International Journal of Academic Accounting, Finance & Management Research (IJAAFMR)
ISSN: ISSN: 2643-976X
Vol. 4, Issue 4, April – 2020, Pages: 28-32

Application of Calculus in Discounting Technique of Financial Management with Laplace Transform

Sanjay Tripathi

M.K. Amin Arts and Science College & College of Commerce, Padra The Maharaja Sayajirao University of Baroda, VADODARA (GUJARAT , INDIA)
E.mail : sanjaymsupdr@yahoo.com

Abstract: *In this paper we have discuss that the Laplace Transform in Mathematics to be equivalent to the Present value of cash flow equation in Financial Management. Further, we have also demonstrate the use of Laplace Transform in evaluation of Consol's Present value under streams of cash flow with example in form of application by using calculus .*

Keywords: Limit, Integration, Present Value, Streams of return, Laplace Transform, Perpetuity, Consol.

Classification (MSC 2010) : 44A10

1. Introduction:

The fundamental goal of Financial Management (FM) is to maximize the wealth of shareholders, which translates into maximising the value of the firm. The different concepts of value are : Book value, Market value, Intrinsic value and Liquidation value. The Basic valuation model (BVM) lies at the hearth of all investment decisions and as such, is an important concept. It is the discounted cash flow model in which the value of any investment is the sum of its future cash flows discounted at an appropriate rate of return. Any asset can be regarded simply as a series of cash flows receivable over a period of the time. Thus, value of any asset can be defined as the present value of these future cash flows. That is the Intrinsic value of an assets is equal to the present value of the benefits associated with it. Symbolically, it can be represented as:

$$P = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots\ldots\ldots\ldots + \frac{CF_n}{(1+r)^n} = \sum_{i=1}^{n} \frac{CF_i}{(1+r)^i} \qquad (1)$$

Where

P = present value of the asset ; CF_i = expected cash flow at the end of period t ;

r = discount rate or the required rate of return on the cash flows

n = expected life of an assets .

A bond or a debenture may be a long-term certificate of indebtedness or security. It is issued by business enterprises or government agencies to raise long term capital. A bond usually carries a hard and fast rate of interest. It is called as coupon payment and the interest rate is called coupon rate. The coupon payment are often either annually or semi-annually. A bond can be irredeemable or redeemable. Redeemable bond have a fixed maturity date and irredeemable bonds have perpetual life with only interest payments periodically. The valuation of bond or a debenture basically, the value of bond is the present value of all the future interest payments and the maturity value return on bond commensurate with the prevailing interest rate and risk. Irredeemable bonds are like perpetuities paying a regular cash flow in the form of interest payment. As the bond is never redeemed, no terminal cash flow is involved. Therefore we can use the present value of the perpetuity equations to determine the present value of the perpetual interest payments. Let us consider a perpetuity of $\$C$ at the end of every period with the interest rate r per period. Now, to derive the formula as follow:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots\ldots\ldots\ldots + \ldots\ldots\ldots\infty$$

$$PV = C\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \ldots\ldots\ldots\ldots + \ldots\ldots\ldots\infty\right]$$

By geometric series sum, we have

$$P = C\left[\frac{1/_{1+r}}{1-1/_{1+r}}\right] = C\left[\frac{1}{1+r-1}\right] = \frac{C}{r}$$

$$P = \frac{C}{r} \qquad (2)$$

In the study of continuous time interest theory a single variable calculus become an important tool. Let us begin with compound interest formula. The future value of an investment of $\$PV$ earning compound interest in discrete time t, with per period rate r, given by

$$FV = PV(1+r)^t \qquad (3)$$

If we compound n times, then (3) becomes

$$FV = PV\left(1+\frac{r}{n}\right)^{nt} \qquad (4)$$

We would like to study what happens for future value as the number of compounding per year approaches to infinity ($n \to \infty$), that is continuous compounding . Taking log on both side of (4), we have

$$\log(FV) = \log[PV\left(1+\frac{r}{n}\right)^{nt}] = logPV + log\left(1+\frac{r}{n}\right)^{nt} = logPV + ntlog\left(1+\frac{r}{n}\right)$$

Taking limit as $n \to \infty$, on both side we get

$$\lim_{n\to\infty} logFV = \lim_{n\to\infty}\left[logPV + ntlog\left(1+\frac{r}{n}\right)\right] = logPV + t\lim_{n\to\infty} n\left(1+\frac{r}{n}\right)$$

$$logFV = logPV + t\lim_{n\to\infty}\frac{\log(1+\frac{r}{n})}{n^{-1}} = logp + tr$$

Raising exponential on both side we get,

$$e^{logFV} = e^{logPV+tr} = e^{logPV}e^{tr} = PVe^{rt}$$

Therefore we get ,

$$FV = PVe^{rt} \qquad (5)$$

and the present value is given by the formula

$$PV = FVe^{-rt} \qquad (6)$$

Now, let us consider a stream of returns (or payments) instead of a single return. This situation might be relevant for large companies with continuous returns such as chain stores, electricity providers, internet commerce and others. To achieve the present value of a stream of returns $C(t)$ over a period of M years, we use concept of limit and integration from calculus as follow:

We divide the interval $[0, M]$ into n subintervals such that:

$$0 = t_1 < t_2 < \ldots\ldots < t_n = n .$$

We assume that in each interval $[t_{i-1}, t_i]$ of a length $\Delta t_i = t_i - t_{i-1}$ a single return $C(t)$ is realizes. Then the future value and present value of the stream of returns $C(t_i)$ with the interest r are given by:

International Journal of Academic Accounting, Finance & Management Research (IJAAFMR)
ISSN: ISSN: 2643-976X
Vol. 4, Issue 4, April – 2020, Pages: 28-32

$$FV = \int_0^M C(t)e^{rt}dt \quad (7)$$

and

$$PV = \int_0^M C(t)e^{-rt}dt \quad (8)$$

In case of console or perpetuity a period interest is paid forever, so the value of M is infinite and from (7) and (8) we obtain

$$FV = \int_0^{+\infty} C(t)e^{rt}dt \quad (9)$$

and

$$PV = p(r) = \int_0^{+\infty} C(t)e^{-rt}dt \quad (10)$$

The Laplace Transform (LT), named after French Mathematician and Astronomer Pierre-Simon Laplace (1749-1827). The main area of applications of LT are: the solution of ordinary and partial differential equations, Signal and image processing, probability theory, the theory of electric circuits and many others. In Economics, the LT can be applied to the analysis of dynamics and shocks in time series of macroeconomic indicators[1] , for pricing barrier options [2] , analysis of continuous time stochastic process such as Brownian motion [3]. The another application of LT is for evaluation of present value of consols [4] .Let us first define Laplace Transform as follow:

If the function $f(t)$ is considered to be continuous, then the Laplace transform of $f(t)$, written as $L\{f(t)\}$, is defined as a function $F(s)$ of the variable s by the integral

$$L\{f(t)\} = F(s) = \int_0^{+\infty} f(t)e^{-st}dt \quad (11)$$

Over the range of value of s for which the integral exists. Replacing s in (11) with continuous compounding rate r simply generates equation (10) (see [5])

Thus we can see from (10) and (11) that the Laplace Transform is shown to be equivalent to the Present value of cash flow equation in Financial Management. That is

$$p(r) = L\{f(t)\} \quad (12)$$

We evaluate the Present value of Concols with different streams of return in the form of applications by Financial method and Laplace transform as follow:

Application 1: Let the stream of return is constant say $f(t) = c$ where c is constant with the interest rate r, then the present value is

$$PV = p(r) = L\{c\} = \int_0^{+\infty} c\, e^{-rt}dt = c \lim_{M\to\infty} \int_0^{M} e^{-rt}dt = c \lim_{M\to\infty} \left(\frac{e^{-rt}}{-r}\right)_0^M = \frac{c}{-r}(0-1) = \frac{c}{r}$$

The ratio $\frac{c}{r}$ (by 2, for $p = c$ constant payment) is well known formula for the Present value of a concols. For example if $c = \$\,100$ and $r = 5\%$ then $PV = \frac{100}{0.05} = \$2000$.

Application 2: Let 10% be interest rate and let the stream of returns $f\{t\}$ be exponentially growing at a rate 5%. The present value of a consol is

$$PV = \int_0^{+\infty} e^{0.05t}\, e^{-0.1t} dt = \lim_{M\to\infty} \int_0^{M} e^{-0.05t} dt = \lim_{M\to\infty} \left(\frac{e^{-0.05t}}{-0.05}\right)_0^M = \frac{1}{-0.05}(0-1) = \$20$$

We can also find the Present value by using the general property of Laplace transform of exponential growth, if we take $f(t) = e^{qt}$,where $q < r$ .The Laplace transform is given by

$$L\{f(t)\} = \int_0^{+\infty} e^{qt} e^{-rt} dt = \int_0^{+\infty} e^{(q-r)t} dt = \lim_{M\to\infty} \left(\frac{e^{(q-r)t}}{q-r}\right)_0^M = \frac{1}{q-r}(0-1) = \frac{1}{r-q}$$

If we take $r = 10\%\ and\ q = 5\%$, then we get the same answer that is

$$PV = \int_0^{+\infty} e^{0.05t}\, e^{-0.1t} dt = \frac{1}{0.1-0.05} = \frac{1}{0.05} = \frac{100}{5} = \$20$$

Application 3 : Let the stream of returns be linearly growing say $f\{t\} = t$ with interest rate 5%. The present value of a consol is given by

$$PV = \int_0^{+\infty} t\, e^{-0.05t} dt = \lim_{M\to\infty} \int_0^{M} t\, e^{-0.05t}\, dt$$

Using integration by part, we have

$$PV = \lim_{M\to\infty} \int_0^{M} t\, e^{-0.05t}\, dt = \lim_{M\to\infty} \left[\left(t \times \frac{e^{-0.05t}}{-0.05}\right)_0^M - \int_0^M \left(\frac{e^{-0.05t}}{-0.05}\right) dt \right]$$

$$= \left(\frac{1}{-0.05}\right) \lim_{M\to\infty} \int_0^{M} e^{-0.05t}\, dt = \left(\frac{1}{0.05}\right)\left(\frac{1}{0.05}\right) = \frac{100\times 100}{5\times 5} = \$400$$

By the above application we can get the general formula for Laplace transform as

$$L\{f(t) = t\} = F(r) = \int_0^{+\infty} f(t) e^{-rt} dt = \frac{1}{r^2}$$

Application 4: Suppose that a company manufactures a certain type of machine and is estimates that each machine will generate a continuously income stream whose rate in the t-th year of operations will be $\$10 + 5t$ per year. Assuming that the lifetime of a machine is about 7 years and that the money can be invested as the annual rate of 10%, compounded continuously. The present value is given by

$$PV = \int_0^{7} (10+5t) e^{-0.1t} dt = \int_0^{7} 10 e^{-0.1t}\, dt + \int_0^{7} 5t e^{-0.1t}\, dt = \frac{10\times 100}{10} + \frac{5\times 100\times 100}{10\times 10}$$

$$= 100 + 500 = \$600.$$

Conclusion: We have seen that, integration and limit concept of calculus is used in the formula of Concol or Laplace Transform. We have also obtained a closed relation between the Present value of perpetuity (or Consol) and Laplace Transform for different stream of return (constant, linearly, exponentially growing etc.) with examples in form of applications.

References:

[1] Buser, S.A. Laplace Transform as present value rules: A note , Journal of Finance, 41(1) (1986), 243-247.

[2] Grubbstrom, R.W. On the application of laplace transform, Berlin, Springer-Verlag,(1974)

[3] Lochowsk, I.R. On the Laplace Transform on some Functionals related to the variation of Brownian motion with drift: International workshop on applied probability, Madrid, (2010).

[4] Pelsser, A. Pricing double barrier options using Laplace Transforms, Finance and Stchastics, 4(1),(2000),95-104.

[5] Park and Sharp-Bettle, Advanced Engineering Economics, Wiley Canada, (1990).