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Using Computer Programming to Find Numerical Solutions for High-Order Differential Equations.

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***Abstract**: In this paper, we presented a survey study about the use of computer systems and programs in numerical calculations resulting from solutions of differential equations (DEs). In many mathematical methods, such as Euler method, Tayler expansion, Runge-Kutta methods (RK) and Linear multistep method (LMM), manual and traditional methods of numerical operations have been replaced by modern computer programs such as Matlab program and Maple software in search of accuracy in calculating results and reducing computational time.*

Keyword: Differential equations (DEs), Runge-Kutta methods (RK), Tayler expansion, Linear multistep method (LMM), Euler method, computer programs, Matlab program and Maple software .

1 Introduction

The differential equations (DEs) are the most important mathematical model for physical phenomena [1-2], for this reason many mathematicians have studied the nature of these equations for hundreds of years and there are many well-developed solution techniques. Often, systems described by DEs are so complex, or the systems that they describe are so large, that a purely analytical solution to the equations is not tractable. It is in these complex systems where computer simulations and numerical methods are useful. The techniques for solving DEs based on numerical approximations were developed before programmable computers existed. It was the first beginning to find solutions to DEs, since Newton, Euler and Taylor time, but many methods do not have any closed-form solutions, so they need to look for approximate solutions by numerical methods so that the problem can be framed mathematical terminology; and structure a mathematical model for this purposed problem [3]. The problems in differential equations occur in several models of non-Newtonian fluid mechanics, mathematical physics, astrophysics, etc. [4], [5]. For example, the theory of internal structure of stars, cluster of galaxies, thermal behavior of a spherical cloud of gas acting under the mutual attraction of its molecules and theories of thermionic currents are modeled by means of differential equations [6].

2 Some mathematical methods in which the solution depended on programming.

Many of the math methods devoted to solving ordinary and partial differential equations were adopted recently in calculating results and finding solutions on the principle of programming for accuracy and time shortening. In this research we will mention one it, which is the Runge- Kutta method for solving ordinary differential equations with high-order.

2.1 Runge-Kutta methods for solving fourth-order ordinary differential equations.

For solving the numerical integration of the special fourth-order ordinary differential equations (ODEs) of the form:

$$y^{(iv)} = f(x, y), \tag{1}$$

with initial conditions,

$$\mathrm{y}(x_0) = \alpha_0,\ \mathrm{y}(x_1) = \alpha_1,\ \mathrm{y}(x_2) = \alpha_2 \ \text{ and } \ \mathrm{y}(x_3) = \alpha_3, \tag{2}$$

where,

$$\mathrm{f} : R \times R^N \to R^N$$

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and, $$y(x) = [y_1(x), y_2(x), \ldots, y_N(x)]$$

where,

$$f(x, y) = [f_1(x, y), f_2(x, y), \ldots, f_N(x, y)],$$

$$\alpha^0 = [\alpha^0_1, \alpha^0_2, \ldots, \alpha^0_N], \alpha^1 = [\alpha^1_1, \alpha^1_2, \ldots, \alpha^1_N],$$

$$\alpha^2 = [\alpha^2_1, \alpha^2_2, \ldots, \alpha^2_{N,}\ \alpha^3 = [\alpha^3_1, \alpha^3_2, \ldots, \alpha^3_N].$$

To solve this type of equation, we must reduce the order of the equation from the fourth-order to the equation of the first-order. But it is known that the method of lowering the orders for differential equations in which the results are less accurate and more time-consuming is used in calculating the results from the direct solution, since the method of the solution goes through indirect stages. While Hussain, Kasim et al 2016 [7] used the computer programming method represented by the Maple software to calculate the order conditions of the Runge-Kutta method for solving ordinary fourth-order differential equations directly and without reduction, thus this method is very appropriate as it is a direct method. While researchers used the Matlab program to solve examples and find numerical solutions to examples that have been solved and calculate the difference between a numerical solution and the exact solution or is called (absolute error).

2.2 Runge-Kutta methods for solving fifth-order ordinary differential equations.

For solving the numerical integration of the special fourth-order ordinary differential equations (ODEs) of the form:

$$y^{(5)} = f(x, y), \tag{3}$$

with initial conditions,

$$y(x_0) = \alpha_0,\ y(x_1) = \alpha_1,\ y(x_2) = \alpha_2\ ,\ y(x_3) = \alpha_3 \text{ and } y(x_4) = \alpha_4, \tag{4}$$

where,

$$f : R \times R^N \rightarrow R^N$$

and, $$y(x) = [y_1(x), y_2(x), \ldots, y_N(x)]$$

where,

$$f(x, y) = [f_1(x, y), f_2(x, y), \ldots, f_N(x, y)],$$

$$\alpha^0 = [\alpha^0_1, \alpha^0_2, \ldots, \alpha^0_N], \alpha^1 = [\alpha^1_1, \alpha^1_2, \ldots, \alpha^1_N],$$

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$$\alpha^2 = [\alpha^2_1, \alpha^2_2, \ldots, \alpha^2_N, \alpha^3 = [\alpha^3_1, \alpha^3_2, \ldots, \alpha^3_N],$$

$$\alpha^4 = [\alpha^4_1, \alpha^4_2, \ldots, \alpha^4_N].$$

Kadhim, Murtaza [8] has used the same method that was used to solve the ordinary fourth-order differential equation in solving the ordinary fifth-order differential equations where the researcher used the method represented by the Maple software to calculate the order conditions of the Runge-Kutta method for solving ordinary fifth-order differential equations directly and without reduction, thus this method is very appropriate as it is a direct method. While researchers used the Matlab program to solve examples and find numerical solutions to examples that have been solved and calculate the difference between a numerical solution and the exact solution or is called (absolute error).

3 Discussion and Conclusions

In this paper, we dealt with a very important topic, which is the use of modern techniques such as computer programming and electronic calculation programs to solve differential equations from the high-order that were previously solved by traditional numerical methods after lowering the orders of these equations so that differential formulas are given from the first-order and here lies the importance of this study of what is enjoyed the accuracy of calculating the results and his speed to obtain them. it is possible in other studies with the same method to prove the use of modern methods in numerical methods such as Tayler expansion and a multi-step method (LMM).

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