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Mathematical Description of Asynchronous Motors

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Abstract* — *In this article, the mathematical representation of the dependence of the change of the main parameters of asynchronous machines on its various modes of operation is applied. The article analyzes the load and load-free operation of asynchronous machines.

Keywords — fixed frequency; asynchronous motor; stator ; rotor; Heyland-Ossanna locus diagram; Speed-torque and current-torque characteristics

Introduction

The mathematical description of an asynchronous motor usually restricts itself to a steady state of operation, i.e. immediately after connecting to the power supply network. The motor is then supplied with a fixed frequency and amplitude AC voltage. For asynchronous motors with round rotor rods and neglecting magnetic hysteresis losses, this results in an equivalent circuit diagram as shown in figure 1[2].

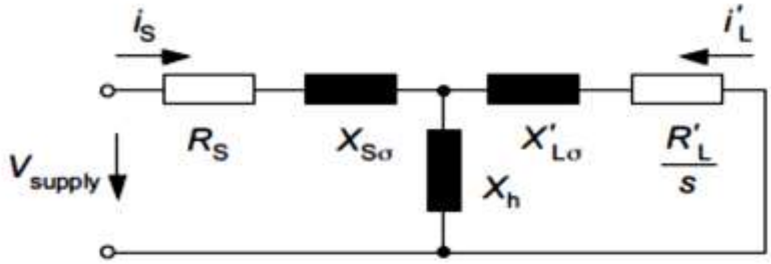

Figure 1. Steady-state equivalent circuit diagram of an asynchronous motor

The following elements are used in the equivalent circuit diagram

V_{supply} - line voltage
i_s - stator current
i_L' - equivalent rotor current
R_s - stator resistance
R_L'- equivalent rotor resistance
$X_{s\sigma}$ - stator stray reactance
$X'_{L\sigma}$ - equivalent rotor stray reactance
X_h - main reactance
s - slip
where $X= L \cdot \omega$; $\omega= 2\pi \cdot 50$ Hz (operation from a 50 Hzline supply frequency)

The equivalent circuit diagram is similar to that for a transformer. The main difference can be found on the secondary side. Whereas in thecase of a transformer the consumer is connected to this side, in an asynchronous motor the secondary side is short-circuited via the slipdependent resistor RL'/s. The slip s is defined by:

The equivalent circuit diagram is limited to a single winding as, due to the symmetrical design of an asynchronous motor, the same equivalent scheme is valid for all other windings.

The equivalent circuit diagram is similar to that for a transformer. The main difference can be found on the secondary side. Whereas in the case of a transformer the consumer is connected to this side, in an asynchronous motor the secondary side is short-circuited via the slip dependent resistor RL'/s. The slip s is defined by:

$$s = \frac{n_d - n}{n_d} \quad ,(1)$$

It describes the difference under the actual operating conditions that exists between the synchronous speed n_d and the mechanical speed n inrelation to the synchronous rotational speed. Slip, as it is a ratio, does not have a unit. It can be compared to the slip of a belt drive: under noload conditions it has less slip, when loaded the slip is larger. For an asynchronous motor, $s = 0$ describes the no-load condition and $s = 1$zero speed. In motoring operation, the slip is in the range $0 < s < 1$.Under rated-operating conditions the slip, depending on the size of the motor, is in the range between 0.03 and 0.10 [2].

Depending on the size of the slip s, the relationships between the currents and voltages in the equivalent circuit diagram of the asynchronous motor change. As it involves sinusoidal alternating quantities, thetreatment and interpretation of the changes as a function of time is notvery transparent and calculations would be very time-consuming.Therefore, the method chosen is to expand the alternating quantities to complex quantities and to treat these as rotating phase vectors. The real physical quantities can be determined from the imaginary or the real parts of the appropriate phasor (see Figure 2).

The vectors rotate with the frequency of the supply voltage. Limiting the treatment to a momentary representation, e.g. at a point in time $t = 0$,results in clear vector diagrams which clearly illustrate the relationships between the currents and voltages. It must not be forgotten that this treatment is a mathematical trick, and that the real physical quantities are sinusoidal alternating quantities. To differentiate the vector quantities from the real quantities they are underlined (v, i) [1].

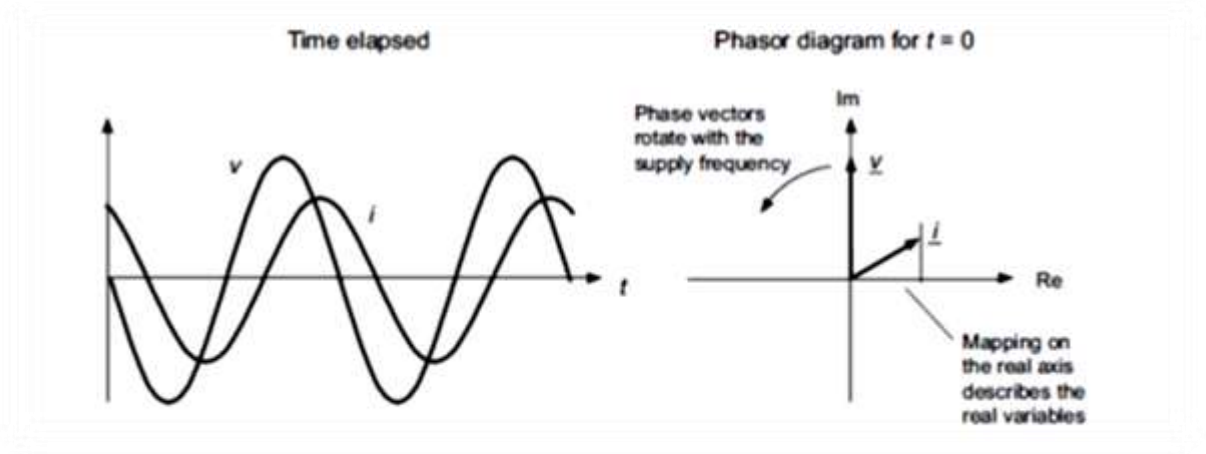

Figure 2. Transformation of sinusoidal quantities to a phasor diagram

All currents and voltages of an asynchronous motor may be represented with the help of vectors. However, for the operational behaviour it is the relationship between the stator voltage and stator current that is of most significance of particular interest is how the vectors change when the motor is loaded. The trace of the vector tips as the load is variedis plotted. This trace is called a locus. The locus, which describes the behaviour of an asynchronous motor with a constant supply voltage for changing loads, is the Heyland-Ossanna locus diagram (see Figure3) [2]. It shows how the stator current vector i_S behaves in relation tothe stator voltage vector v_S as the load is changed. The measure of the loading is the slip s.

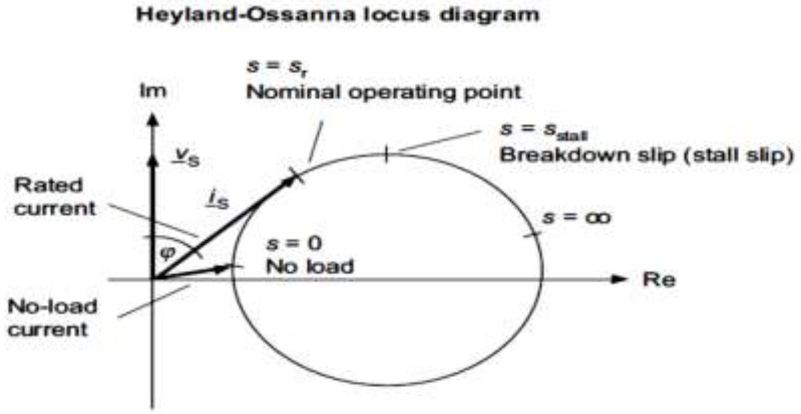

Figure 3. Phasor diagram representation of the operational behaviour of an asynchronous motor

Under no-load conditions the slip $s = 0$. The stator current lags the voltage by 90° and its absolute value is not equal to 0. This means that evenunder no-load conditions an asynchronous motor draws a current i_0.This no-load current is about half of the rated current. The real power of any asynchronous motor is calculated as follows:

$$P=\sqrt{3}\cdot V_S\cdot I_S\cdot\cos\varphi\ ,(2)$$

Where V_s rms value of the stator voltage (conductor-to-conductor voltage),
I_s rms value of the stator current, $\cos\varphi$ power factor

As under no-load conditions φ approaches 90°, no real power is generated under no-load conditions. The asynchronous motor exchanges reactive power with the supply network. As the load increases and with increasing slip s, the stator current vector is "wanders" in the Heyland-Ossanna locus diagram. Its absolute value increases and the angle φ to the stator voltage vector v_s decreases. The rated slip s_r indicates the point to which the stator current vector points when the asynchronous motor is operated with rated load. The power factor cos φ , which is recorded on the rating plate, refers to this operating point. The stall slips_{stall} defines the operating point at which an asynchronous motor delivers its greatest possible torque.

If the torque generated by the asynchronous motor is calculated for every point on the locus, the speed-torque characteristic, as well as the stator current-torque characteristic, can be deduced (see Figure 4) [3].

In the region around the synchronous speed n_d, the speed-torque characteristics almost linear. As a function of the load, the speed n decreases and the stator current, starting at the no-load current, continually increases. In this range, the characteristic of an asynchronous motor is similar to that of a DC motor.

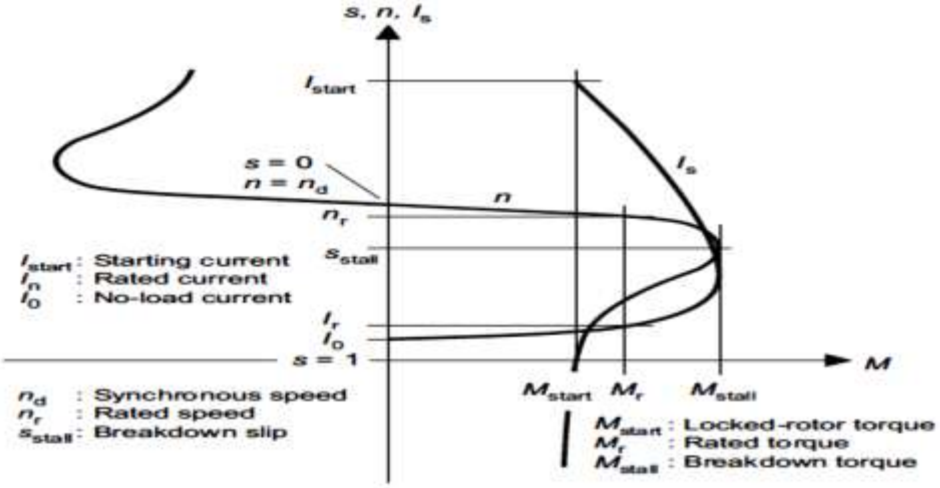

Figure 4. Speed-torque and current-torque characteristics of an asynchronous motor

With increasing load, the curvature of the speed-torque characteristic increases. It finally reaches its maximum possible torque, called the stalling torque or also breakdown torque.

At zero speed ($n = 0$) the motor produces a starting torque M_{start}, also known as the locked-rotor toque, which is often smaller than the rated torque M_r. A very large starting current I_{start} flows. As the speed increases,the torque M of most motors continues to increase until the stalling torque is reached. In some special motors, which have been optimized to have a large starting torque M_{start} , the torque decreases as the speed increases until a saddle point is reached, from which pointonwards it increases until the stalling torque M_{stall} is reached [4].

Above the synchronous speed n_d in the over-synchronous range the torque M changes its sign. The asynchronous motor is now operating as a generator. The speed-torque characteristic is now symmetrical with regard to $s = 0$.Reference values of the various characteristics are shown in Table 1.

Table 1. Operating points of an asynchronous motor

Operating point	**Slip s**	**Power factor cos φ**	**I/I_r**	**M/M_r**
zero speed	1	< 0.4	≈ 10	< 0,5
stall	≈ 0.2	≈ 0.6	≈ 6	> 2
rated speed	≈ 0.02	≈ 0.85	1	1
no-load	≈ 0	< 0.5	≈ 0.3 ... 0.5	≈ 0

The speed-torque characteristic of an asynchronous motor can be specifically influenced by the design of the squirrel-cage rotor bars; Figure5 shows some examples of different rotor-bar designs. The greatest changes are in the range of the starting torque M_{start}.

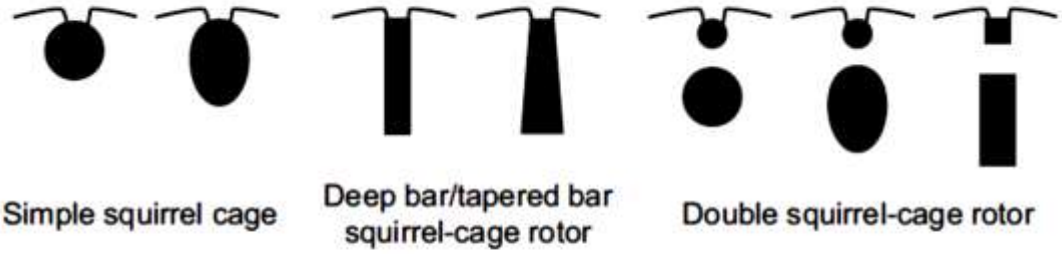

Figure 5. Rotor bar shapes of squirrel-cage asynchronous motors

Designs with deep rotor bars or double cages have a significantly higher starting torque (compare in Figure 6). These motors find particularuse in mills and in conveyor systems as large break away torques are required, particularly following extended downtimes [2].

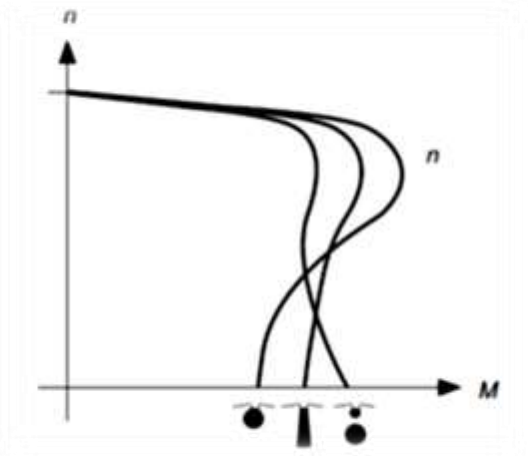

Figure 6. Characteristics of asynchronous motors with different rotor-bar shapes

For varying the speed of an asynchronous motor, the following starting points can be determined from the speed-torque characteristic:

- varying the stator voltage V_s
- varying the stator frequency f
- simultaneously varying the stator voltage V_s and stator frequency f.

Figure 7 shows the influence of the individual system input variableson the speed-torque characteristic.

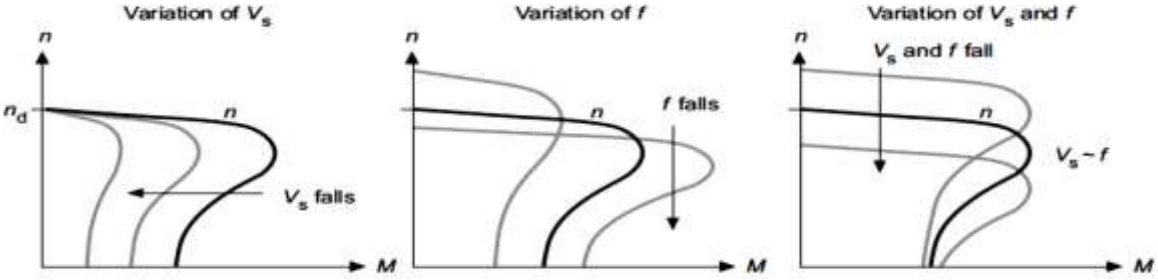

Figure 7. Possibilities for varying the speed of an asynchronous motor

By reducing the stator voltage *V*s the speed-torque characteristic maybe compressed in the direction of the speed axis. The stalling torque isreduced and the asynchronous motor may only be lightly loaded. In therated operating range (n close to n_d) the speed hardly changes. The stator voltage on its own is therefore not a suitable system input for varying the speed of an asynchronous motor [5].

The reduction in the torque and the stator current is proportional to the square of the reduction in the stator voltage.By varying the stator frequency f, the speed-torque characteristic can be shifted in parallel up or down.

At lower frequencies this, theoretically, results in a large increase in the stalling torque. This is based on detailed observation, which shows that the magnetic field of the stator is significantly amplified. However, due to saturation of the iron circuit, this amplification barely takes place. By reducing the frequency, a very large increase in the stator current I_s and increased thermal loading of the motor is traded off for a small amplification in the torque.

Using the stator frequency f as a system input variable for varying the speed of an asynchronous motor is, in principle, suitable, but only if itis increased and not decreased. In this case, the speed increases and the stalling torque decreases. Optimum performance is achieved when the stator voltage and stator frequency are adjusted simultaneously and in the same ratio, observingthe following:

$$\frac{V_s}{f} = const. ,(3)$$

Using this approach, the speed-torque characteristic can be shifted up and down without changing its shape. The stalling torque and the gradient of the characteristic in the rated operating range remain unchanged.Therefore, the parallel adjustment of stator voltage V_s and stator frequency f is chosen as the preferred adjustment method for variable-speed drives with asynchronous motors. Using this method, the speed n cannot be increased beyond the synchronous speed n_d as the maximum value of the stator voltage V_s is limited by the supply voltage. The speed range is therefore limited to $n < n_d$, which is more than sufficient for most applications.

This limitation can be overcome if the stator voltage V_s, once reaching its maximum value, is kept constant and only the stator frequency f is increased. The asynchronous motor is now in “field weakening”. As, however, can be seen from the centre diagram of Figure 7, in this range the full motor torque is no longer available.

Conclusion

When the stator is connected to a 3-phase supply, a sinusoidally distributed, radially directed rotating magnetic flux density wave is set up in the air-gap. The speed ofrotation of the field is directly proportional to the frequency of the supply, andinversely proportional to the pole-number of the winding. The magnitude of the flux wave is proportional to the applied voltage, and inversely proportional to the frequency. When the rotor circuits are ignored (i.e. under no-load conditions), the realpower drawn is small, but the magnetizing current itself can be quite large, giving rise to a significant reactive power demand from the utility supply.

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