International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 4, April – 2020, Pages: 35-37

A Numerical Method for Solving Stochastic Differential Equations.

Jawad Kadhim [1]

[1]Department of Mathematics, Faculty of Computer Science and Mathematics , University of Kufa , Najaf, Iraq.
Corresponding E-mail Address:jawadd.kadhimm@gmail.com

***Abstract**: In this work, is an introduction and survey of numerical solution methods for stochastic differential equations. we are interested with the solve stochastic differential equations (SDEs) by numerical methods. The solutions will be continuous stochastic processes that represent diffusive dynamics, a common modeling assumption for financial systems. These methods are based on the truncated Ito- Taylor expansion. In our study we deal with a nonlinear SDE. We include a review of fundamental concepts, a description of elementary numerical methods and the concepts of convergence and order for stochastic differential equation solvers. Also we describe applications of SDE solvers to Monte-Carlo sampling for financial pricing of derivatives. By using Monte Carlo simulation and exact solution for each method we approximate to numerical solution and obtained from Itos formula. approximation solutions are compared with exact solution to show the effectiveness of the numerical methods.*

Keywords: differential equations (DEs), ordinary differential equations (ODEs), stochastic differential equa- tions (SDEs), Taylor expansion, Ito-Taylor expansion, strong solutions, numerical schemes, Monte- Carlo simulation.

1 Introduction

We are present efficient numerical methods to compute certain quantities depending on the unknown process ($\delta(t)$) with algoritms based on simulations on a computer of the other processes, and this is the main objective of this paper. The numerical analysis of stochastic differential equations is at its very beginning, so it already appears that this field is not at all direct continuation of what has been done for the numerical solving of ordinary differential equations (ODEs). When we wants to to compute a quantity which depends on law it is often unuseful to try to approximate the stochastic differential equation on the space of trajectories , this has been confirmed in numerous research and books such as [1-4].
Consider the simple population growth model

$$\frac{d\delta}{dt} = x(t)\,\delta(t), \qquad \delta(0) = X, \tag{1}$$

Where,

$\delta(t)$ is the size of the population at time t, and $x(t)$ is the relative rate growth at time t. It might happen that $x(t)$ is not completely known, but subject to some random environmental effects, so that we have

$$x(t) = k(t) + \lambda, \tag{2}$$

where λ is noise and $k(t)$ is non-random function. So the Eq (1) becomes

$$\frac{d\delta}{dt} = k(t)\,\delta(t) + \delta(t).\text{"}\lambda\text{"}, \tag{3}$$

Now the general equation can be written as

$$\frac{d\delta_t}{dt}(w) = x(t, \delta_t(w))\,dt + y(t, \delta(w))\,\xi_t(w), \tag{4}$$

where x and y are some given functions, and (ξ_t) were standard Gaussian random variables for each t and $y(t, x)$ a (generally) time-space dependent intensity factor. This symbolic differential was interpreted as an integral equation

$$\delta_t(w) = \delta_{t0}(w) + \int_{t0}^{t} + x\,(s, \delta s(w))ds + \int_{t0}^{t} + y\,(s, \delta s(w))\,\xi_s(w)ds; \tag{5}$$

for each sample path. For the special case of (5) with $x \equiv 0$, $y \equiv 1$ we see that ξ_t should be derivative of pure

Brownian motion, that is the derivative of *a* wiener process W_t, thus we suggest that we could write (5) alternatively as

$$\delta_t(w) = \delta_{t0}(\mathrm{w}) + \int_{t0}^{t} + x\ (\mathrm{s}, \delta\mathrm{s}(\mathrm{w}))\mathrm{ds} + \int_{t0}^{t} + y\ (\mathrm{s}, \delta\mathrm{s}(\mathrm{w}))\ dW\xi_s\ (w)ds; \quad (6)$$

The problem with this is that a wiener process W_t is nowhere differentiable, so strictly speaking the white noise process *t* does not exist as a conventional function of *t*.

2 Numerical methods.

Definition 3.1 *Stochastic differential equation has become standard models for financial quantities such as asset prices, interest rates, and their derivatives. Unlike deterministic models such as ordinary differ- ential equations, which have a unique solution for each appropriate initial condition, SDEs have solutions that are continuous time stochastic processes. Methods for the computational solution of stochastic dif- ferential equations are based on similar techniques for ordinary differential equations, but generalizedto provide support for stochas-tic dynamics. [5]*

The simplest effective computational method for the approximation of ordinary differential equations is Euler's method [6]. The Euler- Maruyama method [7] is the analogue of the Euler method for ordinary differential equations. To develop an approximate solution on the interval [*c*, *d*], assign a grid of points

$$c = \quad t_0 < t_1 < t_2 \ldots < t_n = d$$

Approximate *u* values

$w_0 < w_1 < w_2 \ldots < w_n$

will be determined at the respective *t* points. Given the SDE initial value problem Numerical Solution of Stochastic Differential Equations in Finance

$$d\delta(t) \quad = \quad x(t, \delta)\ dt + \ y(t, \delta)\ dW_t \quad (7)$$

where,

$$\delta(c) \quad = \quad \delta_c \quad (8)$$

3 Stochastic Taylor series expansion.

The Taylor formula plays a very significant role in numerical analysis. We can obtain the approximation of a sufficiently smooth function in a neighborhood of a given point to any desired order of accuracy with the Taylor formula.
Enlarging the increments of smooth functions of Ito processes, it is beneficial to have a stochastic expansion formula with correspondent specialities to the deterministic Taylor formula. Such a stochastic Taylor formula has some possibilities. One of these possibilities is an Ito-Taylor expansion obtained via Itos formula [1].

3.1 Ito-Taylor expansion.

First we can obtain an Ito-Taylor expansion for the stochastic case. Consider

$$d\delta(t) \quad = \quad f\,(\delta(t))d(t) + g(\delta(t))dW(t) \quad (9)$$

where f and g satisfy a linear growth bound and are sufficiently smooth.
Now, let F be a twice continuously differentiable function of $\delta(t)$, then from Itos lemma we have

$$dF[\delta(t)] = \{f[\delta(t)] \frac{\partial F[\delta(t)]}{\partial \delta} + \frac{1}{2} g^2[\delta(t)] \frac{\partial^2 F[\delta(t)]}{\partial \delta^2}\} d(t) + g(\hat{\phi}(t)] \frac{\partial F[\delta(t)]}{\partial \delta} dW(t) \quad (10)$$

4 MonteCarlo simulation

In MonteCarlo simulation, we generate a set of suitable multidimensional sample paths say $\hat{W}(w) = \hat{W}^1(w), \ldots, \hat{W}^d(w)$ on $[0, T]$. We generate a large finite set of paths, each labelled by w. Here the number of paths, say P, must be large enough so that, for example, any statistical information for the solution y_t that we want to extract is sufficiently robust. For each sample path $\hat{W}(w)$, we generate a sample path solution $\hat{y}(w)$ to the stochastic differential equation on $[0, T]$. This can often only be achieved approximately, by using a truncation of the stochastic Taylor expansion for the solution yon successive small subintervals of $[0, T]$. Having generated a set of approximate solutions $\hat{y}t(w_i)$ at time $t \in [0, T]$ for every i for $i = 1, \ldots, P$, we estimate the expectation $Ef(yt)$ by computing

$$\frac{1}{P}\sum_{i=1}^{P} f(yt^{wi})$$

regarded as a suitable approximation for

$$Ef(yt) = f(yt(w))d\,P(w) \quad (11)$$

Now we have to be sample Brownian paths to compute the mean above, or can we choose different paths that will still generate the expectation effectively. We discuss the latter case (weak simulation) briefly next.[7]

References

[1] P. E. Kloeden, E. Platen, Numerical Solution of Stochastic Differential Equations, Springer-Verlag, New York, 1992.

[2] Kloeden, Peter Eris and Platen, Eckhard and Schurz, Henri, Numerical solution of SDE through computer experiments. Springer, 2012.

[3] hrcak.srce.hr and R Horvth Bokor, Numerical solving of stochastic differential equations. Mathemat- ical Communications, Odjel za matematiku, v. 4, pp. 251–256, n. 2, 1999

[4] Schurz, Henri and others, Construction of Nonnegative Numerical Solutions for SDE's: Explosions, Lifetime, Nonnegativity, and Uniform Boundedness. Humboldt University of Berlin, Interdisciplinary Research Project 373, 1994.

[5] Platen, Eckhard and Bruti-Liberati, Nicola, Numerical solution of stochastic differential equations with jumps in finance. Springer Science and Business Media, v. 64, 2010

[6] Sauer, Timothy, Numerical Analysis Pearson Addison Wesley. Boston, 2006

[7] Maruyama, Gisiro, Continuous Markov processes and stochastic equations. Rendiconti del Circolo Matematico di Palermo, Springer, v. 4, pp. 48, n. 1, 1955