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Joint Location– Inventory Model for Petrol Stations Under Stochastic Demand

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Abstract*: We consider a joint location inventory replenishment problem involving a chain of petrol stations at designated locations. Associated with each petrol station is stochastic stationary demand; where inventory replenishment periods are uniformly fixed over the fuel stations. Considering inventory positions of the petrol station chain, we formulate a finite state Markov decision process model where states of a Markov chain represent possible states of demand for diesel product. The unit replenishment cost, shortage cost, demand and inventory positions are used to generate the total inventory cost matrix; representing the long run measure of performance for the Markov decision process problem. The problem is to determine for each petrol station at a specific location an optimal replenishment policy so that the long run inventory costs are minimized for the given state of demand. The decisions of replenishing versus not replenishing at a given location are made using dynamic programming over e finite period planning horizon. We test the model using data from two petrol station locations. The model demonstrates the existence of an optimal state-dependent replenishment policy and inventory costs for two selected petrol station locations.*

Keywords—Inventory; joint location, petrol station; replenishment, stochastic

1. INTRODUCTION

In today's fast-paced and competitive market place, business enterprises need every edge available to them to ensure success in planning and managing inventory levels of items with demand uncertainty. In practice, determining the economic order quantity poses a great challenge for inventory managers when the demand for items follows a stochastic trend. In order to cope with current turbulent market trends of petroleum products; optimal guidelines for EOQ levels at petrol stations are paramount. This improves both customer retention and goodwill in business transactions. To achieve this goal, two major problems are usually encountered: (i) Determining the most desirable period during which to replenish additional fuel and (ii) determining the optimal economic order quantity given a periodic review inventory system when demand is uncertain.

In [1], Oded and Larson examined deliveries of an inventory/routing problem using stochastic dynamic programming. In this paper, the amount of needed product at each customer is a known random process, typically a Wiener process. The objective here is to adjust dynamically the amount of product provided on scene to each customer so as to minimize total expected costs.

Cornillier ey al [2] developed an exact algorithm for the petrol station replenishment problem. The algorithm decomposes the problem into a truck loading and routing problem. The authors determine quantities to deliver within a given interval of allocating products to tank truck compartments and of designing delivery routes to stations. In related work by Cornillier et al([3] , a heuristic for the multi-period petrol station replenishment problem was developed. In this article, the objective is to maximize the total profit equal to the revenue minus the sum of routing costs and of regular and overtime costs. Procedures are provided for the route construction, truck loading and route packing enabling anticipation or the postponement of deliveries. Extension of the same problem by Cornillier et al [4] was made by analyzing the petrol station replenishment problem with time windows. In this article, the aim is to optimize the delivery of several petroleum products to a set of petrol stations using limited heterogeneous fleet of trucks by assigning products to truck compartments, delivery routes and schedules.

In related work by You et al [5]., an Inventory-distribution planning system is considered under uncertainty for industrial gas supply chains. The authors propose a stochastic inventory approach. This is incorporated into a multi-period-two-stage stochastic mixed integer nonlinear programming model to handle uncertainty of demand and loss of addition of customers. Popovic et al [6] presented a simulation approach to analyze the deterministic inventory routing problem solution of the stochastic fuel delivery problem. The method is extended to stochastic problems with planning periods of different lengths by analyzing different process performances. Solutions based on deterministic consumption can be applied to stochastic case by balancing emergency deliveries and safety stocks. In [7], the authors examined the EOQ whose analysis showed

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that a higher EOQ is required to run the retailing station optimally. The paper shows that delays in payment, particularly with fully-backlogged shortage are advantageous for setting the accounts of the retail station over without allowable shortage. According to Oliveira & Hamacher [8] , the application of a stochastic benders decomposition algorithm for the problem of supply chain investment planning under uncertainty applied to the petroleum byproducts supply chain ca be analyzed. In this paper, the uncertainty considered is related to the unknown demand levels for oil products. Triki C[9] in addition suggested solution methods for the petrol station replenishment problem over T-day planning horizon. Four heuristic methods are described for its solution, tested and compared on two instances of real-life test problem. Computational results show encouraging improvements with respect to human planning solution. Related work in [10] reveal the relationship between uncertainty and inventory management is examined. Using monthly data, the impact of uncertainty in oil demand and uncertainty in inventory management is relatively small. Interestingly, then result indicates that uncertainty in future price helps to mitigate uncertainty in inventory management and give support to the existence of the future market.

Most of the literature cited provides interesting insights that are crucial in analyzing the petrol station replenishment problem within the context of transportation and logistics framework. However, a new dynamic approach is sought for petrol stations that relate demand uncertainty with customers and inventory positions as a strategy to optimize economic order quantity and inventory costs in a multi-stage decision setting.

In this paper, an inventory model is considered whose goal is to optimize the inventory replenishment policy and total inventory costs associated with holding inventory. At the beginning of each period, a major decision has to be made, namely whether to replenish additional kerosene or not to replenish and keep kerosene at the current inventory position in order to sustain demand. The paper is organized as follows. After describing the mathematical model in §2, consideration is given to the process of estimating model parameters. The model is solved in §3 and applied to a special case study in §4. Some final remarks lastly follow in §5.

3. MODEL DESCRIPTION

We consider an inventory that consists of a chain of petrol stations at designated locations storing petroleum products for various customers. The demand during each time period over a fixed planning horizon at a given location L is described as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a markov chain. Suppose one is interested in determining an optimal course of action, namely to replenish additional units of petrol (a decision denoted by R=1) or not to replenish additional units of petrol (a decision denoted by R=0) during each time period over the planning horizon, where R is a binary decision variable. Optimality is defined such that the minimum inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon at location L. In this paper, a two-location (L=2) and two-period (N=2) planning period are considered.

3.1 Notation

Sets

i,j Set of states of demand
L Set of locations
R Set of replenishment policies

Parameters

D Demand matrix
M Demand transition matrix
O On-hand inventory matrix
V Inventory cost matrix
e Expected inventory cost
a Accumulated inventory
c_r Unit replenishment cost
c_h Unit holding cost
c_s Unit shortage cost
M^R_{ij} Probability that demand changes from state i to state j given replenishment policy R

Others

n,N Stages
C Customer matrix
F Favorable demand
U Unfavorable demand

3.2 Finite-Period Dynamic Programming Model

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal replenishment policy can be expressed as a finite period dynamic programming model. Assuming Z_n(i , L) denotes the optimal expected inventory costs accumulated at the end of periods *n,n+1*,…………***N*** given that the state of the system at the beginning of period *n* is i ε{F,U}.The recursive equation relating Z_n and Z_{n+1} is

$$Z_n(i,L) = min_R[M^R_{iF}(L)]V^R_{iF}(L)+Z_{n+1}(F,L), M^R_{iU}(L)]V^R_{iU}(L)+Z_{n+1}(U,L)]$$

iε{F,U} , L={1,2} , Rε{0,1} n=1,2,…………….N (1)

together with the conditions

$$Z_{N+1}(F,L) = Z_{N+1}(U,L) = 0$$

This recursive relationship may be justified by noting that the cumulative inventory costs $V^R_{ij}(L)$ + $Z_{N+1}(j)$ resulting from reaching state j E{F,U} at the start of period *n+1* from state i E{F,U} at the start of period *n* occurs with probability $M^R_{ij}(L)$

Clearly

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$e^R(L)=[M^R(L)][V^R(L)]^T$,R ε{0,1}, L={1,2} (2) where "T" denotes matrix transposition. Hence, the dynamic programming recursive equations

$$Z_N(i,L) = min_R[e_i^R(L) + M_{iF}^R(L)Z_{N+1}(F,L) + M_{iU}^R(L)Z_{N+1}(U,L)]$$

(3)

i ε{F,U} , R ε {0,1} , L={1,2}

When demand outweighs on-hand inventory, the inventory cost matrix $V^R(L)$ may be computed by means of the relation

$$V^R(L) = (c_r + c_h + c_s)[D^R(L) - O^R(L)]$$

Therefore,

$$V_{ij}^R(L) = \begin{cases} (c_r + c_h + c_s)[D_{ij}^R(L) - O_{ij}^R(L)] & if \;\; D_{ij}^R(L) > O_{ij}^R(L) \\ c_h[O_{ij}^R(L) - D_{ij}^R(L)] & if \;\; D_{ij}^R(L) \le O_{ij}^R(L) \end{cases}$$

for all i,j ε{F,U} , L={1,2} and Rε{0,1} (6)

The justification for expression (6) is that $D^R_{ij}(L) - O^R_{ij}(L)$ units must be replenished to meet excess demand. Otherwise replenishment is cancelled when demand is less than or equal to on-hand inventory.

The following conditions must, however hold:

1. R=1 when $c_r > 0$ and R=0 when $c_r = 0$
2. $c_s > 0$ when shortages are allowed and $c_s = 0$ when shortages are not allowed

4. OPTIMIZATION

The optimal replenishment policy and inventory costs are found in this section for each period at supermarket location L separately.

4.1 Optimization during period 1

When demand is favorable (ie. in state F), the optimal replenishment policy and associated inventory costs during period 1 are

$$R = \begin{cases} 1 & if \;\; e_F^1(L) < e_F^0(L) \\ 0 & if \;\; e_F^1(L) \ge e_F^0(L) \end{cases}$$

$$Z_1(F,L) = \begin{cases} e_F^1(L) & if \;\; R=1 \\ e_F^0(L) & if \;\; R=0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy and associated inventory costs during period 1 are

$$R = \begin{cases} 1 & if \;\; e_U^1(L) < e_U^0(L) \\ 0 & if \;\; e_U^1(L) \ge e_U^0(L) \end{cases}$$

$$Z_1(U,L) = \begin{cases} e_U^1(L) & if \;\; R=1 \\ e_U^0(L) & if \;\; R=0 \end{cases}$$

4.2 Optimization during period 2

Using (3) and (4) and recalling that $a^R_i(L)$ denotes the already accumulated inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^R(L) = e_i^R(L) + M_{iF}^R(L)min[e_F^1(L), e_F^0(L)] + M_{iU}^R(L)min[e_U^1(L), e_U^0(L)]$$

$$a_i^R(L) = e_i^R(L) + M_{iF}^R(L)Z_2(F,L) + M_{iU}^R(L)Z_2(U,L)$$

Therefore, when demand is favorable (ie. in state F), the optimal replenishment policy and associated inventory costs during period 2 are

$$R = \begin{cases} 1 & if \;\; a_F^1(L) < a_F^0(L) \\ 0 & if \;\; a_F^1(L) \ge a_F^0(L) \end{cases}$$

$$Z_2(F,L) = \begin{cases} a_F^1(L) & if \;\; R=1 \\ a_F^0(L) & if \;\; R=0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy and associated inventory costs during period 2 are

$$R = \begin{cases} 1 & if \;\; a_U^1(L) < a_U^0(L) \\ 0 & if \;\; a_U^0(L) \ge a_U^0(L) \end{cases}$$

$$Z_2(U,L) = \begin{cases} a_U^1(L) & if \;\; R=1 \\ a_U^0(L) & if \;\; R=0 \end{cases}$$

5. A CASE STUDY ABOUT TOTAL PETROL STATION LOCATIONS IN UGANDA

In order to demonstrate use of the model in §3-4, a real case application from two *Total petrol station locations* in Uganda are presented in this section. The demand for petrol fluctuates every week at the two locations. The petrol stations want to avoid excess inventory when demand is unfavorable (state U) or running out of stock when demand is favorable (state F) and hence, seek decision support in terms of an optimal replenishment policy and the associated inventory cost of petrol in a two-week planning period.

5.1 Data collection

Samples of customers, demand, and inventory levels were taken for petrol product (in liters) at Total petrol station locations. The state -transitions and the respective replenishment policies were examined over twelve weeks. At either location, the unit replenishment cost(cr) = 4500UGX, unit holding cost(c_h)=1200 UGX and unit shortage cost(c_s) = 300 UGX . The data is presented in the Tables 1-3 below:

Table 1

Customers versus state-transitions at petrol station locations

		Replenishment policy 1		Replenishment policy 0	
Petrol station location	State	F	U	F	U
1	F	182	142	164	60
	U	126	26	100	50
2	F	90	118	108	80
	U	118	26	90	22

Table 2

Demand versus state-transitions at petrol station locations

		Replenishment policy 1		Replenishment policy 0	
Petrol station	State	F	U	F	U

location					
1	F	156	15	190	186
	U	107	11	186	188
2	F	123	78	87	90
	U	78	15	93	91

Table 3
On hand inventory versus state-transitions at petrol station locations

Petrol station location	State	Replenishment policy 1 F	Replenishment policy 1 U	Replenishment policy 0 F	Replenishment policy 0 U
1	F	93	60	290	80
	U	59	11	71	159
2	F	72	77	162	157
	U	75	11	154	157

5.2 Computation of Model Parameters

Using (5) and (6), the state-transition matrices and inventory costs (in million UGX) at each respective location are:

$$M^1(1) = \begin{bmatrix} 0.5617 & 0.4383 \\ 0.8312 & 0.1688 \end{bmatrix} \quad V^1(1) = \begin{bmatrix} 333.5 & 78 \\ 77 & 53 \end{bmatrix}$$

$$M^1(2) = \begin{bmatrix} 0.4327 & 0.5673 \\ 0.8194 & 0.1806 \end{bmatrix} \quad V^1(2) = \begin{bmatrix} 52 & 110 \\ 125.26 & 68.5 \end{bmatrix}$$

for the case when additional units were replenished(R=1) during week 1; while these matrices are

$$M^0(1) = \begin{bmatrix} 0.7322 & 0.2678 \\ 0.6875 & 0.3125 \end{bmatrix} \quad V^0(1) = \begin{bmatrix} 437.26 & 181.5 \\ 173.26 & 39.5 \end{bmatrix}$$

$$M^0(2) = \begin{bmatrix} 0.5745 & 0.4255 \\ 0.8036 & 0.4964 \end{bmatrix} \quad V^0(2) = \begin{bmatrix} 9.0 & 1.5 \\ 4.5 & 67.5 \end{bmatrix}$$

for the case when additional units were not replenished(R=0) during week 1. Using (2) and (3), the expected inventory costs and accumulated inventory costs (in million UGX) are calculated at petrol station locations, states of demand and replenishment policies. Results are summarized in Table 4 below:

Table 4
The Expected and Accumulated Inventory Costs of Petrol Product at Total Petrol Station Locations

Petrol Station Location (L)	State of Demand (i)	Expected Inventory Costs $e^R_i(L)$ R=1	R=0	Accumulated Inventory Costs $a^R_i(L)$ R=1	R=0
1	F	222.64	368.50	381.89	659.2
	U	75.02	121.77	276.24	406.46
2	F	84.90	5.80	225.85	250.70
	U	118.28	16.88	314.89	328.32

5.3 The Optimal Inventory Replenishment Policy at Petrol Station Locations

Week 1:

At location 1, since 222.64<368.50, it follows that R=1 is an optimal replenishment policy for week 1 with associated inventory costs of 222.64 M.UGX for the case of favorable demand. Since 75.02<121.77, it follows that R=1 is an optimal replenishment policy for week 1 with associated inventory costs of 75.02 M.UGX for the case when demand is unfavorable. At location 2, since 5.80<84.90 follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 5.80 M.UGX for the case of favorable demand. Since 16.88<118.28, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 16.88 M.UGX for the case when demand is unfavorable.

Week 2:

At location 1, since 381.8<659.2, it follows that R=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 381.8 M.UGX for the case of favorable demand. Since 276.24<406.46, it follows that R=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 278.24 M.UGX for the case when demand is unfavorable. At location 2, since 225.88<250.70 follows that R=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 225.88 M.UGX for the case of favorable demand. Since 314.80<328.32, it follows that R=1 is an optimal replenishment policy for week 2 with associated inventory costs of 314.89 M.UGX for the case when demand is unfavorable.

6. CONCLUSIONS AND DISCUSSION

A location-inventory model for petrol stations with stochastic demand was presented in this paper. The model determines an optimal replenishment policy and inventory costs of petrol under demand uncertainty. The decision of whether or not to replenish additional petrol at a specific location is made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal replenishment policies and inventory costs for the given problem at each location. As a cost minimization strategy for petrol station location-inventory problems, computational efforts of using Markov decision process approach provide promising results. However, further extensions of the research are vital in order to analyze the impact of non-stationary demand on replenishment policies. In the same spirit, the model developed raises a number of salient issues to consider: lead time of petrol during the replenishment cycle and customer response to abrupt changes in price of petrol at the chosen location. Special interest is also sought in further extending the model by considering replenishment policies for minimum inventory costs in the context of continuous time Markov Chains (CTMC). The model developed is therefore expected to be amenable to some formal, logical or systematic analysis. The stochastic location-inventory model developed is to allow its application in the specific form in order to assist users achieve specific purposes. As noted in the study, inventory cost comparisons were vital in determining the optimal replenishment policy for the two petrol station locations. By the same token, classification of demand as a two-state Markov chain facilitated modeling and optimization of replenishment policies for the chosen locations.

Managerial implications of the study give noteworthy recommendations to petrol station managers that handle random demand for petroleum products. If the petrol station manager keeps too little inventory, he risks loss of sales and customers from stock outs. However, keeping too much inventory will require more space, transportation, handling, labor and capital. Therefore, simplistic inventory stocking policies within a petrol station chain can lead to substantial inefficiencies. Because inventory ties up a lot of capital, the location-inventory model proposed provides a basis for sound inventory policies that govern petrol station locations. Managers can reconsider the flaws in supplier logistics process and minimize stock outs or excessive inventories as a cost minimization strategy.

In light of the current inventory management practice at petrol stations, the demand and inventory are likely to increase in the next five years. Petroleum giants like Shell and Total have established several branches in Uganda with immediate expansion plans. Random demand patterns in such outlets are driven by individual characteristics like income, education and household size. External factors like quality, quantity, variety and convenience of shopping are also crucial.

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