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Applications of Elementary Mathematics in Bond Theory

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Abstract: *In this article we will discuss the elementary mathematical concept in comparing the two of more Projects/Bonds. We know that, a decision is the conclusion of a process by which one chooses between two or more available alternative courses of action for the purpose of attaining a goal(s), which is refer as decision making. So, if a firm or company want to do the investment in the market where there are different options are available, it is always to take a proper decision by the firm before investment the money in the projects, which will give them better return and maximum profit provided the cash flow is given.*

Keywords— Financial Management, Sequence and Series, Geometric series, Present value, Future value, Annuities**.**

1. INTRODUCTION

The Financial Management (FM) is a combination of two words "Finance" and "Management". Finance means to arrange payments for it or in other words Finance means the study of money, its nature, creation and behavior, it is true that one needs money to make money, therefore finance is a lifeblood of business enterprise and there must be a continuous flow of funds in and out of a business enterprise. Money makes the wheels of business run smoothly. Proper planning, efficient production system and excellent marketing network are all hampered in the absence of an adequate and timely supply of funds. Efficient management of business is closely linked with efficient management of its finance. Hence, we conclude that FM is concerned with planning, directing, monitoring, organizing and controlling monetary resources of an organization. The Interest rate is a percentage measure of interest, the cost of money, which accumulates to the lender and compound interest is the addition of interest to the principal sum of a loan or deposit, or, interest on interest. An annuity is a sequence of payments, usually equal in size and made at equal intervals of time. The examples are premiums of life insurance, monthly deposits in bank, instalment loan payments etc. The types of annuities are, Ordinary annuity (Immediate annuity) is an annuity where the first payment of which is made at the end of first payment interval, Annuity Due is annuity where the first payment of which is at the beginning of the first payment intervals. The Time Value of Money play very important roll in FM. We all know that Money has Time Value that is a rupee today is more valuable than a year hence, this means that the value of money is different at different time and the main reasons are: risk and uncertainty, inflation consumption investment opportunity and many more. Thus we can say that time value of money play a vital role in the firm/company for making financial decision. If the timing and risk of cash flow is not consider by the firm/company than the firm will miss its objective of maximization the owner welfare because the primary goal of the firm is to earn profit which is the objective of the FM. The Finance Manager has to make his decisions in such a manner that the profit of the concerned firm are maximized. There are generally two types of techniques for the adjustment of the time value of money they are : Compounding Technique (Future Value Technique) and Discounting Technique (Present Value Technique). In order to find the sum of amount of present value or future value, the mathematical concept called as sequence and series is very much useful. We need the following concept of sequence and series before presenting the Geometric series: If we denote infinite sequence by $\{a_n\}$, then

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + -- + a_n + \cdots \qquad (1)$$

is called an infinite series.

Notes:

1. The n^{th} partial sum of infinite series (1) is denoted by S_n and given by

$$S_n = a_1 + a_2 + a_3 + -- + a_n.$$

2. If the sequence of partial sums $\{S_n\}$ converges to S, then the series $\sum_{n=1}^{\infty} a_n$ converges and limit S is the sum of the series and can be written as:

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} (a_1 + a_2 + \cdots + a_n) = \sum_{n=1}^{\infty} a_n.$$

3. If the sequence $\{S_n\}$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

We will now define Geometric series and its sum as follow: Consider the first term, second term, third term and the nth term as : a , ar , ar^2, − − − − − − − − , ar^{n-1}, where a is the first term and r is the common ratio. The Geometric series is given by:

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + - - - - - - - - + ar^{n-1} + .. (2)$$

Notes:

1. The geometric series converges if and only if $|r| < 1$.

When $|r| < 1$, the sum of the Geometric series (2) (the value the series converges to) is given by

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad (3)$$

If $|r| \geq 1$, then the geometric series diverges.

2. The ratio r is the factor you multiply the previous term by to get the next one. That is,

$$r = \frac{n^{th}\text{ term}}{(n+1)^{th}\text{ term}}$$

During the duration of an investment, the value of an investment can vary in function of time. The study of an investment at different dates produces a sequence of values. For example, the application of geometric sequences is found in bank transactions (loans, investments, recurring deposit). For example, a person open a saving account in a bank and deposits an amount of 1000 $ at the bank. The bank offers this person an annual return of 6 % on his investment, If the person leaves the interest in the account, the annual evolution of the investment is given in the following table:

Years	Deposits	Interest	Balances
0-year	**1000**	**0**	**a_0 =1000**
1-year	**0**	**1000 x 0.06=60**	**a_1 =1060**
2-years	**0**	**1060 x 0.06= 63.60**	**a_2 =1123.60**
3-years	**0**	**1123.60 x 0.06=67.42**	**a_3 =1191.02**
4-years	**0**	**1191.02 x 0.06=71.46**	**a_4 =1262.48**

The above evolution of the investment form a geometric sequence as follows:

$$a_0 = 1000$$
$$a_1 = 1000 + 0.06 \times 1000 = 1060$$
$$a_2 = 1060 + 0.06 \times 1060 = 1123.60$$
$$a_3 = 1123.60 + 0.06 \times 1123.60 = 1191.016$$
$$a_4 = 1191.016 + 0.06 \times 1191.016 = 1262.47696$$

In general,

$$a_n = a_{n-1} + 0.06a_{n-1} = 1.06a_{n-1,}\ n \geq 1.$$

$$r = \frac{n^{th}\text{ term}}{(n+1)^{th}\text{ term}} = \frac{1.06a_n}{a_n} = 1.06.$$

Thus, the sequence of accumulated values of the investment form a geometric sequence with common ratio 1.06.

Let us consider an investment situation in which there are two times "now" and "later" , symbolized by times t_0 and t_1. An amount of money $A_0 = A(t_0)$ is invested at time t_0 and its value at time t_1 is $A_1 = A(t_1)$ .In general we will use a

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function $A(t)$ to represent the total accumulated value of an investment at time. The present and future value is given by the formula $A(t_0) = \frac{A(t_1)}{1+r}$ and $A(t_1) = (1+r)A(t_0)$. We will now in position to state the theorem by which we can compare the two projects and decide which one is batter as follow:

Theorem 1 : Let $A_1, A_2, A_3, ---, A_n$ and $B_1, B_2, B_3, ---, B_n$ be two cash flow sequences. Let r be a positive interest rate if $A_k \geq B_k$, (for $k = 1,2,3 \ldots \ldots n$) , then the present value of the cash flow $A_1, A_2, A_3, ---, A_n$ is atleast as large as that of the cash flow sequence $B_1, B_2, B_3, ---, B_n$.

Proof :

The present value of the cash flow of the sequence $A_1, A_2, A_3, ---, A_n$ is given by:

$$\frac{A_1}{1+r} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} ---- + \frac{A_n}{(1+r)^n} \qquad (4)$$

and present value of cash flow of the sequence $B_1, B_2, B_3, ---, B_n$ is given by :

$$\frac{B_1}{1+r} + \frac{B_2}{(1+r)^2} + \frac{B_3}{(1+r)^3} + --- + \frac{B_n}{(1+r)^n} \qquad (5)$$

Since we are given that $A_k \geq B_k$, (for all $k = 1,2,3 \ldots \ldots n$). This means that

$$A_1 \geq B_1, A_2 \geq B_2, \ldots A_n \geq B_n \qquad (6)$$

Therefore for > 0 , we have

$$\frac{A_1}{(1+r)^1} \geq \frac{B_1}{(1+r)^1}, \frac{A_2}{(1+r)^2} \geq \frac{B_2}{(1+r)^2}, \ldots \frac{A_n}{(1+r)^n} \geq \frac{B_n}{(1+r)^n} \qquad (7)$$

Hence from for (6) and (7) with positive r and ($i = 1,2, \ldots, n$) we have,

$$\frac{A_1}{(1+r)^i} \geq \frac{B_1}{(1+r)^i} \Rightarrow \sum_{i=1}^{n} \frac{A_i}{(1+r)^i} \geq \sum_{i=1}^{n} \frac{B_i}{(1+r)^i} \qquad (8)$$

Thus the present value of sequence $A_1, A_2, A_3 --- A_n$ is large than that of sequence $B_1, B_2, B_3 --- B_n$, which proves the theorem 1 .

The above Theorem 1 can be applied to compare the Projects/Bonds whose cash flows are given in form of sequence. It should be noted that, this is not the general method for accepting or rejecting the Projects/Bond, Theorem 1 is applicable only when the conditions of the theorem is satisfied. We have consider two applications which is self-created of this Theorem1 which we have put in the form of applications as follows:

Application 1 : Suppose a Firm want to invest money in the projects. The following projects are available in the markets : The return cash flow sequence for the Projects which give in first, second, third , fourth and fifth years (in thousand) are given in the following table below:

Project	First Year	Second Year	Third Year	Fourth Year	Fifth Year
Project 1	35	45	55	65	75
Project 2	34	44	54	64	74
Project 3	34	42	25	40	60

The company want to invest in one of these Project. Which Project will Firm preferred ?. If the nominal rate of interest is 10% .

Proof:

By Theorem 1, if we take

$A_1 = 35, A_2 = 45, A_3 = 55, A_4 = 65, A_5 = 75$

$B_1 = 34, B_2 = 44, B_3 = 54, B_4 = 64, B_5 = 74$

$C_1 = 34, C_2 = 42, C_3 = 25, C_4 = 40, C_5 = 60,$ then the condition of Theorem1 is satisfied. That is

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$$\sum_{i=1}^{n}\frac{A_i}{(1+r)^i} \geq \sum_{i=}^{n}\frac{B_i}{(1+r)^i} \geq \sum_{i=}^{n}\frac{C_i}{(1+r)^i} \quad , \quad i = 1,2,3,4,5.$$

Hence by Theorem 1, Project 1 is acceptable by the Firm. Also, by Financial Management technique it should be clear that, the cash flow sequence with largest present value will be preferred/acceptable. We will compute the present value of the above cash flow as follows:

The Present Value of Project 1 is

$$= \frac{35}{1+r} + \frac{45}{(1+r)^2} + \frac{55}{(1+r)^3} + \frac{65}{(1+r)^4} + \frac{75}{(1+r)^5}$$

$$= \frac{35}{1+0.1} + \frac{45}{(1+0.1)^2} + \frac{55}{(1+0.1)^3} + \frac{65}{(1+0.1)^4} + \frac{75}{(1+0.1)^5}$$

= 35x0.9091+45x0.8264+55x0.7513+65x0.6830+75x0.6209

= ⍰201.2905

The Present Value of Project 2 is

$$= \frac{34}{1+r} + \frac{44}{(1+r)^2} + \frac{54}{(1+r)^3} + \frac{64}{(1+r)^4} + \frac{74}{(1+r)^5}$$

$$= \frac{34}{1+0.1} + \frac{44}{(1+0.1)^2} + \frac{54}{(1+0.1)^3} + \frac{64}{(1+0.1)^4} + \frac{74}{(1+0.1)^5}$$

= 34x0.9091+44x0.8264+54x0.7513+64x0.6830+74x0.6209

= ⍰197.4998

The Present Value of Project 3 is

$$= \frac{34}{1+r} + \frac{42}{(1+r)^2} + \frac{25}{(1+r)^3} + \frac{40}{(1+r)^4} + \frac{60}{(1+r)^5}$$

$$= \frac{34}{1+0.1} + \frac{42}{(1+0.1)^2} + \frac{25}{(1+0.1)^3} + \frac{40}{(1+0.1)^4} + \frac{60}{(1+0.1)^5}$$

= 34x0.9091+42x0.8264+25x0.7513+40x0.6830+60x0.6209

= ⍰148.9747

Conclusion 1: As the conditions of Theorem1 is satisfied, so we directly say without calculation that Project 1 will be accepted by the Firm. Also by Financial Management, the present value of Project 1 is the largest so Project 1 is acceptable.

Application 2 : Suppose a Firm want to invest money in the Bonds.. The following Bonds are available in the markets. The return that the Bond will give in first, second, third, fourth and fifth years (in thousand) are given by the following table:

Bonds	First Year	Second Year	Third Year	Fourth Year	Fifth Year
Bond A	15	25	35	45	55
Bond B	20	16	14	12	10
Bond C	25	25	25	25	25

The Firm want to invest in one of these Bond. Which Bond will Firm preferred ?. If the nominal rate of interest is 10%.

Proof:

By Theorem 1 , if we take

$A_1 = 15 , A_2 = 25 , A_3 = 35 , A_4 = 45 , A_5 = 55$

$B_1 = 20 , B_2 = 16, B_3 = 14, B_4 = 12, B_5 = 25$

$C_1 = 25, C_2 = 25, C_3 = 25, C_4 = 25, C_5 = 25,$ then the condition of Theorem1 is not satisfied. That is

$$\sum_{i=1}^{n}\frac{A_i}{(1+r)^i} \ngeq \sum_{i=}^{n}\frac{B_i}{(1+r)^i} \leq \sum_{i=}^{n}\frac{C_i}{(1+r)^i} \quad , \quad i = 1,2,3,4,5.$$

Hence by Theorem 1, we cannot take any decision about the Bond. Now we calculate the Present values of each Bond as follows:

The Present Value of Bond A is

$$= \frac{15}{1+r} + \frac{25}{(1+r)^2} + \frac{35}{(1+r)^3} + \frac{45}{(1+r)^4} + \frac{55}{(1+r)^5}$$

$$= \frac{15}{1+0.1} + \frac{25}{(1+0.1)^2} + \frac{35}{(1+0.1)^3} + \frac{45}{(1+0.1)^4} + \frac{55}{(1+0.1)^5}$$

= 15x0.9091+25x0.8264+35x0.7513+45x0.6830+55x0.6209

= ⍰125.4765.

The Present Value of Bond B is

$$= \frac{20}{1+r} + \frac{16}{(1+r)^2} + \frac{14}{(1+r)^3} + \frac{12}{(1+r)^4} + \frac{10}{(1+r)^5}$$

$$= \frac{20}{1+0.1} + \frac{16}{(1+0.1)^2} + \frac{14}{(1+0.1)^3} + \frac{12}{(1+0.1)^4} + \frac{10}{(1+0.1)^5}$$

= 20x0.9091+16x0.8264+14x0.7513+12x0.6830+10x0.6209

= ⍰56.3276

The Present Value of Bond C is

$$= \frac{25}{1+r} + \frac{25}{(1+r)^2} + \frac{25}{(1+r)^3} + \frac{25}{(1+r)^4} + \frac{25}{(1+r)^5}$$

By using the Geometric series, we have

$$= \frac{25}{1+r}\left\{\frac{(1+r)^4 - 1}{1+r-1}\right\} = \frac{25}{1.1}\left\{\frac{(1.1)^4 - 1}{0.1}\right\} = ⍰105.4773$$

Conclusion 2: By Financial Management technique Bond A is accept by the Firm as its Present value is the largest. By Theorem 1 we can compare Bond B and Bond C as it satisfied the condition of Theorem 1. That is

$$\sum_{i=}^{n} \frac{B_i}{(1+r)^i} \leq \sum_{i=}^{n} \frac{C_i}{(1+r)^i} \quad , \quad i = 1,2,3,4,5.$$

and hence by Theorem 1 Bond C is accepted, Now the Present value of Bond A is larger than Bond C and hence the Firm will invest the money in Bond A.

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