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Stochastic Joint Replenishment Model For The Petroleum Supply Chain

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***Abstract**: In this paper, a stochastic two-echelon supply chain model is proposed for petroleum products; consisting of a fuel depot, petrol stations and end customers. The petrol stations face stochastic stationary demand; where inventory replenishment periods are uniformly fixed over the echelons. A finite state Markov decision model is formulated where states of a Markov chain represent possible states of demand for kerosene and diesel products. The inventory cost matrix is determined for each petroleum product by multiplying the unit (replenishment, holding and shortage costs) by the demand and inventory positions of products. The objective is to determine over each echelon of the planning horizon, an optimal replenishment policy so that the long run inventory costs are minimized for a given state of demand. Using weekly equal intervals, the decisions of whether to replenish or not replenish additional fuel are made using dynamic programming over a finite period planning horizon. Furthermore, use of the model is tested on a real case extracted from Oilcom fuel depot with petrol stations in Kampala, Uganda. The model demonstrates the existence of an optimal state-dependent replenishment policy and inventory costs for kerosene and diesel products in managing the fuel supply chain*.

Keywords—Joint replenishment, petroleum, supply chain, stochastic demand

1. INTRODUCTION

Supply chain networks demand concerted efforts to procure raw materials, transform materials into intermediate goods and then to final products. The final product must be delivered to customers through a distribution system that include an echelon inventory system. In the petroleum inventory-distribution system, optimal replenishment policies of products are paramount for increased sales revenue, lower inventory costs and improved cash flows. This is a critical issue that requires vigilant attention and daily monitoring in order to maintain business continuity. Despite the fluctuating demand patterns of petroleum products, optimal replenishment policies are vital otherwise high inventory costs in maintaining stock levels may affect company performance. On the other hand, running out of stock is a hindrance to customer retention and goodwill in business transactions of petroleum products. Inventory management of petroleum products is usually affected by one major challenge: determining the most desirable period during which to replenish additional units of the petroleum product in question given a periodic review inventory system under demand uncertainty. In this paper, a two-echelon inventory system is considered whose goal is to optimize replenishment policies and inventory costs associated with kerosene and diesel products. At the beginning of each period, a major decision has to be made, namely whether to replenish additional units of kerosene and diesel or not to replenish and utilize the prevailing inventory positions of petroleum products in order to sustain demand at a given echelon. The paper is organized as follows. After reviewing the relevant literature, a mathematical model is described where consideration is given to the process of estimating model parameters. The model is solved and applied to a special case study. Some final remarks finally follow.

2. LITERATUE REVIEW

According to Rodney and Roman [1] , the optimal policies can be examined in the context of a capacitated two-echelon inventory system. This model includes installations with production capacity limits, and demonstrates that a modified base stock policy is optimal in a two-stage system when there is a smaller capacity at the downstream facility. This is shown by decomposing the dynamic programming value function into value functions dependent upon individual echelon stock variables. The optimal structure holds for both stationary and non-stationary customer demand. In [2] Axsater formulated a simple decision rule for decentralized two-echelon inventory control. A two-echelon distribution inventory system with a central warehouse and a number of retailers is considered. The retailers face stochastic demand and the system is controlled by continuous review installation stock policies with given batch quantities. A back-order cost is provided to the warehouse and the warehouse chooses the reorder point so that the sum of the expected holding and backorder costs are minimized. Given the resulting warehouse policy, the retailers similarly optimize their costs with respect to the reorder points. The study provides a simple technique for determining the backorder cost to be used by the warehouse. In [3], a simplified two-echelon supply chain system with one supplier and one retailer that choose different replenishment policies is considered. The bullwhip effect causes excessive inventory due to information distortion where the order amount is exaggerated while a minor demand variation occurs. This information is amplified as the supply chain moves to the upstream. Cornillier et al [4] developed an exact algorithm for the petrol station replenishment problem. The algorithm decomposes the problem into a truck loading and routing problem. The authors determine quantities to deliver within a given interval of allocating products to tank truck compartments and of designing delivery routes to stations. In related work of Cornillier et al [5] , a heuristic for the multi-period petrol station replenishment problem was developed. In this article, the objective is to maximize the total profit equal to the revenue minus the sum of routing costs and of regular and overtime costs. Procedures are provided for the route construction, truck loading and route packing enabling anticipation or the postponement of deliveries. Extension of the solution procedure was made in [6] where authors analyzed the petrol station replenishment problem with time windows. In this article, the aim is to optimize the delivery of several petroleum products to a set of petrol stations using limited heterogeneous fleet of trucks by assigning products to truck compartments, delivery routes and schedules. Results in [7] revealed that the variability in planning a supply schedule as it renders the process of determining a supply decision as ineffective. Results of the paper indicate that the uncertainty level is too high to be ignored and the ability to create plans that cover the company's vision are decreased. Haji [8] considered a two-echelon inventory system consisting of one central warehouse and a number of non-identical retailers. The warehouse uses a one-for-one policy to replenish its inventory, but the retailers apply a new policy that is each retailer orders one unit to central warehouse in a predetermined time interval; thus, retailer orders are deterministic not random. In [9], the authors considered a vendor managed Two-Echelon inventory system for an integrated production procurement case. Joint economic lot size models are presented for the two supply situations, namely staggered supply and uniform supply. Cases are employed that describe the inventory situation of a single vendor supplying an item to a manufacturer that is further processed before it is supplied to the end user. Using illustrative examples, the comparative advantages of a uniform sub batch supply over a staggered alternative are investigated and uniform supply models are found to be comparatively more beneficial and robust than the staggered sub batch supply. In [10], the problem of optimizing the investment planning process of logistics infrastructure for the distribution of petroleum products under uncertainty is addressed. A two-stage stochastic model is developed using the Sample Average Approximation (SAA) method to produce approximations of optimal solution. In [11], Awudu and Zhang proposed a stochastic production model for biofuel supply chain under demand and price uncertainties. Using stochastic linear programming, a model is proposed within a single period planning framework to maximize the expected profits. Related work in [12] explores the optimal design of supply chain biofuel market price, feedstock yields, and logistic costs. A two-stage stochastic programming model was developed in which conditional value at risk is utilized as a risk measure to control the amount of shortage in demand zones.

Tonjg et al [13] addressed the optimal design and strategic planning of the integrated biofuel and petroleum supply chain system in the presence of pricing and quantity uncertainties. To achieve a higher modelling resolution and improve the overall economic performance, the model explicitly considers equipment units and material streams and proposes a multi-planning model to coordinate the various activities in the petroleum refineries. Liu et al [14] addressed management of risks in Chinese chemical supply chain. The paper defines the supply chain risk management problem within the context of a Chinese chemical industry. A stochastic chance-constrained programming model is developed to select optimal mitigation strategies to effectively achieve risk reduction goals without exceeding the excess budget. Mubiru [15] proposed a mathematical model to optimize replenishment policies and inventory costs of a two-echelon supply chain system for kerosene product under demand uncertainty. The system consists of a fuel depot and petrol stations under stochastic stationary demand. Optimal replenishment policies and inventory costs of kerosene product are determined using Markov decision process approach.

2.1 The Stochastic Joint Replenishment Model versus existing Fuel Supply Chain Models

The literature cited provides profound insights by authors that are crucial in analyzing two-echelon inventory systems. Existing models that address the fuel supply chain problem are similarly presented. As noted by Cornillier et al [4],[5],[6], the three models address the petrol station replenishment problem using transportation and logistics perspectives. The source (depot) is not vividly known and the overall goal is to minimize transportation costs of petroleum products. Randomness of demand is not a salient issue or not discussed at all. However, demand uncertainty has a direct bearing in answering the inventory question of "when to deliver or replenish" at minimum inventory costs. A closer look at findings in [10], depict analysis from an investment planning perspective before product distribution. By the same token, the study in [11] considers demand and price uncertainties during the production cycle using profits as a chosen measure of effectiveness. Comparatively, profound insights in terms of randomness of demand, logistic cost and shortages in demand zones which have to be filled are well explained in [12], where authors tackle the problem as an inventory-distribution problem. As earlier noted by Mubiru [15], a single product (kerosene) was considered in this paper.

Based on the existing models by scholars, a new stochastic dynamic programming approach is sought in order to relate state-transitions with customers, demand and inventory

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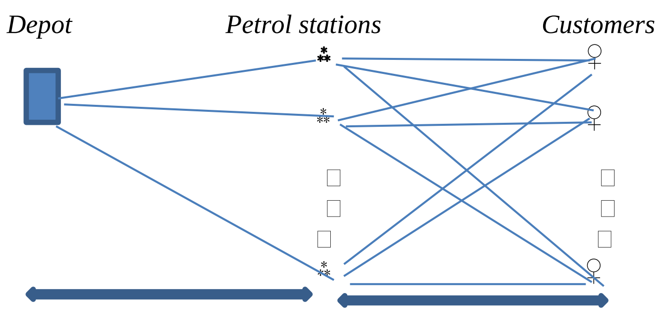
positions of the products over the echelons. This is done with a view of optimizing replenishment policies and inventory costs of the supply chain in a multi-stage decision setting. The stochastic fuel supply chain model developed incorporates demand uncertainty in determining optimal replenishment decisions where "shortage" or "no shortage" conditions are catered for when calculating total inventory costs over the echelons. The model can assist inventory managers of petroleum products in answering the question of "when to replenish" at minimum costs under demand uncertainty. Petrol stations within a supply chain framework that share a common fuel depot can consider adopting the proposed model as a cost minimization strategy.

3.MODEL DESCRIPTION

We consider a two-echelon inventory system consisting of a fuel depot storing kerosene and diesel products for a designated number of petrol stations at echelon 1. At echelon 2; customer demand is realized for the two petroleum products at petrol stations. A general schematic diagram of a two-echelon fuel supply chain network is presented below:

Figure 1
General Model Schematic Diagram of a Two-Echelon Fuel Supply Chain Network\



Echelon 1[h=1] Echelon 2[h=2]

The demand during each time period over a fixed planning horizon for a given echelon (h) is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to replenish additional units of kerosene or diesel (a decision denoted by R=1) or not to replenish additional units of kerosene (a decision denoted by R=0) during each time period over the planning horizon, where R is a binary decision variable. Optimality is defined such that the minimum inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-echelon (h =2), two product (m=2) and two-period (N=2) planning horizon are considered.

3.1 Notation

Sets

i,j States of demand

h Set of echelon

R Set of replenishment policies

Parameters

D Demand matrix
Q Demand transition matrix
O On hand inventory matrix
C Inventory cost matrix
e Expected inventory costs
a Accumulated inventory costs
c_r Unit replenishment cost
c_h Unit holding cost
c_s Unit shortage cost
Q^R_{ij} Probability that demand changes from state i to state j, given replenishment policy R

Others

F Favorable demand
U Unfavorable demand
n,N Stages
m Petroleum product
S Customer matrix

i,j ε {F,U} h ε {1,2} R ε {0,1} m ε {1,2} n=1,2,…………………….N

3.2 Finite - period dynamic programming problem formulation

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal replenishment policy may be expressed as a finite period dynamic programming model.

Let g_n(i,h,m) the optimal expected inventory costs of product *m* accumulated during the periods *n,n+1,…**N*** over echelon *h* given that the state of the system at the beginning of period *n* is *i*ϵ{ F, U }.The recursive equation relating gn and $gn+1$ is

$$g_n(i,h,m) = min_R\,[Q^R_{iF}(h,m)\,C^R_{iF}(h,m)\,g_{n+1}(F,h,m) + min_R\,[Q^R_{iU}(h,m)\,C^R_{iU}(h,m) + g_{n+1}(U,h,m) \quad (1)$$

together with the conditions

$$g_{N+1}(F,h,m) = g_{N+1}(U,h,m) = 0$$

This recursive relationship may be justified by noting that the cumulative inventory costs

C_{ij} (h,m)+ g_{N+1} (j,h,m) resulting from reaching state j ϵ{F, U} at the start of period n+1 from state i ϵ{ F, U } at the start of period *n* occurs with probability Q^R_{ij} (h,m).

Clearly, e^Z(h,m) = [Q^R(h,m)] [C^R(h,m)]^T

Where "T" denotes matrix transposition, and hence the dynamic programming recursive equations

$$g_n(i,h,m) = \min_R\,[e_i^R(h,m)] + \min_R[Q^R_{iF}(h,m)\,g_{n+1}(F,m)] + \min_R[Q^R_{iU}(h,m)\,g_{n+1}(U,m)] \quad (3)$$

$$g_N(i,h,m) = min_R[e_i^R(h,m)] \quad (4)$$

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result where (4) represents the Markov chain stable state.

3.2.1 Computing $Q^R(h,m)$ and $C^R(h,m)$

The demand transition probability from state iϵ{ F, U } to state j ϵ{ F, U },given replenishment policy R ϵ{ 0,1 } may be taken as the number of product *m* customers observed over echelon *h* with demand initially in state i and later with demand changing to state j, divided by the sum of customers over all states. That is,

$$Q^R_{ij}(h,m) = S^R_{ij}(h,m)/[S^R_{iF}(h,m) + S^R_{iU}(h,m)] \qquad (5)$$

i, j ε {F,U}   h ε {1,2}   R ε {0,1}   m ε {1,2}

When demand outweighs on-hand inventory, the inventory cost matrix $C^R(h,m)$ may be computed by means of the relation

$$C^R(h,m) = [c_r(m) + c_h(m) + c_s(m)][D^R(h,m) - O^R(h,m$$

Therefore,

$$C^R_{ij}(h,m) = \begin{cases} [c_r(m) + c_h(m) + c_s(m)][D^R_{ij}(h,m) - O^R_{ij}(h,m)] & if \quad D^R_{ij}(h,m) > O^R_{ij}(h \\ c_h(m)[D^R_{ij}(h,m) - O^R_{ij}(h,m)] & if \quad D^R_{ij}(h,m) \leq O^R_{ij}(h,m) \end{cases} \qquad (6)$$

i, j ε {F,U}   h ε {1,2}   R ε {0,1}   m ε {1,2}

The justification for expression (6) is that $D^R_{ij}(h,m)$ – $O^R_{ij}(h,m)$ units must be replenished to meet excess demand. Otherwise replenishment is cancelled when demand is less than or equal to on-hand inventory.

The following conditions must however hold:

i) R=1 when $c_r(m) > 0$ and R=0 when $c_r(m) = 0$

ii) $c_r(m) > 0$ when shortages are allowed and $c_r(m) = 0$ when shortages are not allowed

4. OPTIMIZATION

The optimal replenishment policy and inventory costs are found in this section for product *m* during each period over echelon *h* separately

4.1 Optimization during period 1

When demand is favorable (ie. in state F), the optimal replenishment policy during period 1 is

$$R = \begin{cases} 1 & if \quad e^1_F(h,m) < e^0_F(h,m) \\ 0 & if \quad e^1_F(h,m) \geq e^0_F(h,m) \end{cases}$$

The associated inventory costs are then

$$g_1(F,h,m) = \begin{cases} e^1_F(h,m) & if \quad R = 1 \\ e^0_F(h,m) & if \quad R = 0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy during period 1 is

$$R = \begin{cases} 1 & if \quad e^1_U(h,m) < e^0_U(h,m) \\ 0 & if \quad e^1_U(h,m) \geq e^0_U(h,m) \end{cases}$$

In this case, the associated inventory costs are

$$g_1(U,h,m) = \begin{cases} e^1_U(h,m) & if \quad R = 1 \\ e^0_U(h,m) & if \quad R = 0 \end{cases}$$

4.2 Optimization during period 2

Using (2) ,(3) and recalling that $a^R_i(h,m)$ denotes the already accumulated inventory costs of product *m* at the end of period 1 over echelon *h* as a result of decisions made during that period, it follows that

$$a^R_i(h,m) = e^R_i(h,m) + Q^R_{iF(}(h,m)\min[e^1_F(h,m), e^0_F(h,m)] + Q^R_{iU(}(h,m)\min[e^1_U(h,m), e^0_U(h,m)]$$

Therefore, when demand is favorable (ie. in state F), the optimal replenishment policy during period 2 is

$$R = \begin{cases} 1 & if \quad a^1_F(h,m) < a^0_F(h,m) \\ 0 & if \quad a^1_F(h,m) \geq a^0_F(h,m) \end{cases}$$

while the associated inventory costs are

$$g_2(F,h,m) = \begin{cases} a^1_F(h,m) & if \quad R = 1 \\ a^0_F(h,m) & if \quad R = 0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy during period 2 is

$$R = \begin{cases} 1 & if \quad a^1_U(h,m) < a^0_U(h,m) \\ 0 & if \quad a^1_U(h,m) \geq a^0_U(h,m) \end{cases}$$

In this case, the associated inventory costs are

$$g_2(U,h,m) = \begin{cases} a^1_U(h,m) & if \quad R = 1 \\ a^0_U(h,m) & if \quad R = 0 \end{cases}$$

5.A CASE STUDY ABOUT KEROSENE AND DIESEL PRODUCTS AT OILCOM SUPPLY CHAIN

In order to demonstrate use of the model in §3-4, real case applications from *Oilcom*, a fuel company for kerosene and diesel products including four petrol stations are presented in this section. Oilcom fuel depot supplies kerosene and diesel at petrol stations (echelon 1) while end customers come to petrol stations for the two petroleum products (echelon 2). The demand for kerosene and diesel fluctuates every week at both echelons. The fuel depot and petrol stations want to avoid excess inventory when demand is unfavorable (state U) or running out of stock when demand is Favorable (state F) and hence seek decision support in terms of an optimal replenishment policy and the associated inventory costs of kerosene and diesel in a two-week planning period

5.1 Data collection

Samples of customers, demand and inventory levels at state-transitions were taken for kerosene and diesel products (in thousand liters). This was done over the echelons for the respective replenishment policies in twelve weeks. The unit inventory costs (in US\$) were similarly determined for the two petroleum products. The data is presented in the Tables below:

Table 1

Unit inventory costs (in US$) of Kerosene and Diesel

Petroleum Product (m)	Unit replenishment Cost (c_r)	Unit holding cost (c_h)	Unit shortage cost (c_s)
Kerosene	1.00	0.75	0.50

Diesel	1.50	0.75	0.50

Table 2

Customers versus state-transitions at supply chain echelons for Kerosene Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	101	81	92	60
	U	74	23	66	45
2	F	58	65	64	56
	U	69	23	55	21

Table 3

Customers versus state-transitions at supply chain echelons for Diesel Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	67	72	46	63
	U	72	19	66	19
2	F	81	61	72	40
	U	54	13	46	25

Table 4

Demand versus state-transitions at supply chain echelons for Kerosene Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	166	125	133	88
	U	117	21	80	25
2	F	83	50	62	67
	U	49	21	65	13

Table 5

Demand versus state-transitions at supply chain echelons for Diesel Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	70	80	41	60
	U	75	10	62	10
2	F	146	105	113	68
	U	97	15	60	30

Table 6

On hand Inventory versus state-transitions at supply chain echelons for Kerosene Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	105	103	53	55
	U	103	104	56	58
2	F	135	150	71	68
	U	68	70	69	66

Table 7

On hand Inventory versus state-transitions at supply chain echelons for Diesel Product

		Replenishment policy 1		Replenishment policy 0	
Echelon	State	F	U	F	U
1	F	52	58	95	90
	U	62	58	90	40
2	F	85	83	43	45
	U	83	70	40	38

5.2 Computation of Model Parameters

Using (5) and (6), the demand transition matrices and inventory costs (in million UGX) of kerosene and diesel over echelons 1 and 2 during week 1 are as follows:

Demand Transition Matrices of Petroleum Products

Kerosene

$$Q^1(1,1) = \begin{bmatrix} 0.5549 & 0.4951 \\ 0.7628 & 0.2372 \end{bmatrix} \quad Q^0(1,1) = \begin{bmatrix} 0.6053 & 0.3047 \\ 0.5946 & 0.4054 \end{bmatrix}$$

$$Q^1(2,1) = \begin{bmatrix} 0.4820 & 0.5180 \\ 0.7500 & 0.2500 \end{bmatrix} \quad Q^0(2,1) = \begin{bmatrix} 0.5333 & 0.4667 \\ 0.7237 & 0.2763 \end{bmatrix}$$

Diesel

$$Q^1(1,2) = \begin{bmatrix} 0.4820 & 0.5180 \\ 0.7912 & 0.2088 \end{bmatrix} \quad Q^0(1,2) = \begin{bmatrix} 0.4220 & 0.5780 \\ 0.7765 & 0.2235 \end{bmatrix}$$

$$Q^1(2,2) = \begin{bmatrix} 0.5704 & 0.4296 \\ 0.8060 & 0.1940 \end{bmatrix} \quad Q^0(2,2) = \begin{bmatrix} 0.6429 & 0.3571 \\ 0.6479 & 0.3521 \end{bmatrix}$$

Total Inventory Cost Matrices of Petroleum Products

Using the relation in (6), the following total inventory cost matrices (in hundred USD) are obtained for each product

Kerosene

$$C^1(1,1) = \begin{bmatrix} 137.75 & 49.5 \\ 31.5 & 62.25 \end{bmatrix} \quad C^0(1,1) = \begin{bmatrix} 87.75 & 11.25 \\ 22.5 & 9.75 \end{bmatrix}$$

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$$C^1(2,1) = \begin{bmatrix} 39 & 69 \\ 14.25 & 36.75 \end{bmatrix} \quad C^0(2,1) = \begin{bmatrix} 6.75 & 0.75 \\ 3.00 & 39.75 \end{bmatrix}$$

Diesel

$$C^1(1,2) = \begin{bmatrix} 35.75 & 60.5 \\ 35.75 & 36 \end{bmatrix} \quad C^0(1,2) = \begin{bmatrix} 36.75 & 60.5 \\ 35.75 & 36 \end{bmatrix}$$

$$C^1(2,2) = \begin{bmatrix} 167.75 & 60.5 \\ 38.5 & 41.25 \end{bmatrix} \quad C^0(2,2) = \begin{bmatrix} 192.5 & 63.25 \\ 55 & 6 \end{bmatrix}$$

Using (2) and (3), the expected inventory costs and accumulated inventory costs (in hundred USD) are calculated over the echelons for the respective replenishment policies. Results are summarized in Tables 8-11 below:

Table 8
Expected inventory costs of kerosene product

Inventory Echelon (h)	State of Demand (i)	Expected Inventory Costs	
		R=1	R=0
1	F U	98.47 38.79	57.56 17.33
2	F U	50.10 19.88	13.95 13.15

Table 9
Expected inventory costs of Diesel product

Inventory Echelon (h)	State of Demand (i)	Expected Inventory Costs	
		R=1	R=0
1	F U	48,57 35.80	50.48 35.21
2	F U	121.68 39.03	146.34 37.75

Table 10
Accumulated inventory costs of Kerosene product

Inventory Echelon (h)	State of Demand (i)	Accumulated Inventory Costs	
		R=1	R=0
1	F U	138.12 86.81	99.24 58.58
2	F U	86,40 67.38	42.73 59.59

Table 11
Accumulated inventory costs of Diesel product

Inventory Echelon (h)	State of Demand (i)	Accumulated Inventory Costs	
		R=1	R=0
1	F U	90.53 81.70	91.67 81.55
2	F U	164.76 85.12	190.55 81.78

5.3 The Optimal; Replenishment Policy

Week 1: Echelon 1

Kerosene

Since 57.56 < 98.47, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 57.56 USD for the case of favorable demand. Since 17.33 < 38.79, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 17.33 USD for the case of when demand is unfavorable.

Diesel

Since 48.57 < 50.48, it follows that R=1 is an optimal replenishment policy for week 1 with associated inventory costs of 48.57 USD for the case of favorable demand. Since 35.21 < 35.81, it follows that R=1 is an optimal replenishment policy for week 1 with associated inventory costs of 35.21 USD for the case when demand is unfavorable.

Week 1: Echelon 2

Kerosene

Since 13.95 < 50.10, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 13.95 USD for the case of favorable demand. Since 13.15 < 19.88, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 13.15 USD for the case when demand is unfavorable.

Diesel

Since 121.68 < 146.34, it follows that R=1 is an optimal replenishment policy for week 1 with associated inventory costs of 121.68 USD for the case of favorable demand. Since 37.75 < 39.03, it follows that R=0 is an optimal replenishment policy for week 1 with associated inventory costs of 37.75 USD for the case of unfavorable demand.

Week 2: Echelon 1

Kerosene

Since 99.24 < 138.12, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 99.24 USD for the case of favorable demand. Since 58.58 < 86.81, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 58.58 USD for the case when demand is unfavorable.

Diesel

Since 90.53 < 91.67, it follows that R=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 90.53 USD for the case of favorable demand. Since 81.55 < 81.70, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 81.53 USD for the case when demand is unfavorable.

Week 2: Echelon 2

Kerosene

Since 42.73 < 86.40, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 42.73 USD for the case of favorable demand. Since 59.59 < 67.38, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 59.59 USD for the case of unfavorable demand.

Diesel

Since 164.76 < 190.35, it follows that R=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 164.76USD for the case of favorable demand . Since 81.78 < 85.12, it follows that R=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 81.78 USD for the case of unfavorable demand.

6. CONCLUSIONS AND DISCUSSION

A two-echelon fuel supply chain model with stochastic demand was presented in this paper. The model determines an optimal replenishment policy and inventory costs of kerosene and diesel products under demand uncertainty. The decision of whether or not to replenish additional units is modelled as a multi-period decision problem using dynamic programming over a finite period planning horizon. Results from the model indicate optimal replenishment policies and inventory costs over the echelons for the two selected petroleum products. As a cost minimization strategy in echelon-based fuel supply chains, computational efforts of using Markov decision process provide promising results. However, further extensions of the research are sought in order to determine replenishment policies that minimize inventory costs under varying stochastic demand. In the same spirit, the model developed raises a number of salient issues to reconsider: Lead time of kerosene and diesel during the replenishment cycle, customer response to abrupt changes in price of kerosene and diesel as well as limitations of the truck load in fuel delivery. Finally, special interest is sought in further extending the model by considering replenishment policies for minimum inventory costs within the context of Continuous Time Markov Chain (CTMC).

REFERENCES

[1] Rodney P & Roman K, 2004," Optimal Policies for a capacitated Two-Echelon Inventory system", *Operations Research*, 152(5), 739-747

[2] Axsater S, 2005, A simple decision rule for decentralized two echelon inventory control, *International Journal of Production Economics*, 93-94(1), 53-59.

[3] Liu L, Li S & Wu Y, 2014, Supply Chain Risk Management in Chinese Chemical Industry based on stochastic chance constrained programming Model, *Applied Math and Information Science*, vol 8. No, 3, 1201-1206

[4] Cornillier F,Boctor F,Laporte G & Renand J,2008,"An Exact Algorithm for the Petrol station Replenishment Problem", *Journal of Operations Research Society*, vol.59, no.5, pp.607-615

[5] Cornillier F,Boctor F,Laporte G & Renand J,2009,"A Heuristic for the multi-period Petrol station Replenishment Problem", *European Journal of Operations Research*, vol.19, no.2, pp.295-305

[6] Cornillier F,Boctor F,Laporte G & Renand J,2009,"The Petrol station Replenishment Problem with Time Windows", *Computers and Operations Research*, vol.56,no.3,pp.919-935.

[7] Simatupang T & Padhana H, 2010, Improving Management and Supply chain of Diesel in Peptamina main depots, *Asian Journal of Technology Management*, vol3, no.1, pp.34-48

[8] Haji R & Tayebi H, 2011," Applying a new ordering policy in a two-echelon inventory system with Poisson demand rate retailers and transportation cost", *International Journal of Business Performance and Supply Chain Modelling*, 20-27.

[9] Abhijeet S & Saroj K, 2011, A Vendor-managed Two-Echelon Inventory system for an integrated procurement case, *Asia Pacific Journal of Operations Research*, 28(2), 301-322.

[10] Oliviera F & Hamacher S, 2011, Optimization of the Petroleum product supply chain under uncertainty: A Case study of northern Brazil, *Industrial Engineering and Chemical Research*, 51, 4279-4287.

[11] Awudu I & Zhang J, 2013, Stochastic Production Planning for a biofuel supply chain under demand and Price uncertainties, *Applied Energy*1030, 189-196.

[12] Kazemzadeh N & Hu G, 2013, Optimization Models for Bio Refinery Supply Chain network design under uncertainty, *Journal of Renewable and Sustainable Energy*, vol.5, No.5

[13] Tong K,Gong J,Yue D & You F,2014,Stochastic Programming Approach to Optimal Design and Operations of Integrated hydrocarbon biofuel and petroleum supply chains, *ACS sustainable chemistry and Engineering*, vol2, no.1,.49-61.

[14] Lee H & Wu J, 2005, A Study on Inventory Replenishment Policies in a Two-Echelon Supply Chain system, *Computers and Industrial Engineering*, vol.5, no.2, pp.257-263.

[15] Mubiru K, 2015, A Stochastic Two-Echelon Supply Chain Model for the Petrol Station Replenishment Problem, *Journal of Operations and Supply Chain Management*, vol.8, no.2, pp.67-76.