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Modeling Treasury Bill Purchasing Options Under Markovian Demand

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***Abstract**: We consider a multi-period purchasing decision problem for treasury bills at investment firms under demand uncertainty. Associated with purchase of treasury bills is stochastic stationary demand; where purchasing decisions are uniformly fixed over the planning horizon. The purchasing price, selling price and demand for treasury bills are considered in order to determine the profit matrix; representing the long run measure of performance for the markov decision process problem. We formulate a finite-state markov decision process model where states of a markov chain represent possible states of demand for treasury bills. The problem is to determine an optimal purchasing decision for treasury bills so that the long run profits are maximized over a given state of demand at investment firms. The decisions of purchasing versus not purchasing additional treasury bills are made using dynamic programming over a finite period planning horizon. We test the model using data from two investment firms in Uganda. The model demonstrates the existence of an optimal state-dependent purchasing decision and profits for the two selected investment firms used in this study.*

Keywords— Bills; markov; modeling; purchasing; treasury

1. INTRODUCTION

Government securities are a set of instruments used by the government to borrow money from the public. Government borrows money when its income falls short of public spending needs. In Uganda, treasury bills are used to borrow money for short term periods that do not exceed one year. Currently, the government borrows money through treasury from three specific periods; that is 91,182 and 364 days. The sale of treasury bills is conducted through an auction. This means that interest rates are market determined. Participation in the primary auctions is done through a set of commercial banks that are designated as primary dealers. Investors who conduct their banking with non-primary dealer banks can still invest in government securities through their specific commercial banks, which work with the primary dealers to facilitate the opening of investor's accounts on central securities depository.

Treasury bills are safe to invest in because they are backed by the governments. These are short term debt obligations backed by the treasury department with a maturity of one year or less. In practice, the longer the maturity date, the higher the interest rate that the treasury bill will pay to the investor. Treasury bills are normally held until the maturity date; although some holders may wish to cash out before maturity and realize the short-term gains by reselling the investment into the secondary market.

Although treasury bills have zero default risk, their returns are lower than corporate bonds. Treasury bills don't pay periodic interest payments so they are sold at a discounted rate to the face value of the bond. The gain is realized when the bond matures, which is the difference between purchase price and the face value. However, if they are sold early, there could be a gain or loss depending on where bond prices are trading at the time of sale. Several factors influence treasury bill prices; including macroeconomic conditions, monetary policy and the overall supply and demand of treasuries.

2.LITERATURE REVIEWS

In [1], the authors provided a normal return benchmark to improve pricing errors made by the market trading on the basis of demand mispricing. An estimate was derived and implemented to show how much of the typical deviation consists of mispricing and misestimation. Goldstein et al [2] studied the link between secondary market liquidity for a corporate bond and the bond's yield spread at issuance. An economically large impact on yield spread was found. According to Dungey et al [3] modeling trade duration in treasury markets revealed how trade duration exhibits significant clustering and the time taken to expand trade volume decreases the time between initiation and consecutive traders. In [4] the authors consolidated on corporate bond markets by capturing the interaction of market behavior, fund trading strategies and cash allocation by investors. This model explored the impact of shocks with greater risk aversion. In order to determine whether time variation in the movements of daily stock and treasury bond returns can be linked to measure of stock market uncertainty. According to Connolly et al [5], an investigation into this search. concluded negative correlation between the uncertainty measures and the future correlation of stock and bond returns. Baviera[6] explained the basis of the bond market model who shows the possibility of pricing with black-like formulas the three classes of plain vanilla options; by deriving prices, bond options and swap options. In reference to bond yield modeling, Diebold et al [7] showed how a joint macro-finance strategy can provide comprehensive understanding of the term structure of interest rates. The authors discuss various questions that arise and examine the relationship with dynamic latent factor model. An empirical analysis of stock and bond market liquidity by Tarun et al [8] shows that innovations in liquidity are positively correlated across stock and bond markets. Order imbalances in the stock market impact bond and stock liquidity, suggesting a common liquidity factor in a stock and bond markets.

3. MODEL DEVELOPMENT

We consider a set of investment firms engaged in buying and selling treasury bonds based on future business expectations. The demand for treasury bills during each time period over a fixed planning horizon at investment firm *m* is described as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining the optimal course of action; namely to purchase additional treasury bills (a decision denoted by Z=1) or not to purchase additional treasury bills (a decision denoted by Z=0) during each time period over the planning horizon where Z is a binary decision variable. Optimality is defined such that the maximum profits are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, two investment firms(m=2) and a two-period (N=2) planning horizon are considered

3.1 Notation

Sets

i,j	Set of states of demand
m	Set of investment firms
Z	Set of purchasing decisions

Parameters

D	Demand matrix
T	Transaction matrix
P	Profit matrix
p_r	Purchasing price
p_s	Selling price

Others

n,N	Stages
F	Favorable demand
U	Unfavorable demand
e	Expected profits
a	Accumulated profits

i,j ε{F,U} , Z ε {0,1} , m= {1,2}

3.2 Finite Period Dynamic Programming Model

Recalling that the demand for treasury bills can either be in state F or in state U, the problem of finding an optimal purchasing decision can be expressed as a finite period dynamic programming model. Assuming g_n(i,m) denotes the optimal expected profits accumulated at the end of periods n,n+1…………………..N given that the state of the system at the beginning of period *n* is i ε{F,U}, The recursive equation relating g_n and g_{n+1} is

$$g_n(i,m) = max_Z[Q^Z_{iF}(\text{m})P^Z_{iF}(\text{m}) + g_{n+1}(\text{F,m}), Q^Z_{iU}(\text{m})P^Z_{iU}(\text{m}) + g_{n+1}(\text{U,m})]$$

i,j ε{F,U} , Z ε {0,1} , m= {1,2} n = 1,2,………………N (1)

together with the conditions

$g_{n+1}(F,m) = g_{n+1}(U,m) = 0$

This recursive relationship may be justified by noting that the cumulative profits

$P^Z_{ij}(m) + g_{n+1}(j)$ resulting from reaching state j ε{F,U} at the start of period *n+1* form state i ε{F,U} at the start of period *n* occurs with probability Q^Z_{ij}(m).

Clearly,

$$e^Z(\text{m}) = [Q^Z(\text{m})]\ [P^Z(\text{m})]$$

Z ε {0,1} , m= {1,2} (2)

$$g_N(\text{i,m}) = max_Z[e^Z_i(\text{m}) + Q^Z_{iF}(m)g_{n+1}(\text{F}) + Q^Z_{iU}(m)g_{n+1}(\text{U})] \quad (3)$$

$$g_n(\text{i,m}) = max_Z[e^Z_i(m)] \quad (4)$$

result where (4) represents the markov chain stable state.

3.3 Computing $Q^Z(m)$ and $P^Z(m)$

The demand transition probability from state i ε{F,U} to sate j ε{F,U} given purchasing decision Z ε {0,1} may be taken as the number of transactions for investment firm *m* with demand initially in state i and later with demand changing to state j, divided by the sum of transactions over all states. That is

$$Q^Z_{ij}(\text{m}) = T^Z_{ij}(\text{m}) / [T^Z_i\text{F}(\text{m}) + T^Z_{iU}(\text{m})] \quad (5)$$

i,j ε{F,U} , Z ε {0,1} , m= {1,2}

We assume uniform purchasing price(p_r) and selling price(p_s) of treasury bills over the planning horizon. Calculating the profits for investment firm *m* given purchasing decision Z,

$$P^Z(\text{m})=(p_s–p_r)[D^Z_{ij}(\text{m})] \quad (6)$$

for all i,j ε{F,U} , Z ε {0,1} , m= {1,2}

4. OPTIMIZATION

The optimal purchasing decision and profits are found in this section for each period at investment firm *m* separately.

4.1 Optimization During Period 1

When demand is favorable (ie. in state F), the optimal purchasing decision during period 1 is

$Z = 1\ if\ e^1_F(m) > e^0_F(m)$otherwise $Z = 0\ if\ e^1_F(m) \leq e^0_F(m)$. The associated profits are then

g_1 (F,m) = $e^1_F(m)$ if Z=1

otherwise g_1 (F,m) = $e^0_F(m)$ if Z=0

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Similarly, when demand is unfavorable (ie. in state U), the optimal purchasing decision during period 1 is
$Z = 1\ if\ e_U^1\ (m) > e_U^0\ (m)$
otherwise $Z = 0\ if\ e_U^1\ (m) \leq e_U^0\ (m)$. In this case, the associated profits are g_1 (U,m)= $e_U^1\ (m)$ if Z=1 otherwise g_1 (U,m) = $e_U^0\ (m)$ if Z=0

4.2 Optimization during period 2

Using (3) and (4) and recalling that a_i^Z (m) denotes the already accumulated profits at the end of period 1 as a result of decisions made during that period, it follows that
$a_i^Z(m) = e_i^Z(m) + Q_{iF}^Z$(m) max$[e_F^1(m), e_F^0(m)]$ $+Q_{iU}^Z$(m) max$[e_U^1(m), e_U^0(m)]$
$a_i^Z(m) = e_i^Z(m) + Q_{iF}^Z(m)g_2(F,m) + Q_{iU}^Z(m)g_2(U,m)$
Therefore, when demand is favorable (ie. in state F). the optimal purchasing decision during period 2 is $Z = 1\ if\ a_F^1\ (m) > a_F^0\ (m)$otherwise $Z = 0\ if\ a_F^1\ (m) \leq a_F^0\ (m)$.. The associated profits are then $g_2(F,m)$= $a_F^1\ (m)$ if Z=1
otherwise $g_2(F,m)$= $a_F^0\ (m)$ if Z=0,
Similarly, when demand is unfavorable (ie. in state U), the optimal purchasing decision during period 2 is
Z= 1 $if\ a_U^1\ (m) > a_U^0\ (m)$otherwise Z=0 if $a_U^1\ (m) \leq a_U^0\ (m)$
$g_2(U,m) = a_U^1\ (m)$ if Z=1
otherwise $g_2(U,m) = a_U^0\ (m)$ if Z=0

5. A CASE STUDY ABOUT FINTECH AND FTS INVESTMENT FIRMS IN UGANDA

In order to demonstrate use of the model in §3-4, a real case application from *Fintech and FTS* investment firms in Uganda are presented in this section. The demand for treasury bills fluctuates every month at the firms. Decision support is sought in terms of an optimal purchasing decision and the associated profits in a two-month planning period.

5.1 Data Collection

Samples of transactions and demand for treasury bills were taken at *Fintech* and *FTS* companies. The state-transitions, transactions, demand and the respective purchasing decisions were examined over six months. The data is presented in Tables 1 – 3.

Table 1: Transactions versus transitions at Investment firms

Investment Firm (m)	States of Demand (F/U)	Purchasing Decision 1 F	U	Purchasing Decision 0 F	U
Fintech (1)	F	91	71	82	30
	U	63	13	55	25
FTS (2)	F	45	59	54	40
	U	59	13	45	11

Table 2: Demand (treasury bills) versus transitions at Investment firms

Investment Firm (m)	States (F/U)	Purchasing Decision 1 F	U	Purchasing Decision 0 F	U
Fintech (1)	F	78	15	61	39
	U	54	11	39	15
FTS (2)	F	43	30	36	38
	U	29	11	38	20

5.2 Computation of Model Parameters

Using (5) and (6), the state-transition matrices and profits (in USD) are calculated for the investment firms

Fintech

$$\mathbf{Q^1(1)} = \begin{bmatrix} \mathbf{0.562} & \mathbf{0.438} \\ \mathbf{0.829} & \mathbf{0.171} \end{bmatrix}$$

$$\mathbf{P^1(1)} = \begin{bmatrix} \mathbf{117} & \mathbf{22.5} \\ \mathbf{81} & \mathbf{16.5} \end{bmatrix}$$

$$\mathbf{Q^0(1)} = \begin{bmatrix} \mathbf{0.732} & \mathbf{0.268} \\ \mathbf{0.688} & \mathbf{0.312} \end{bmatrix}$$

$$\mathbf{P^0(1)} = \begin{bmatrix} \mathbf{91.5} & \mathbf{58.5} \\ \mathbf{58.5} & \mathbf{22.5} \end{bmatrix}$$

FTS

$$\mathbf{Q^1(2)} = \begin{bmatrix} \mathbf{0.433} & \mathbf{0567} \\ \mathbf{0.819} & \mathbf{0.181} \end{bmatrix}$$

$$\mathbf{P^1(2)} = \begin{bmatrix} \mathbf{64.5} & \mathbf{45} \\ \mathbf{43.5} & \mathbf{16.5} \end{bmatrix}$$

$$\mathbf{Q^0(2)} = \begin{bmatrix} \mathbf{0.574} & \mathbf{0.426} \\ \mathbf{0.804} & \mathbf{0.196} \end{bmatrix}$$

$$\mathbf{P^0(1)} = \begin{bmatrix} \mathbf{54} & \mathbf{57} \\ \mathbf{57} & \mathbf{30} \end{bmatrix}$$

Using (2), the expected profits (in USD) are calculated when demand is favorable (state F) or unfavorable (state U) for purchasing decision Z ε {0,1} during month 1. The results are presented in Table 3.

Table 3: Expected profits (in USD) versus state transitions for investment firms

Investment Firm (m)	States of Demand (i)	Expected Profits e^Z_i(m) Z=1	Z=0
Fintech (1)	F	75.6	82.7
	U	70.0	47.3
FTS (2)	F	53.4	55.3
	U	38.6	51.7

Using (4), the accumulated profits (in USD) are similarly calculated for favorable demand (state F) and unfavorable demand (state U) given purchasing decision Z ε {0,1} during month 2. The results are presented in Table 4.

Table 4: Accumulated profits (in USD) versus state transitions for investment firms

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Investment Firm (m)	States of Demand (i)	Accumulated Profits $a^Z_i(m)$ Z=1	Z=0
Fintech (1)	F	152.7	161.9
	U	150.5	126.0
FTS (2)	F	106.7	109.0
	U	93.2	106.2

5.3 *The Optimal Purchasing Decision and Profits for Investment Firms*

Week 1:

At Fintech, since 82.7 >75.6 it follows that Z=0 is an optimal purchasing decision for month 1 with associated expected profits of 82.7 USD for the case of favorable demand. Since 70.0 > 47.3, it follows that Z=1 is an optimal purchasing decision for month 1 with associated expected profits of 70.0 USD for the case of unfavorable demand.

At FTS, since 55.3.>53.4 it follows that Z=0 is an optimal purchasing decision for month 1 with associated expected profits of 55.3 USD for the case of favorable demand. Since 51.7 > 38.6, it follows that Z=0 is an optimal purchasing decision for month 1 with associated expected profits of 51.7 USD for the case of unfavorable demand

Week 2:

At Fintech, since 161.9 > 152.7 it follows that Z=0 is an optimal purchasing decision for month 2 with associated accumulated profits of 161.9 USD for the case of favorable demand. Since 150.5.>126.0, it follows that Z=1 is an optimal purchasing decision for month 2 with associated accumulated profits of 150.5 USD for the case of unfavorable demand.

At FTS, since 109.0.>106.7 it follows that Z=0 is an optimal purchasing decision for month 2 with associated accumulated profits of 109.0 USD for the case of favorable demand. Since 106.3 > 93.2, it follows that Z=0 is an optimal purchasing decision for month 2 with associated accumulated profits of 106.3 USD for the case of unfavorable demand

6. CONCLUSIONS AND DISCUSSION

An optimization model for treasury bill purchasing decisions was presented in this paper. The model determines an optimal purchasing decision and profits for treasury bills under demand uncertainty. The decision of whether or not to purchase additional treasury bills for a given firm is made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal purchasing decisions and profits for the investment firms used in the case study. As a profit maximization strategy for treasury bill purchasing decisions, computational efforts of using markeov decision process provide promising results. However, extending the proposed model is deemed vital in order to analyze the impact of non-stationary demand on purchasing decisions of treasury bills. Special interest is also sought in further extending the model to analyze purchasing decisions for optimal profits in the context of Continuous Time Markov Chains (CTMC).

As noted in the study, profit comparisons were vital in determining the optimal purchasing decision of treasury bills for the firms considered in this study. By the same token, classification of demand as a two-state markov chain facilitated modeling and optimization procedure at the selected investment firms.

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