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Fuzzy Magnified Translation QS-ideals of QS-algebras

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Abstract *— Fuzzy magnified translation of QS-subalgebras and fuzzy magnified translation of QS-ideals in QS-algebras are discussed. Relations among fuzzy magnified translations, them are investigated.*

Keywords—component; QS-algebra, QS -subalgebras, QS -ideal, Fuzzy QS -subalgebras, Fuzzy QS -ideal, (β,α)-magnified translations of fuzzy QS-subalgebras, (β,α)-magnified translations of fuzzy QS-ideal,

1. Introduction

Several authors ([8]) have introduced of BCK-algebras as a generalization of the concept of set-theoretic difference and propositional calculus and studied some important properties. K.B. Lee and et al [7]introduced translation fuzzy and fuzzy multiplication of BCK/BCI -algebras. S.M. Mostafa and et al [9] and A.T. Hameed [6], introduced KUS-ideals on KUS-algebras and A.T. Hameed and et al [2], introduced the notion fuzzy QS-ideals of QS-algebras investigated relations among them. A.T. Hameed, and et al ([3-5]) discuss translation and multiplication of fuzzy ideals in some algebras.

In this paper, we discuss (β,α)-magnified translations of fuzzy QS-subalgebras in QS-algebras and we discuss (β,α)-magnified translationsof fuzzy QS-ideals in QS-algebras. Also, we investigate relations among of fuzzy QS-subalgebra and fuzzy QS-ideals in QS-algebras.

2. Preliminaries

Now, we introduced an algebraic structure of QS-algebra and we give some results and theorems of it .

Definition 2.1([1]). Let $(X; *, 0)$ be an algebra of type (2,0) with a single binary operation $(*)$. X is called a **QS-algebra** if it satisfies the following identities: for any x, y, z∈X ,

(QS_1) : $(z*y) * (z*x) = x*y$,

(QS_2) : $x * 0 = x$,

(QS_3) : $x * x = 0$,

(QS_4): $(x*y)*z = (x*z)*y$.

In X we can define order relation (≤) by : $x \le y$ if and only if, $x * y = 0$.

Proposition 2.2([2]). In any QS-algebra $(X; *, 0)$, the following properties hold: for all x, y, z ∈X;

a) $x*y=0$ *and* $y*x=0$ *imply* $x=y$,

b) $(x*y)*x=0*y$,

c) $y*x=0$ *imply* $0*x=0*y$,

d) $0*(x*y)=y*x$,

e) $0*x=0$ *implies* $x=0$,

f) $x=(x*0)*0$,

g) $(x*y)*0=(x*0)*(y*0)$.

Definition 2.3([1]) . Let X be a QS-algebra and let S be a nonempty subset of X. S is called a **QS-subalgebra** of X, if $x * y \in S$, whenever x , y ∈ S.

Definition 2.4([1]). A nonempty subset I of a QS-algebra X is called a **QS-ideal** of X if it satisfies: for x , y, z ∈ X,

(IQS_1) $(0 \in I)$,

(IQS_2) $(z*y) \in I$ *and* $(x*y) \in I$ *imply* $(z*x) \in I$.

Definition 2.5([6]). Let X be a QS-algebra, a fuzzy subset μ in X is called a **fuzzy QS-subalgebra** of X if for all x , y ∈X ,
$\mu(x * y) \ge \min\{\mu(x), \mu(y)\}$.

Definition 2.6([2]). Let X be a QS-algebra, a fuzzy subset μ in X is called a **fuzzy QS-ideal** of X if it satisfies the following conditions: , for all x , y, z ∈ X ,

(FQS_1) $\mu(0) \geq \mu(x)$,

(FQS_2) $\mu(z * x) \geq \min\{\mu(z * y), \mu(x * y)\}$.

3. (β,α)-magnified translations Fuzzy QS-subalgebras of QS-algebra

We shall define the notion of (β,α)-magnified translation fuzzy QS-subalgebras of QS-algebra X and studied its properties as [11] .

Definition 3.1. Let μ be a fuzzy subset of a QS-algebra X and let α ∈[0,T] and β ∈(0, 1].
A mapping $\mu^C_{(\beta,\alpha)}$:X→[0,1] is called a **(β,α)-magnified translation** of μ if it Satisfies: $\mu^C_{(\beta,\alpha)}(x) = \beta.\mu(x)+\alpha$, for all x ∈X.

Definition 3.2. Let X be a QS-algebra, a fuzzy subset μ in X is called a **(β,α)-magnified translation fuzzy QS-subalgebra** of X if for all x , y ∈X, $\mu^C_{(\beta,\alpha)}(x * y) \geq \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$.

Theorem 3.3. Let μ be a fuzzy QS-subalgebra of QS-algebra X and α ∈[0,T], β ∈(0,1]. Then the (β,α)-magnified translation fuzzysubset $\mu^C_{(\beta,\alpha)}$ of μ is the (β,α)-magnified translation fuzzy QS- subalgebra of X.

Proof. Assume μ be a fuzzy QS-subalgebra of X and α ∈[0,T], β ∈(0,1], let x, y ∈X.
Then $\mu^C_{(\beta,\alpha)}(x * y) = \beta.\mu(x * y)+\alpha \geq \beta.\min\{\mu(x), \mu(y)\}+\alpha = \min\{\beta.\mu(x), \beta.\mu(y)\}+\alpha$

$= \min\{\beta.\mu((x)+\alpha, \beta.\mu((y)+\alpha\} = \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$.

Hence $\mu^C_{(\beta,\alpha)}$is a (β,α)-magnified translation fuzzy QS- subalgebra of X. ⌂

Theorem 3.4. Let μ be a fuzzy subset of QS-algebra X such that the (β,α)- magnified translation fuzzy subset $\mu^C_{(\beta,\alpha)}$ of μ is a fuzzy QS-subalgebra of X for some α ∈ [0,T], β∈ (0,1]. Then μ is a fuzzy QS-subalgebra of X .

Proof. Assume $\mu^C_{(\beta,\alpha)}$ be a (β,α)-magnified translation fuzzy QS-subalgebra of X for some α ∈ [0,T], β ∈ (0,1]. Let x, y ∈X, then

$\beta.\mu(x * y)+\alpha = \mu^C_{(\beta,\alpha)}(x * y) \geq \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$

$= \min\{\beta.\mu(x)+\alpha, \beta.\mu(y)+\alpha\} = \min\{\beta.\mu(x), \beta.\mu(y)\}+ \alpha$

$= \beta.\min\{\mu(x), \mu(y)\}+\alpha$ and so $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.

Hence μ is a fuzzy QS-subalgebra of X .⌂

Definition 3.5([10]). For a fuzzy subset μ of a QS-algebra X, α ∈ [0,T] , β, t ∈ [0, 1] with t ≥ α, let $U_{(\beta,\alpha)}(\mu; t) := \{x \in X \mid \mu(x) \geq \frac{t-\alpha}{\beta}\}$, where β ≠0 .

If μ is a fuzzy QS- subalgebra of X, then it is clear that $U_{(\beta,\alpha)}(\mu; t)$ is a QS-subalgebra of X for all t ∈ Im(μ) with t ≥ α. But, if we do not give a condition that μ is a fuzzy QS-subalgebra of X, then $U_{(\beta,\alpha)}(\mu; t)$ is not a QS-subalgebra of X as seen in the following example.

Example 3.6. by Consider a QS-algebra X = {0, a, b, c} with the following Cayley table:

∗	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a fuzzy subset μ of X by:

X	0	a	b	c
μ	0.8	0.5	0.6	0.5

Define a fuzzy subset λ of X

X	0	a	b	c
λ	0.7	0.6	0.4	0.3

Then λ is not a fuzzy QS- subalgebra of X since $\lambda(a * b) = 0.3 < 0.4 = \min\{\lambda(a), \lambda(b)\}$. For α = 0.1 , β = 1 and t = 0.5, we obtain $U_{(\beta,\alpha)}(\lambda; t) = \{0, a, b\}$ which is not a QS- subalgebra of X since $a * b = c \notin U_{(\beta,\alpha)}(\lambda; t)$.

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Proposition 3.7. Let μ be a fuzzy subset of a QS-algebra X and $\alpha \in [0,T]$, $\beta \in (0,1]$. Then the (β,α)-magnified translation fuzzy subset $\mu^C_{(\beta,\alpha)}$ of μ is a fuzzy QS-subalgebra of X if and only if $U_{(\beta,\alpha)}(\mu; t)$ is a QS-subalgebra of X for all $t \in \mathrm{Im}(\mu)$ with $t \geq \alpha$ and $\beta \neq 0$.

Proof. Necessity is clear{assume that $\mu^C_{(\beta,\alpha)}$ is a fuzzy QS-algebra by Theorem(4.4),then μ is a fuzzy QS-algebra, by Definition(4.5) then $U_{(\beta,\alpha)}(\mu; t)$ is a fuzzy QS-sub algebra}. To prove the sufficiency, assume that there exist $x, y \in X$ such that $\mu^C_{(\beta,\alpha)}(x * y) < \gamma \leq \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$. Then $\mu(x) \geq \frac{\gamma-\alpha}{\beta}$ and $\mu(y) \geq \frac{\gamma-\alpha}{\beta}$, but $\mu(x * y) < \frac{\gamma-\alpha}{\beta}$. This shows that $x, y \in U_{(\beta,\alpha)}(\mu; \gamma)$ and $x * y \notin U_{(\beta,\alpha)}(\mu; \gamma)$. This is a contradiction, and so $\mu^C_{(\beta,\alpha)}(x * y) \geq \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$, for all $x, y \in X$. Hence $\mu^C_{(\beta,\alpha)}$ is the (β,α)-magnified translation fuzzy QS-subalgebra of X. ⌂

Theorem 3.8. Let $f : (X; *, 0) \to (Y; *', 0')$ be a onto homomorphism between QS-algebras X and Y respectively. For every (β,α)-magnified translation fuzzy QS-subalgebra μ of X with sup property, $f(\mu)$ is a (β,α)-magnified translation fuzzy QS-subalgebra of Y. **Proof:** By definition $\lambda^C_{(\beta,\alpha)}(y') = f(\mu^C_{(\beta,\alpha)})(y') = \sup_{x \in f^{-1}(y')} \beta \cdot \mu(t) + \alpha$, for all $y' \in Y$ ($\sup\varnothing = 0$). We have to prove that $\lambda^C_{(\beta,\alpha)}(x' * y') = f(\mu^C_{(\beta,\alpha)})(x' * y') \geq \min\{\lambda^C_{(\beta,\alpha)}(x'), \lambda^C_{(\beta,\alpha)}(y')\}$, for all $x', y', z' \in Y$.

Let $f : (X; *, 0) \to (Y; *', 0')$ be a onto homomorphism of QS-algebras, μ is a (β,α)-magnified translation fuzzy QS-subalgebra of X with sup property and $\lambda^C_{(\beta,\alpha)}$ the image of $\mu^C_{(\beta,\alpha)}$ under f. For any $x', y' \in Y$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$ be such that: $f(\mu^C_{(\beta,\alpha)}(x_0 * y_0)) = \lambda^C_{(\beta,\alpha)}(x_0 * y_0) = \sup_{(x_0 * y_0) \in f^{-1}(x' * y')} \beta \cdot \mu(x_0 * y_0) + \alpha = \sup_{t \in f^{-1}(x' * y')} \beta \cdot \mu(t) + \alpha$. Then $\lambda^C_{(\beta,\alpha)}(x' *' y') =$

$$\sup_{(x_0 * y_0) \in f^{-1}(x' * y')} \beta \cdot \mu(x_0 * y_0) + \alpha = \mu^C_{(\beta,\alpha)}(x_0 * y_0) \geq \min\{\mu^C_{(\beta,\alpha)}(x_0), \mu^C_{(\beta,\alpha)}(y_0)\}$$

$$= \min\{\sup_{t \in f^{-1}(x')} \beta \cdot \mu(t) + \alpha, \sup_{t \in f^{-1}(y')} \beta \cdot \mu(t) + \alpha\} = \min\{\lambda^C_{(\beta,\alpha)}(x'), \lambda^C_{(\beta,\alpha)}(y')\}.$$

Hence $\lambda^C_{(\beta,\alpha)} = f(\mu^C_{(\beta,\alpha)})$ is a (β,α)-magnified translation fuzzy QS-subalgebra of Y.⌂

Theorem 3.9. An onto homomorphic pre-image of a (β,α)-magnified translation fuzzy QS-subalgebra of QS-algebra is also a (β,α)-magnified translation fuzzy QS-subalgebra of QS-algebra.

Proof: Let $f : (X; *, 0) \to (Y; *', 0')$ be an onto homomorphism of QS- algebras , λ the (β,α)-magnified translation fuzzy QS-subalgebra of Y and μ the pre-image of λ under f, then $\mu^C_{(\beta,\alpha)}(x) = \lambda^C_{(\beta,\alpha)}(f(x))$, for all $x \in X$. Since $f(x) \in Y$ and λ is a (β,α)-magnified translation fuzzy QS-subalgebra of Y, let $x, y \in X$ such that for any $x', y' \in Y$, since f is onto, then $f(x) = x'$, $f(y) = y'$. Then $\mu^C_{(\beta,\alpha)}(x * y) = \lambda^C_{(\beta,\alpha)}(f(x * y)) = \lambda^C_{(\beta,\alpha)}(f(x) *' f(y)) \geq \min\{\lambda^C_{(\beta,\alpha)}(f(x)), \lambda^C_{(\beta,\alpha)}(f(y))\} = \min\{\beta.\lambda(x') + \alpha, \beta.\lambda(y') + \alpha\}$ $= \min\{\mu^C_{(\beta,\alpha)}(x), \mu^C_{(\beta,\alpha)}(y)\}$. Hence $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-subalgebra of X. ⌂

4. (β,α)-magnified Translation fuzzy QS-ideals of QS-algebra

We shall define the notion of (β,α)-magnified translation fuzzy QS-ideals, of QS-algebra X and then studied its properties .

Definition 4.1. Let X be a QS-algebra, a (β,α)-magnified translation fuzzy subset μ of X is called a **(β,α)-magnified translation fuzzy QS-ideal** of X if it satisfies the following conditions: for all $x, y, z \in X$,

(FQS_1) $\mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$,

(FQS_2) $\mu^C_{(\beta,\alpha)}(z * x) \geq \min\{\mu^C_{(\beta,\alpha)}(z * y), \mu^C_{(\beta,\alpha)}(x * y)\}$.

Theorem 4.2. Let μ is a fuzzy QS-ideal of a QS-algebra X, then the (β,α)-magnified translation fuzzy QS-ideal $\mu^C_{(\beta,\alpha)}$ of μ is a fuzzy QS-ideal of X for all $\alpha \in [0,T]$, $\beta \in (0,1]$.

Proof. Assume μ be a fuzzy QS-ideal of X and let $\alpha \in [0,T]$, $\beta \in [0,1]$. Then for all $x, y, z \in X$. Then

1- $\mu^C_{(\beta,\alpha)}(0) = \beta.\mu(0) + \alpha \geq \beta.\mu(x) + \alpha = \mu^C_{(\beta,\alpha)}(x)$.

2- $\mu^C_{(\beta,\alpha)}(z * x) = \beta.\mu(z * x) + \alpha \geq \beta.\min\{\mu(z * y), \mu(x * y)\} + \alpha$
$= \min\{\beta.\mu(z * y), \beta.\mu(x * y)\} + \alpha = \min\{\beta.\mu(z * y) + \alpha, \beta.\mu(x * y) + \alpha\}$

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$= \min\{\mu^C_{(\beta,\alpha)}(z*y),\ \mu^C_{(\beta,\alpha)}(x*y)\}$.

Hence $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X .⌂

Theorem 4.3. Let μ be a fuzzy subset of QS-algebra X such that the (β,α)-magnified translation fuzzy $\mu^C_{(\beta,\alpha)}$ subset of μ is a fuzzy QS-ideal of X for some $\alpha \in [0,T]$, $\beta \in (0,1]$. Then μ is a fuzzy QS-ideal of X.

Proof. Assume $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X for some $\alpha \in [0,T]$, $\beta \in (0,1]$. Let $x, y, z \in X$, we have $\beta.\mu(0) + \alpha = \mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x) = \beta.\mu(x) + \alpha$ and so

$\mu(0) \geq \mu(x)$. $\beta.\mu(z*x) + \alpha = \mu^C_{(\beta,\alpha)}(z*x) \geq \min\{\mu^C_{(\beta,\alpha)}(z*y),\ \mu^C_{(\beta,\alpha)}(x*y)\}$

$= \min\{\beta.\mu(z*y) + \alpha,\ \beta.\mu(x*y) + \alpha\}$

$= \min\{\beta.\mu(z*y),\ \beta.\mu(x*y)\} + \alpha$

$= \beta.\min\{\mu(z*y),\ \mu(x*y)\} + \alpha$

$\mu(z*x) \geq \min\{\mu(z*y),\ \mu(x*y)\}$. Hence μ is a fuzzy QS-ideal of X .⌂

Theorem 4.4. For $\alpha \in [0,T]$ and $\beta \in (0,1]$, let $\mu^C_{(\beta,\alpha)}$ be the (β,α)-magnified translation fuzzy subset μ of QS-algebra X. Then the following are equivalent:

(1) $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X.

(2) $\forall t \in Im(\mu)$, $t > \alpha \Rightarrow U_{(\beta,\alpha)}(\mu; t)$ is QS-ideal of X .

Proof. Assume that $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X and let $t \in Im(\mu)$ be such that $t > \alpha$.

Since $\mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$ for all $x \in X$, we have $\beta.\mu(0) + \alpha = \mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x) = \beta.\mu(x) + \alpha \geq t$, for $x \in U_{(\beta,\alpha)}(\mu; t)$.

Hence $0 \in U_{(\beta,\alpha)}(\mu; t)$.

Let $x, y, z \in X$, $\beta \neq 0$ such that $(z*y) \in U_{(\beta,\alpha)}(\mu; t)$ and $(x*y) \in U_{(\beta,\alpha)}(\mu; t)$. Then $\mu(z*y) \geq \frac{t-\alpha}{\beta}$ and $\mu(x*y) \geq \frac{t-\alpha}{\beta}$, i.e., $\mu^C_{(\beta,\alpha)}(z*y) = \beta.\mu(z*y) + \alpha \geq t$ and $\mu^C_{(\beta,\alpha)}(x*y) = \beta.\mu(y*x) + \alpha \geq t$. Since $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X, it follows that $\beta.\mu(z*x) + \alpha = \mu^C_{(\beta,\alpha)}(z*x) \geq \min\{\mu^C_{(\beta,\alpha)}(z*y),\ \mu^C_{(\beta,\alpha)}(x*y)\} \geq t$, that is, $\mu(z*x) \geq \frac{t-\alpha}{\beta}$ so that $(z*x) \in U_{(\beta,\alpha)}(\mu; t)$ Therefore $U_{(\beta,\alpha)}(\mu; t)$ is QS-ideal of X.

Conversely, suppose that $U_{(\beta,\alpha)}(\mu; t)$ is QS-ideal of X for every $t \in Im(\mu)$ with $t > \alpha$. If there exists $x \in X$ such that $\mu^C_{(\beta,\alpha)}(0) < \lambda \leq \mu^C_{(\beta,\alpha)}(x)$, then $\mu(x) \geq \frac{\lambda-\alpha}{\beta}$, but $\mu(0) < \frac{\lambda-\alpha}{\beta}$ This shows that $x \in U_{(\beta,\alpha)}(\mu; t)$ and $0 \notin U_{(\beta,\alpha)}(\mu; t)$. This is a contradiction, and so $\mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$ for all $x \in X$.

Now, assume that there exist $x, y, z \in X$ such that $\mu^C_{(\beta,\alpha)}(z*x) < \gamma \leq \min\{\mu^C_{(\beta,\alpha)}(z*y),\ \mu^C_{(\beta,\alpha)}(x*y)\}$.

Then $\mu(z*y) \geq \frac{\gamma-\alpha}{\beta}$ and $\mu(x*y) \geq \frac{\gamma-\alpha}{\beta}$, but $\mu(z*x) < \frac{\gamma-\alpha}{\beta}$. Hence $(z*y) \in U_{(\beta,\alpha)}(\mu; \gamma)$ and $(x*y) \in U_{(\beta,\alpha)}(\mu; \gamma)$, but $(z*x) \notin U_{(\beta,\alpha)}(\mu; \gamma)$. This is a contradiction, and therefore as $\mu^C_{(\beta,\alpha)}(z*x) \geq \min\{\mu^C_{(\beta,\alpha)}(z*y),\ \mu^C_{(\beta,\alpha)}(x*y)\}$, for all $x, y, z \in X$. Hence $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-ideal of X. ⌂

In Theorem (5.4(2)), if $t \leq \alpha$, then $U_{(\beta,\alpha)}(\mu; t) = X$.

Proposition 4.5. Let μ be a fuzzy QS-ideal of a QS-algebra X and let $\alpha \in [0,T]$, $\beta \in (0,1]$, then the (β,α)-magnified translation fuzzy QS-ideal $\mu^C_{(\beta,\alpha)}$ of μ is a (β,α)-magnified translation fuzzy QS-subalgebra of X.

Proof: Since μ be a fuzzy QS-ideal of a QS-algebra X and let $\alpha \in [0,T]$, $\beta \in (0,1]$, then by Theorem (5.4), the (β,α)-magnified translation fuzzy QS-ideal $\mu^C_{(\beta,\alpha)}$ of X. By Proposition (2.8), for all $\alpha \in [0,T]$, $\beta \in (0,1]$, then the (β,α)-magnified translation fuzzy QS-subalgebra $\mu^C_{(\beta,\alpha)}$ of X. Hence by Proposition (4.7), $\mu^C_{(\beta,\alpha)}$ is a (β,α)-magnified translation fuzzy QS-subalgebra of QS-algebra X. ⌂

Theorem 4.5. Let $f : (X; *, 0) \to (Y; *', 0')$ be a onto homomorphism between QS-algebras X and Y respectively. For every μ (β,α)-magnified translation fuzzy QS-ideal of X with sup property, $f(\mu)$ is a (β,α)-magnified translation fuzzy QS-ideal of Y.

Proof: Let $f: (X; *, 0) \to (Y; *', 0')$ be a onto homomorphism of QS-algebras, μ is a (β,α)-magnified translation fuzzy QS-ideal of X with sup property and $\lambda^C_{(\beta,\alpha)}$ the image of $\mu^C_{(\beta,\alpha)}$ under f. Since μ is a (β,α)-magnified translation fuzzy QS-ideal of X, we have $\mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$, for all $x \in X$. Note that $0 \in f(0')$, where 0 and 0' are the zero elements of X and Y respectively. Thus $\lambda^C_{(\beta,\alpha)}(0') = f(\mu^C_{(\beta,\alpha)})(0') = \sup_{t \in f^{-1}(0')} \beta \cdot \mu(t) + \alpha = \mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$, for all $x \in X$, which implies that $\lambda^C_{(\beta,\alpha)}(0') \geq \sup_{t \in f^{-1}(0')} \beta \cdot \mu(t) + \alpha = \lambda^C_{(\beta,\alpha)}(x')$.

For any $x', y', z' \in Y$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be such that:

$f(\mu^C_{(\beta,\alpha)})(z_0 * y_0) = \sup_{t \in f^{-1}(z' *' y')} \beta \cdot \mu(t) + \alpha$, $f(\mu^C_{(\beta,\alpha)})(x_0 * y_0) = \sup_{t \in f^{-1}(y' *' x')} \beta \cdot \mu(t) + \alpha$.

Then $f(\mu^C_{(\beta,\alpha)})(z_0 * x_0) = \lambda^C_{(\beta,\alpha)}(z_0 * x_0) = \sup_{(z_0 * x_0) \in f^{-1}(z' *' x')} \beta \cdot \mu(z_0 * x_0) + \alpha = \sup_{t \in f^{-1}(z' *' x')} \beta \cdot \mu(t) + \alpha$.

Then
$$\begin{aligned}
\lambda^C_{(\beta,\alpha)}(z' *' x') &= \sup_{t \in f^{-1}(z' *' x')} \beta \cdot \mu(t) + \alpha \\
&= \mu^C_{(\beta,\alpha)}(z_0 * x_0) \geq \min\{\mu^C_{(\beta,\alpha)}(z_0 * y_0), \mu^C_{(\beta,\alpha)}(x_0 * y_0)\} \\
&= \min\{\sup_{t \in f^{-1}(z' *' y')} \beta \cdot \mu(t) + \alpha, \sup_{t \in f^{-1}(x' *' y')} \beta \cdot \mu(t) + \alpha\} \\
&= \min\{\lambda^C_{(\beta,\alpha)}(z' *' y'), \lambda^C_{(\beta,\alpha)}(x' *' y')\}.
\end{aligned}$$

Hence $\lambda^C_{(\beta,\alpha)} = f(\mu^C_{(\beta,\alpha)})$ is a fuzzy QS-ideal of Y.⌂

Theorem 4.6. An onto homomorphic pre-image of a (β,α)-magnified translation fuzzy QS-ideal of QS-algebra X is also a (β,α)-magnified translation fuzzy QS-ideal.

Proof: Let $f : (X; *, 0) \to (Y; *', 0')$ be an onto homomorphism of QS- algebras, λ the (β,α)-magnified translation fuzzy QS-ideal of Y and μ the pre-image of λ under f, then $\mu^C_{(\beta,\alpha)}(x) = \lambda^C_{(\beta,\alpha)}(f(x))$, for all $x \in X$. Since $f(x) \in Y$ and λ is a (β,α)-magnified translation fuzzy QS-ideal of Y, it follows that $\beta.\lambda(0') + \alpha \geq \lambda^C_{(\beta,\alpha)}(f(x)) = \mu^C_{(\beta,\alpha)}(x)$, for every $x \in X$, where 0' is the zero element of Y. But $\beta.\lambda(0') + \alpha = \beta.\lambda(f(0)) + \alpha = \mu^C_{(\beta,\alpha)}(0)$ and so $\mu^C_{(\beta,\alpha)}(0) \geq \mu^C_{(\beta,\alpha)}(x)$, for $x \in X$. Since λ is a (β,α)-magnified translation fuzzy QS-ideal of Y, let $x, y, z \in X$ such that $\mu^C_{(\beta,\alpha)}(z * x) = \lambda^C_{(\beta,\alpha)}(f(z * x)) = \lambda^C_{(\beta,\alpha)}(f(z) *' f(x)) \geq \min\{\lambda^C_{(\beta,\alpha)}(f(z) *' f(y)), \lambda^C_{(\beta,\alpha)}(f(x) *' (f(y))\} = \min\{\beta.\lambda(z' *' y') + \alpha, \beta.\lambda(x' *' y') + \alpha\}$, for any $x', y', z' \in Y$, since f is onto, then $f(x) = x'$, $f(y) = y'$, $f(z) = z'$. Then

$$\begin{aligned}
\mu^C_{(\beta,\alpha)}(z * x) &\geq \min\{\beta.\lambda(z' *' y') + \alpha, \beta.\lambda(x' *' y') + \alpha\} \\
&= \min\{\beta.\lambda(f(z) *' f(y)) + \alpha, \beta.\lambda(f(x) *' f(y)) + \alpha\} \\
&= \min\{\beta.\lambda(f(z * y)) + \alpha, \beta.\lambda(f(x * y)) + \alpha\} \\
&= \min\{\lambda^C_{(\beta,\alpha)}(f(z * y)), \lambda^C_{(\beta,\alpha)}(f(x * y))\} \\
&= \min\{\mu^C_{(\beta,\alpha)}(z * y)), \mu^C_{(\beta,\alpha)}(x * y)\}.
\end{aligned}$$

Since x', y' and z' are arbitrary element of Y, the above result is true for all $x, y, z \in X$.

i.e., $\mu^C_{(\beta,\alpha)}(z * x) \geq \min\{\mu^C_{(\beta,\alpha)}(z * y)), \mu^C_{(\beta,\alpha)}(x * y)\}$, for all $x, y, z \in X$. ⌂

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