O) set and $x_0 \in U$, so there exists a (S-O) set V; $x_0 \in V \subseteq U$, and $O(x) \subseteq U$, $\forall x \in V$. V° is open set containing x_0, so it is (S-O) set; $V^\circ \subseteq V \subseteq U$ and $O(x) \subseteq V^\circ, \forall x \in V^\circ$.

Hence, x_0 is stable .

proof $\Leftarrow$:

Let U ba any (S-O) set containing $x_\circ$. Note that U° is open. Since x_0 is stable, then there exists an open set V, $V \subseteq U^\circ$ such that $O(x) \subseteq U^\circ, \forall x \in V$. Now, V is (S-O) set with $O(x) \subseteq U^\circ \subseteq U, \forall x \in V$.

Hence ,x_0 is semi-stable.

Definition 3.2 *Let $(\mathbb{X},\tau)$ be a topological space, in a $D.D.S.$ $\{f^n\}_{n\in N_\circ}$, and let x_0 be a fixed point of f. We say that x_0 is called semi-c-stable if for any (S-O) set U containing x_0, there exists (S-O) set $V \subseteq U$ containing x_0, such that $O(x) \subseteq \overline{U}, \forall x \in V$.*

Otherwise, x_0 is called not semi-c-stable fixed point.

Example 3.4 *Consider the topological space (R,τ_u), and $f: R \longrightarrow R$ is the function defined by $f(x) = 2x + 1$. The fixed point of f is -1, and the $D.D.S.$ is $\{2^n x + (2^{n-1} + 2^{n-2} + \dots + 1)\}_{n\in N_\circ}$.*

$U = [-2,0)$ is a (S-O) set; $-1 \in U$.

Let V be any (S-O) subset of U containing -1. V contains (S-O) set of the form $(-\epsilon - 1, \epsilon - 1)$; $0 < \epsilon \leq 1$. Let $x \in V$; $x < -1$. Then, $O(x) \nsubseteq \overline{U}$.

So, -1 is not semi-c- stable fixed point .

Example 3.5 *Consider the topological space (R,τ_u), and $f: R \longrightarrow R$ is the function defined by $f(x) = \frac{2}{3}x$. The fixed point of f is 0, and the $D.D.S.$ is $\{(\frac{2}{3})^n x\}_{n\in N_\circ}$.*

Let U be any (S-O) sets containing 0 of the form $U = (x_0,x_1), [x_0,x_1)$, $(x_0,x_1] and [x_0,x_1]$ also a union of any of these forms, U must contain (S-O) set of the form $U^* = (x_0,x_1)$.

Put $V = (-a,a)$, $a = \min\{|x_0|, x_1\}$, $0 \in V \subseteq U^* \subseteq U$.

Then, $O(x) \subseteq \overline{U}, \forall x \in V$.

So, 0 is semi-c- stable fixed point .

Theorem 3.2 *Let $(\mathbb{X},\tau)$ be a topological space and $\{f^n\}_{n\in N_\circ}$ be a $D.D.S.$ with a fixed point x_0 such that every open set contains x_0.*

Then x_0 is semi-c-stable if and only if it is c-stable.

proof $\Rightarrow$: Let x_0 be a semi-c-stable fixed point .

Let U be any open set containing x_0. Then U is a (S-O) set and $x_0 \in U$. So, there exists a (S-O) set V; $x_0 \in V \subseteq U$, and $O(x) \subseteq \overline{U}, \forall x \in V$.

V° is open set containing x_0 with $V^\circ \subseteq V \subseteq U$.

So, $O(x) \subseteq \overline{U}, \forall x \in V^\circ$.

Hence, x_0 is c-stable .

$\Leftarrow$: Let x_0 be a c-stable fixed point and let U be a (S-O) set containing x_0.

since U° is an open set containing x_0 and x_0 is c-stable fixed point, then there exists an open set V containing x_0 such that $O(x) \subseteq U^\circ, \forall x \in V$. Since V open set, then it is a (S-O) set. We have $O(x) \subseteq U^\circ \subseteq U, \forall x \in V$. Hence, x_0 is a semi-c-stable fixed point.

Theorem 3.3 *Let $(\mathbb{X}, \tau)$ be a topological space, $\{f^n\}_{n \in N_\circ}$ be a $D.D.S.$ with fixed point x_0 .If x_0 is a semi-stable then it is semi-c-stable.*

proof: Let x_0 be a semi-stable fixed point and let U be a (S-O) set; $x_0 \in U$. Since x_0 is semi-stable, then there exists semi open set V ; $x_0 \in V \subseteq U$ such that $O(x) \subseteq U, \forall x \in V$.

So, $O(x) \subseteq \overline{U}, \forall x \in V$.Hence, x_0 semi-c-stable.

Definition 3.3 *Let $(\mathbb{X}, \tau)$ be a topological space in a $D.D.S.$ $\{f^n\}_{n \in N_\circ}$, and let x_0 be a fixed point of f. We say that x_0 is semi-ic-stable if for any (S-O) set U containing x_0, there exists a (S-O) set $V \subseteq U$ containing x_0, such that $O(x) \subseteq \overline{U}^\circ, \forall x \in V$.*

Otherwise, x_0 is called not semi-ic-stable fixed point.

Example 3.6 *Consider the topological space (R, τ_u) and $\mathbb{X} = (0, \infty) \subset R$, and $f: \mathbb{X} \longrightarrow \mathbb{X}$ is the function defined by $f(x) = \sqrt{x}$. the fixed point of f are 0 and 1, and the $D.D.S.$ is $\{x^{\frac{n}{2}}\}_{n \in N_\circ}$. $B_{\tau_u} = \{(a, b); a, b \in R\}$ is a basis for τ_u.*

B_{τ_u} induces the following basis for the relative topology on X.

$$B_{\mathbb{X}} = \begin{cases} \phi & if \;\; a, b \notin \mathbb{X} \\ (0, b) & if \;\; only \;\; b \in \mathbb{X} \\ (a, b) & if \;\; a, b \in \mathbb{X}. \end{cases}$$

The (S-O) sets in $\mathbb{X}$ are of the forms $(0, b), (a, b), [a, b), (a, b], (0, b], [a, b]$, and any union of any these forms.

Any (S-O) set U of the above forms, $1 \in U$, containing a (S-O) set of the form (c, d); $0 < c < 1$ and $d > 1$.

$1 \in V = (c, d)$. V is a (S-O) subset of U and $O(x) \subseteq \overline{V}^\circ \subseteq \overline{U}^\circ, \forall x \in V$.

Then, 1 is semi-ic-stable

Example 3.7 *Consider the topological space (R, τ_u), and $f: R \longrightarrow R$ is the function defined by $f(x) = 4x(1 - x)$. The fixed points of f are 0 and $\frac{3}{4}$.*

0 is not semi-ic-stable:

$U = (-2,1)$ is a (S-O) set; $0 \in U$. Let V be any (S-O) set containing 0 and $V \subseteq U$. There exists $x \in V; 0 < x < 1$ such that, $O(x) \nsubseteq \overline{U}^\circ$. So, 0 is not semi-ic-stable.

$\frac{3}{4}$ is not semi-ic-stable: Let $U = (0,1)$ is a (S-O) set; $\frac{3}{4} \in U$.

Let V any (S-O) sets contains 0 and $V \subseteq U$. There exist $x \in V, 0 < x < 1$ such that, $O(x) \nsubseteq \overline{U}^\circ$. So, $\frac{3}{4}$ is not semi-ic-stable.

Theorem 3.4 *Let $(\mathbb{X}, \tau)$ be a topological space and $\{f^n\}_{n \in N_\circ}$ be a $D.D.S.$ a with fixed point x_0 such that every open set contains x_0.* Then x_0 is semi-ic-stable if and only if it is ic-stable.

proof ⇒: Let x_0 be a semi-ic-stable fixed point and let U be any open set containing x_0. Then U is a semi open set with $x_0 \in U$.

So, there exists (S-O) set V; $x_0 \in V \subseteq U$ and $O(x) \subseteq \overline{U}^{\circ}, \forall x \in V$.

V° is an open set containing with $V^{\circ} \subseteq V \subseteq U$. So, $O(x) \subseteq \overline{U}^{\circ}, \forall x \in V^{\circ}$. Hence, x_0 is ic-stable fixed point.

proof ⇐ : let x_0 be an ic-stable fixed point and let U be a (S-O) set containing x_0.

Since $U^{\circ}$is an open set containing x_0 and x_0 is an ic-stable fixed point, then there exists an open set V containing x_0 such that $O(x) \subseteq \overline{U^{\circ}}^{\circ}, \forall x \in V$.

Now, V is a (S-O) set containing x_0 such that $O(x) \subseteq \overline{U^{\circ}}^{\circ} \subseteq \overline{U}^{\circ}, \forall x \in V$. Hence, x_0 is semi-ic-stable fixed point.

Example 3.8 *Consider the topological space* $(R, \tau), \tau = \{(-n, n)\}_{n=1,2,3,\dots} \cup \{R, \phi\}$, *and* $f: R \longrightarrow R$ *is the function defined by* $f(x) = \frac{-1}{4}x$. *The fixed point of* f *is* 0, *and the* $D.D.S.$ *is* $\{(\frac{-1}{4})^n x\}_{n \in N_\circ}$.

Note that each open set contains 0 and 0 is stable fixed point .

So it is ic-stable. Then x_0 is semi-ic-stable.

Remark 3.2 *A semi-stable fixed point needs not be semi-ic-stable:*

Example 3.9 *Let* $(\mathbb{X}, \tau)$ *be a topological space and* $\mathbb{X} = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$ *and* $f: \mathbb{X} \longrightarrow \mathbb{X}$ *is the function defined by* $f(2) = f(3) = 3$, $f(1) = 1$. *The fixed point of* f *are* 1 *and* 3. *The* $D.D.S.$ *generated by* f *is.*

x	$f(x)$	f^2	$f^3(x)$	...	$f^n(x)$	...
1	1	1	1	...	1	...
2	3	3	3	...	3	...
3	3	3	3	...	3	...

$SO(\mathbb{X}) = \tau \cup \{\{1,3\}, \{2,3\}\}$.

3 is semi-stable fixed point but not semi-ic-stable:

The (S-O) sets containing 3 are $U = \{1,3\}$ and $U = \{2,3\}$. In each case, choose $V = U$. $O(x) \subseteq U, \forall x \in V$.

So, 3 is semi-stable.

$U = \{2,3\}$ is a (S-O) set containing 3 and the only (S-O) subset of U containing 3 is $V = U$, and $2 \in V$ but $O(2) \nsubseteq \overline{U}^{\circ}$.

So, 3 is not semi-ic-stable.

Theorem 3.5 *Let* $(\mathbb{X}, \tau)$ *be a topological space,* $\{f^n\}_{n \in N_\circ}$ *be a* $D.D.S.$ *with a fixed point* x_0. *If* x_0 *is a semi-ic-stable, then it is semi-c-stable.*

proof: Let x_0 be a semi-ic-stable fixed point and let U be a (S-O) set, $x_0 \in U$. Since x_0 is semi-ic-stable , then there exists a (S-O) set V ; $x_0 \in V \subseteq U$ such that $O(x) \subseteq \overline{U}^{\circ}, \forall x \in V$.

So, $O(x) \subseteq \overline{U}, \forall x \in V$.Hence, x_0 semi-c-stable.

4 Conclusion

Certain types of stability which depend on the (S-O) sets had been discussed. Since every open set is a semi open set, so these types of stability had been discussed the stability in phase spaces in which the collection of (S-O) sets is at most finer than the collection of open sets. This means that we gave a stability in larger phase spaces.

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