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Fuzzy β-magnified AB-ideals of AB-algebras

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Abstract— *In this paper, we introduce the notion of fuzzy magnified, fuzzy extensions magnified of fuzzy AB-subalgebra and fuzzy AB-ideal on AB-algebras and investigate some of their properties.*

Keywords— AB-algebra , fuzzy AB-subalgebra, fuzzy AB-ideal ,fuzzy magnified, fuzzy extension.

1. Introduction

Several authors ([8,10]) have introduced of BCK-algebras as a generalization of the concept of set-theoretic difference and propositional calculus and studied some important properties. The concept of a fuzzy set, was introduced by L.A. Zadeh [12]. A.T. Hameed and others, ([5,13,15]) introduced KUS-ideals in KUS-algebras and introduced the notions fuzzy KUS-subalgebras, fuzzy KUS-ideals of KUS-algebras and investigated relations among them. In ([11]), they applied the concept of fuzzy set to BCK/BCI-algebras and gave some of its properties. A.T. Hameed and others, ([1,2,7,9,14]) discussed fuzzy α-translation, (normalized, maximal) fuzzy S-extension of fuzzy (KUS/CI/QS)-subalgebras on (KUS/CI/QS)-algebra. They discussed fuzzy α-translation and fuzzy extension of fuzzy (KUS/CI/QS)-ideals in (KUS/CI/QS)-algebra. Dr. Areej Tawfeeq Hameed and others, ([3,4,6]) introduced AB-ideals on AB-algebras and introduced the notions fuzzy AB-subalgebras, fuzzy AB-ideals of AB-algebras and investigated relations among them .

In this paper, we define fuzzy multiplication AB-ideal of AB-algebras and look for some of their properties accurately by using the concepts of fuzzy AB-subalgebra and fuzzy AB-ideal.

2. Preliminaries:

We review some definitions and properties that will be useful in our results.

Definition 2.1.([3,4]) Let X be a set with a binary operation "$*$" and a constant 0.Then $(X, *, 0)$ is called **an AB-algebra** if the following axioms satisfied: for all $x, y, z \in X$:
(i) $((x * y) * (z * y)) * (x * z) = 0$,
(ii) $0 * x = 0$,
(iii) $x * 0 = x$,

Note that: Define a binary relation ($\leq$) on X by $x * y = 0$ if and only if, $x \leq y$.

Proposition 2.2.([3,4]) In any AB-algebra X, for all $x, y, z \in X$, the following properties hold:
(1) $(x * y) * x = 0$.
(2) $x \leq y$ implies $x*z \leq y*z$.
(3) $x \leq y$ implies $z*y \leq z*x$.

Remark 2.3.([3,5]) An AB-algebra is satisfies for all $x, y, z \in X$
(1) $(x * y) * z = (x * z) * y$,
(2) $(x * (x * y)) * y = 0$.

Definition 2.4. .([3,5]) Let X be an AB-algebra and $I \subseteq X$. I is called **an AB-ideal of X** if it satisfies the following conditions:
(i) $0 \in I$,
(ii) $(x * y)*z \in I$ and $y \in I$ imply $x*z \in I$.

Definition 2.5 [6]. A fuzzy μ is called a fuzzy relation on any set S, if μ is a fuzzy subset $\mu: X\times X \to [0,1]$.

Definition 2.6.([3,6]) A fuzzy subset μ of AB-algebra X is called **a fuzzy AB-subalgebra of X** if $\mu(x * y) \geq \min\{ \mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 2.7.([3,6]) A fuzzy subset μ of AB-algebra X is called **a fuzzy AB-ideal of X** if it satisfies :
$FAB_1)$ $\mu(0) \geq \mu(x)$;
$FAB_2)$ $\mu(x * z) \geq \min\{ \mu(x * (y * z)), \mu(y)\}$, for all $x, y, z \in X$.

Proposition 2.8 ([3,6]):
1- Every AB-ideal of AB-algebra X is an AB-subalgebra of X **.**
2- Every fuzzy AB-ideal of AB-algebra X is a fuzzy AB-subalgebra of X **.**

Definition 2.9 [3]: Let f :(X;∗,0)⟶ (Y;∗`, 0`) be a mapping from an AB-algebra X into an AB-algebra Y. If μ is a fuzzy subset of X, then $f(\mu)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu(x), & f^{-1}(y) = \{x\in X | f(x) = y\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$**,** is said to be **the image of μ under *f*** and is denoted by $f(\mu)$.

Definition 2.10 [3]: Let f :(X;∗,0)⟶ (Y;∗`, 0`) be a mapping from an AB-algebra X into an AB-algebra Y. If β is a fuzzy subset of AB-algebra Y, then the fuzzy subset μ = β o f of X (i. e, the fuzzy subset defined by μ (x) = β (f (x)), for all x ∊ X) is called **the pre-image of β under *f* .**

Proposition 2.11 [3]: Let f:(X;∗,0)→(Y;∗`,0`) be a homomorphism from X into Y and A be a fuzzy AB-subalgebra of X. Then the image f(A) is a fuzzy AB-subalgebra of Y.

Proposition 2.12 [3]: Let f:(X;∗,0)→(Y;∗ `,0`) be a homomorphism from X into Y and B be a fuzzy AB-subalgebra of Y. Then the inverse image f^{-1}(B) is a fuzzy AB-subalgebra of X.

Definition 2.13 [1,7,11]: Let μ be a fuzzy subset of X and β∊ [0, 1]. A **multiplication fuzzy subset** of μ, denoted by μ_β^M is defined to be a mapping μ_β^M: X→[0,1] such that : $\mu_\beta^M(x) = \beta\,.\,\mu(x)$, for all x ∊ X.

3. Fuzzy β-magnified AB-subalgebra of AB-algebras:

In this section, we discuss β-magnified on AB-algebras and we get some of relations, theorems, propositions and give examples of β-magnified of fuzzy AB-subalgebra. We show the notion of β-magnified fuzzy AB-subalgebras of AB-algebra and investigate some of their properties.

In what follows, let (X; ∗ ,0) denote an AB-algebra, and for any fuzzy subset μ of X, we denote $T = 1 - sup\{\mu(x) \mid x \in X\}$.

Definition 3.1:
Let μ be a fuzzy subset of an AB-algebra X and let β ∊(0,1] A mapping
μ_β^M:X→[0,1] is called a **β-magnified** of μ if it satisfies: $\mu_\beta^M(x) = \beta\,.\,\mu(x)$, for all x∊X.

Definition 3.2: Let X be an AB-algebra, a fuzzy subset μ in X is called a **β-magnified fuzzy AB-subalgebra** of X , if for all x , y ∊X, $\mu_\beta^M(x * y) \geq \min\{\mu_\beta^M(x), \mu_\beta^M, (y)\}$.

Theorem 3.3: Let μ be a fuzzy AB-subalgebra of AB-algebra X and β ∊ (0,1]. Then μ_β^M is a fuzzy AB-subalgebra of X .

Proof: Assume that μ is a fuzzy AB-subalgebra of X, and β ∊ (0,1]. Let x, y ∊X, then $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.Thus $\mu_\beta^M(x * y) = \beta\,.\,\mu(x * y) \geq \beta\,.\,\min\{\mu(x), \mu(y)\} = \min\{\beta\,.\,\mu(x, \beta.\mu(y)\} = \min\{\mu_\beta^M(x), \mu_\beta^M(y)\}$ and so $\mu_\beta^M(x * y) \geq \min\{\mu_\beta^M(x), \mu_\beta^M(y)\}$. Hence μ_β^M, is a fuzzy AB-subalgebra of X.□

Theorem 2.4: Let μ be a fuzzy subset of AB-algebra X such that μ_β^M of μ is a fuzzy AB-subalgebra of X, for some β ∊ (0,1]. Then μ is a fuzzy AB-subalgebra of X .

Proof. Assume that μ_β^M is a fuzzy AB-subalgebra of X for some β ∊ (0,1]. Let x, y ∊X, then $\beta\,.\,\mu(x * y) = \mu_\beta^M(x * y) \geq \min\{\mu_\beta^M(x), \mu_\beta^M(y)\} = \min\{\beta\,.\,\mu(x), \beta\,.\,\mu(y)\}$
$= \beta\,.\,\min\{\mu(x), \mu(y)\}$ and so $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy AB-subalgebra of X . □

Definition 3.5 : For a fuzzy subset μ of an AB-algebra X, β ∊ (0,1] and t∊ Im(μ) with t ≤ β, let $U_\beta(\mu; t) := \{x \in X \mid \mu(x) \geq t/\beta\}$.

Remark 3.6: If μ is a fuzzy AB-subalgebra of X, then it is that $U_\beta(\mu; t)$ is an AB-subalgebra of X, for all t ∊ Im(μ) with t ≤ β. Let x , $y \in U_\beta(\mu; t)$, then μ(x) ≥ t/β , and μ(y) ≥ t/β , then min { μ(x) , μ(y)}≥ t/β , since μ is a fuzzy AB-subalgebra, then μ(x∗ y)≥ $min\{\mu(x), \mu(y)\}$ ≥ t/β ,therefor x∗ $y \in U_\beta(\mu; t)$} .

But if we do not give a condition that μ is a fuzzy AB-subalgebra of X, then $U_\beta(\mu; t)$ is not an AB-subalgebra of X as seen in the following example.

Example 3.7: Let X = {0, 1, 2 , 3} in which (∗) be a defined by the following table:

∗	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

Then (X;∗,0) is an AB-algebra. It is easy to show that S_1 ={0,1}, S_2 ={0,2}, S_3={0,3} and S_4 ={0, 1, 2, 3}are AB-subalgebras of X .

Define a fuzzy subset λ of X:

X	0	1	2	3
λ	0.7	0.6	0.4	0.3

Then λ is not a fuzzy AB-subalgebra of X.
Since λ (1∗2) = 0.3 < 0.4 = min{λ (1), λ (2)}. For α = 0.1, β = 0.9 and t = 0.5, we obtain U_β (λ; t) = {0, 1, 2} which is not an AB-subalgebra of X since $1_*2 = 3 \notin U_\beta$λ; t).

Proposition 3.8: Let μ be a fuzzy subset of an AB-algebra X and β ∈ (0,1]. Then μ_β^M is a fuzzy AB-subalgebra of X if and only if, U_β (μ; t) is an AB-subalgebra of X, for all t ∈ Im(μ) with t ≤ β.

Proof: Necessity is clear { assume that μ_β^M is a fuzzy AB-subalgebra by Theorem (3.4), then μ is a fuzzy AB-subalgebra, by Remark (3.6), then U_β (μ; t) is a fuzzy AB-subalgebra}.

To prove the conversely, assume that $x, y \in U_\beta$ (μ; t) and μ_β^M of μ is not a fuzzy AB-subalgebra of X, therefore $\mu_\beta^M(x * y) < t \leq min\{\mu_\beta^M(x), \mu_\beta^M(y)\}$. Then $\mu(x) \geq t/\beta$ and $\mu(y) \geq t/\beta$, but $\mu(x * y) < t/\beta$. This shows that $x * y \notin U_\beta(\mu;\ t)$.

This is a contradiction, and so $\mu_\beta^M(x * y) \geq \min\{\mu_\beta^M(x), \mu_\beta^M(y)\}$, for all x, y ∈ X.

Hence μ_β^M is a fuzzy AB-subalgebra of X.□

Theorem 2.9 : Let f: (X; ∗ ,0) → (Y; ∗ `, 0`) be an onto homomorphism between AB-algebras X and Y. or every β-magnified fuzzy AB-subalgebra μ_β^M of X, $f(\mu_\beta^M)$ is a β-magnified fuzzy AB-subalgebra of Y.

Proof: By definition $\lambda_\beta^M(y') = f(\mu_\beta^M)(y') = sup_{x \in f^{-1}(y')} \beta . \mu(x)$, for all y'∈ Y (sup ∅ =0). By Proposition (2.10). Hence $f(\mu_\beta^M)$ is a fuzzy AB-subalgebra of Y.□

Theorem 3.10: An homomorphic pre-image of a β-magnified fuzzy AB-subalgebra of AB-algebra is also a β-magnified fuzzy AB-subalgebra of AB-algebra.

Proof: Let f: (X;∗ ,0) → (Y;∗ `, 0`) be a homomorphism of AB-algebras , λ the β-magnified fuzzy AB-subalgebra of Y and μ the pre-image of λ under f, then $\mu_\beta^M(x) = \lambda_\beta^M(f(x))$, for all $x \in X$. By Proposition (3.11). Hence μ_β^M is a fuzzy AB-subalgebra of X. □

Proposition 3.11: Let μ be a fuzzy subset of AB-algebra X, α∈[0,T] and β∈(0, 1]. Then every translation fuzzy AB-subalgebra μ_α^T of μ is a fuzzy S-extension of the multiplication fuzzy AB-subalgebra μ_β^M of μ.

Proof: For every x∈X, we have $\mu_\alpha^T(x) = \mu(x)+\alpha \geq \mu(x) \geq \beta.\ \mu(x) = \mu_\beta^M(x)$,
and so μ_α^T is a fuzzy extension of μ_β^M. Assume that μ_β^M is a multiplication fuzzy AB-subalgebra of X. Then μ is a fuzzy AB-subalgebra of X by Proposition (3.10).

It follows from Theorem (3.2) that μ_α^T is a fuzzy AB-subalgebra of X, for all α ∈[0,T]. Hence every translation fuzzy AB-subalgebra μ_α^T is a fuzzy S-extension of the multiplication fuzzy AB-subalgebra μ_β^M. □

Proposition 3.12: For any fuzzy subset μ of AB-algebra X, the following are equivalent:
(A) μ is a fuzzy AB-subalgebra of X.
(B) For any β ∈(0, 1], μ_β^M is a fuzzy AB-subalgebra of X.

Proof: let β∈(0, 1] be such that μ_β^M is a fuzzy AB-subalgebra of X. For any x, y∈X, where β=1
μ (x∗y) ≥ min{μ(x), μ(y)}. Hence μ is a fuzzy AB-subalgebra of X. □

4. β-magnified of Fuzzy AB-ideals

In this section, we shall define the notion of β-magnified of fuzzy AB-ideals, and we study some of the relations, theorems, propositions and examples of β-magnified of fuzzy AB-ideals of AB- algebra.

Definition 4.1. : Let X be an AB-algebra, a α-translation fuzzy subset μ of X is called a **β-magnified fuzzy AB-ideal** of X if it satisfies the following conditions: for all $x , y, z \in$ X ,

(FAB_1) $\mu_\beta^M(0) \geq \mu_\beta^M(x)$,

(FAB_2) $\mu_\beta^M(x * z) \geq min\{\mu_\beta^M(x * (y * z)), \mu_\beta^M(y)\}$.

Theorem 4.2.: Let μ is a fuzzy AB-ideal of an AB-algebra X , then μ_β^M is a fuzzy AB-ideal of X, for all β ∈ (0,1].

Proof : Assume that μ is a fuzzy AB-ideal of X and let β ∈ (0,1]. Then, for all $x, y, z \in$ X.

1- $\mu_\beta^M(0) = \beta.\mu(0) \geq \beta.\mu(x) = \mu_\beta^M(x)$.

2- $\mu_\beta^M(x * z) = \beta.\mu(x * z) \geq \beta.min\{\mu(x * (y * z)), \mu(y)\}$

$= min\{\beta\,.\,\mu(z * y)\,,\beta\,.\,\mu(x * y)\} = min\{\mu_\beta^M(z * y), \mu_\beta^M(x * y)\}$. Hence μ_β^M is a fuzzy AB-ideal of X . □

Theorem 4.3.: Let μ be a fuzzy subset of AB-algebra X such that μ_β^M is a fuzzy AB-ideal of X for some β ∈ (0,1]. Then μ is a fuzzy AB-ideal of X.

Proof : Assume that μ_β^M is a β-magnified fuzzy AB-ideal of X for some β ∈ (0,1]. Let $x, y, z \in$ X, we have $\beta\,.\,\mu(0) = \mu_\beta^M(0) \geq \mu_\beta^M(x) = \beta\,.\,\mu(x)$
and so $\mu(0) \geq \mu(x)$.
$\beta\,.\,\mu(x * z) = \mu_\beta^M(x * z) \geq min\{\mu_\beta^M(x * (y * z)), \mu_\beta^M(y)\}$,
$= min\{\beta\,.\,\mu(x * (y * z))\,,\beta\,.\,\mu(y)\} = \beta\,.\,min\{\mu(x * (y * z)), \mu(y)\}$, then
$\mu(x * z) \geq min\{\mu(x * (y * z)), \mu(y)\}$.
Hence μ is a fuzzy AB-ideal of X. □

Theorem 4.4. : For β ∈ (0,1], let μ_β^M be the β-magnified fuzzy subset μ of AB-algebra X. Then the following are equivalent:

(1) μ_β^M is a fuzzy AB-ideal of X.

(2) ∀ t ∈ Im(μ), t > α ⇒ $U_\beta(\mu; t)$ is an AB-ideal of X .

Proof: Assume that μ_β^M is a β-magnified fuzzy AB-ideal of X and let $t \in Im(\mu)$ be such that t ≤ β. Since $\mu_\beta^M(0) \geq \mu_\beta^M(x)$, for all $x \in$ X, we have $\beta\,.\,\mu(0)=\mu_\beta^M(0) \geq \mu_\beta^M(x)= \beta\,.\,\mu(x) \geq$ t, for all $x \in U_\beta(\mu; t)$. Hence $0 \in U_\beta(\mu; t)$.
Let $x, y, z \in$ X, such that $(x * (y * z)) \in U_\beta(\mu; t)$ and $(y) \in U_\beta(\mu; t)$. Then $\mu(x * (y * z)) \geq t - \alpha$ and $\mu(y) \geq t - \alpha$,
i.e., $\mu_\beta^M(x * (y * z))=\beta\,.\,\mu(x * (y * z)) \geq t$ and $\mu_\beta^M(y)=\beta\,.\,\mu(y) \geq t$.
Since μ_β^M is a β-magnified fuzzy AB-ideal of X, it follows that
$\beta\,.\,\mu(x * z) = \mu_\beta^M(x * z) \geq min\{\mu_\beta^M(x * (y * z)), \mu_\beta^M(y)\} \geq t$, that is
$\mu(x * z) \geq t/\beta$, so that $(x * z) \in U_\beta(\mu; t)$, therefore $U_\beta(\mu; t)$ is AB-ideal of X.
Conversely, suppose that $U_\beta(\mu; t)$ is an AB-ideal of X, for every $t \in Im(\mu)$ with t ≤ β. If there exists $x \in$ X such that $\mu_\beta^M(0) < t \leq \mu_\beta^M(x)$, then $\mu(x) \geq t/\beta$, but $\mu(0) < t/\beta$. This shows that $x \in U_\beta(\mu; t)$ and $0 \notin U_\beta(\mu; t)$. This is a contradiction, and so $\mu_\beta^M(0) \geq \mu_\beta^M(x)$, for all x ∈ X .
Now, assume that there exist x, y, z ∈ X such that $\mu_\beta^M(x * z) < t \leq min\{\mu_\beta^M(x * (y * z)), \mu_\beta^M(y)\}$.
Then $\mu(x * (y * z)) \geq t/\beta$ and $\mu(y) \geq t/\beta$, but $\mu(x * z) < t/\beta$.
Hence $(x * (y * z)) \in U_\beta(\mu; t)$ and $(y) \in U_\beta(\mu; t)$, but $(x * z) \notin U_\beta(\mu; t)$ and this is a contradiction.
Therefore as $\mu_\beta^M(x * z) \geq min\{\mu_\beta^M(x * (y * z)), \mu_\beta^M(y)\}$, for all $x, y, z \in$ X.
Hence μ_β^M is a fuzzy AB-ideal of X. □
In Theorem (4.4.(2)), if t ≤ α, then $U_\beta(\mu; t)$ = X.

Theorem 4.5.: Let f: (X; $_*$,0) →(Y; ∗`,0`) be an onto homomorphism between AB-algebras X and Y . For every β-magnified fuzzy AB-ideal μ of X , f(μ) is a β-magnified fuzzy AB-ideal of Y.

Proof: Let f :(X;$_*$,0) →(Y; ∗`,0`) be an onto homomorphism of AB-algebras, μ is a β-magnified fuzzy AB-ideal of X and λ_β^M the image of μ_β^M under f. Since μ is a β-magnified fuzzy AB-ideal of X, we have $\mu_\beta^M(0) \geq \mu_\beta^M(x)$, for all $x \in$ X.
Note that 0 ∈ f(0`), where 0 and 0' are the zero elements of X and Y respectively.
Thus $\lambda_\beta^M(0') = f(\mu_\beta^M)(0') = sup_{t \in f^{-1}(0')}\beta\,.\,\mu(t) = \mu_\beta^M(0) \geq \mu_\beta^M(x)$, for all x∈X, which implies that
$\lambda_\beta^M(0') \geq sup_{t \in f^{-1}(x')}\beta\,.\,\mu(t) = \lambda_\beta^M(x')$.
For any $x', y', z' \in$ Y, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be such that:
$f(\mu_\beta^M(x' *' (y' *' z'))= sup_{t \in f^{-1}(x' *' (y' *' z'))}\beta\,.\,\mu(t)$, $f(\mu_\beta^M)(y')= sup_{t \in f^{-1}(y')}\beta\,.\,\mu(t)$.
Then $f(\mu_\beta^M)(x' *' (y' *' z')) = \lambda_\beta^M(x' *' (y' *' z')) = sup_{x_0 * (y_0 * z_0) \in f^{-1}(x' *' (y' *' z'))}\beta\,.\,\mu(x_0 * (y_0 * z_0))$
$= sup_{t \in f^{-1}(x' *' (y' *' z'))}\beta\,.\,\mu(t)$
Then $\lambda_\beta^M(x' *' z') = sup_{t \in f^{-1}(x' *' z')}\beta\,.\,\mu(t) = \mu_\beta^M(x_0 * z_0)$
$\geq min\{\mu_\beta^M(x_0 * (y_0 * z_0)), \mu_\beta^M(y_0)\}$
$= min\{sup_{t \in f^{-1}(x' *' (y' *' z'))}\beta\,.\,\mu(t)\,, sup_{t \in f^{-1}(y')}\beta\,.\,\mu(t)\}$.
$= min\{\lambda_\beta^M(x' *' (y' *' z'))\,, \lambda_\beta^M(y')\}$.
Hence $\lambda_\beta^M = f(\mu_\beta^M)$ is a fuzzy AB-ideal of Y. □

Theorem 4.6.: An homomorphic pre-image of a β-magnified fuzzy AB-ideal of AB-algebra X is also a β-magnified fuzzy AB-ideal.

Proof: Let f: (X;$_*$,0) → (Y;$_*$`,0`) be a homomorphism of AB-algebras , λ the β-magnified fuzzy AB-ideal of Y and μ the pre-image of λ under f, then $\mu_\beta^M(x) = \lambda_\beta^M(f(x))$, for all $x \in$ X .

By Theorem (4.5), we have that μ_β^M is a fuzzy AB-ideal of X.□

Definition 4.6: Let μ_1 and $\mu_2$be fuzzy subsets of an AB-algebra X. Then μ_2 is called a **fuzzy extension AB-ideal** of μ_1 if the following assertions are valid:

(I_i) μ_2 is a fuzzy extension of μ_1.

(I_{ii}) If $\mu_1$is a fuzzy AB-ideal of X, then μ_2 is a fuzzy AB-ideal of X.

Proposition 4.7: For any fuzzy subset μ of an AB-algebra X, the following are equivalent:

(i) μ is a fuzzy AB-ideal of X.

(ii) For all $\beta \in (0, 1]$, $\mu_\beta^M$is a multiplication fuzzy AB-ideal of X.

Proposition 4.8: Let μ be a fuzzy subset of an AB-algebra X, $\alpha \in [0,T]$ and $\beta \in (0, 1]$. Then every translation fuzzy subset μ_α^T of μ is a fuzzy extension AB-ideal of the multiplication fuzzy AB-ideal μ_β^M of μ.

Proof:

For every $x \in X$, we have $\mu_\alpha^T(x) = \mu(x)+\alpha \geq \mu(x) \geq \beta.\mu(x) = \mu_\beta^M(x)$, and so μ_α^T is a fuzzy extension AB-ideal of μ_β^M. Assume that μ_β^M is a fuzzy AB-ideal of X. Then μ is a fuzzy AB-ideal of X by Proposition (4.3).

It follows that μ_α^T is a fuzzy AB-ideal of X, for all $\alpha \in [0,T]$.

Hence every fuzzy translation subset μ_α^T is a fuzzy extension AB-ideal of the fuzzy multiplication AB-ideal μ_β^M . □

The following example illustrates Proposition (4.8).

Example 4.9:

Let X={0, 1, 2, 3} be a AB-algebra which is given in Example (3.2) . Define a fuzzy subset μ of X by:

X	0	1	2	3
μ	0.8	0.5	0.3	0.3

Then μ is a fuzzy AB-ideal of X. If we take β = 0.1, then the multiplication fuzzy subset $\mu_{0.1}^M$ of μ is given by:

X	0	1	2	3
$\mu_{0.1}^M$	0.08	0.05	0.03	0.03
$\mu_{0.3}^M$	0.24	0.15	0.09	0.09

Clearly $\mu_{0.1}^M$ and $\mu_{0.3}^M$ are multiplication fuzzy AB-ideals of X. Also, for any $\alpha \in [0,0.2]$, the translation fuzzy $\mu_\alpha^T$of μ is given by:

X	0	1	2	3
μ_α^T	0.8+α	0.5+α	0.3+α	0.3+α

Then μ_α^T is a fuzzy extension AB-ideal of $\mu_{0.3}^M$ and $\mu_{0.1}^M$.

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