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The Rational valued characters table of the group (Q_{2n}×D_4) when n is an odd number

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Abstract *: The goal of this paper is to find the rational valued characters table of the group(Q_{2n}×D_4) when n is an odd number, which is denoted by $\equiv^*(Q_{2n}\times D_4)$, where Q_{2n} is denoted to Quaternion group of order 4n, such that for each positive integer n, there are two generators a and b for Q_{2n} satisfies $Q_{2n}=\{a^h b^k, 0\leq \square \leq 2n-1, k=0,1\}$which has the following pronerties$\{a^{2n}=y^4=I, ba^nb^{-1}=a^{-n}\}$ and D_4 is the Dihedral group of order 8 is generate by a rotation t of order 4 and reflection f of order 2 then 8 elements of D_4 can be written as: $\{I^*,t,t^2, t^3,f,ft,ft^2,ft^3\}$.*

Keywords*:* Rational, characters ,Group,Q_{2n} ,D_4, odd number.

1. Introduction

Let F be a field and G be a group .A matrix representation of G is a homomorphism T:G →GL(n,F), n is called the degree of representation T. T is called a unit representation (principal) T(g)=1 for all g∈G .[3]

Let T be a matrix representation of G over the field F.The character χ of a matrix representation T is the mapping χ: G →F defined by χ(g) =tr(T(g)), for all g ∈G.The degree of T is called th degree of χ.Recall that the trace of an nxn matrix A is the sum of main diagonal elements.tr(A)=$\sum_{i=1}^{n} a_{ii.}$ [6]

Let G be a finite group ,two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes . [2]

In 1980, M.S. Kirdar [8] studied "The factor Group of the Z-valued class function Modulo the group of the Generalized characters", University of Birmingham.

In 1994, H.H. Abass [5] studied "On The Factor Group Of Class Function Over The Group Of Generalized Characters Of D_n", M. Sc.

In 1995, N.R. Mahamood [9] studied"The cyclic Decomposition of the factor group of cf$(Q_{2m},z)/\bar{R}(Q_{2m})$ ",M.SC. thesis University of Technology.In 1994, Abass.H. H [4] studied "On The Factor Group Of Class Function Over The Group Of Generalized Characters Of D_n", M. Sc. thesis, Technology University.

In this work we find the valued characters table of the group(Q_{2n}×D_4) when n is an odd number .

2.Preliminaries:

2.1 The general form of the characters table of Q_{2n} when n is an odd number is given in the following table:[1]

$\equiv(Q_{2n})$=

CL_α	[I]	$[a^2]$	$[a^4]$	⋯	$[a^{n-1}]$	$[a^n]$	[a]	$[a^3]$	⋯	$[a^{n-2}]$	[b]	[ab]
$\lvert CL_\alpha\rvert$	1	2	2	⋯	2	1	2	2	⋯	2	n	n
$\lvert C_{Q_{2n}}(CL\alpha)\rvert$	4n	2n	2n	⋯	2n	4n	2n	2n	⋯	2n	4	4
λ_1	1	1	1	⋯	1	1	1	1	⋯	1	1	1
μ_2	2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^8+\upsilon^{2n-8}$	⋯	$\upsilon^{2(n-1)}+\upsilon^2$	2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^6+\upsilon^{2n-6}$	⋯	$\upsilon^{2(n-2)}+\upsilon^4$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
μ_{n-1}	2	$\upsilon^{2(n-1)}+\upsilon^2$	$\upsilon^{4(n-1)}+\upsilon^4$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	2	$\upsilon^{n-1}+\upsilon^{n+1}$	$\upsilon^{n-3}+\upsilon^{n+3}$	⋯	$\upsilon^2+\upsilon^{2(n-1)}$	0	0
λ_2	1	1	1	⋯	1	1	1	1	⋯	1	-1	-1
μ_1	2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^4+\upsilon^{4(n-1)}$	⋯	$\upsilon^{n-1}+\upsilon^{n+1}$	-2	$\upsilon+\upsilon^{2n-1}$	$\upsilon^3+\upsilon^{2n-3}$	⋯	$\upsilon^{n-2}+\upsilon^{n+2}$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
μ_{n-2}	2	$\upsilon^{2n-4}+\upsilon^4$	$\upsilon^{2n-8}+\upsilon^8$	⋯	$\upsilon^2+\upsilon^{2(n-1)}$	-2	$\upsilon^{n-2}+\upsilon^{n+2}$	$\upsilon^{n-6}+\upsilon^{n+6}$	⋯	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	0	0
λ_3	1	1	1	⋯	1	-1	-1	-1	⋯	-1	i	-i
λ_4	1	1	1	⋯	1	-1	-1	-1	⋯	-1	-i	i

Table(1)

The characters table of matrix from degree(n+3)×(n+3)where $\upsilon=e^{2\pi i/2n}$, $\upsilon^n = -1$

2.2 The Characters Table Of The Dihedral Group D_4 :[5]

There are $\frac{n-2}{2}+4$ conjugate classes of D_4

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≡(D_4)=

CL_α	[I*]	$[t^2]$	[t]	[f]	[ft]
$\|CL_\alpha\|$	1	1	2	2	2
$\|C_{D_4}(CL_\alpha)\|$	8	8	4	4	4
λ_1'	1	1	1	1	1
λ_2'	1	1	-1	1	-1
λ_3'	1	1	1	-1	-1
λ_4'	1	1	-1	-1	1
λ_5'	2	-2	0	0	0

Table(2)

Theorem 2.2.1:[2]
1-Sum of characters is a character.
2-Product of characters is a character.
2.3The Rational valued characters table:
Definition(2.3.1):[4]
The group generated by all characters on C is called the group of the generalized characters of G, and it is denoted by R(G).
Definition(2.3.2):[4]
The intersection of cf(G,Z) with R(G) forms an abelian group is called the group of Z-valued generalized characters of G, denoted by $\bar{R}$(G).
Definition(2.3.3) [1]
A rational valued character θ of G is a character whose values are in Z,which is θ(g)∈Z for all g∈ G.
Corollary (2.3.4) [1]
The rational valued characters $\rho_i = \sum_{\sigma \epsilon Gal(Q(\lambda_i)/Q)} \sigma(\lambda_i)$Form a basis for $\bar{R}(G)$,whereλ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of a cyclic subgroup of G.
Proposition(2.3.5) [2]
The number of all rational valued characters of finite G is equal to the number of all distinct Γ-classis.
Proposition(2.3.6):[8]
The rational valued characters table of the cyclic group C_{p^s} of the rank s+1 where p is a prime number, which is denoted by (≡*(C_{p^s})), and given as follows:
≡*(C_{p^s}) =

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^{p^2}]$	$[x^p]$	[x]
θ_1	$P^{s-1}(p-1)$	$-p^{s-1}$	0	0	⋯	0	0	0
θ_2	$P^{s-2}(p-1)$	$P^{s-2}(p-1)$	$-p^{s-2}$	0	⋯	0	0	0
θ_3	$P^{s-3}(p-1)$	$P^{s-3}(p-1)$	$P^{s-3}(p-1)$	$-p^{s-3}$	⋯	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
θ_{s-1}	P(p-1)	P(p-1)	P(p-1)	P(p-1)	⋯	P(p-1)	-p	0
θ_s	p-1	p-1	p-1	p-1	⋯	p-1	p-1	-1
θ_{s+1}	1	1	1	1	⋯	1	1	1

Table(3)
Where its rank s+1 which represents the number of all distinct Γ-classes.

Proposition(2.3.7):[8]

For n = $p_1^{\alpha_1}.p_2^{\alpha_2}....p_n^{\alpha_n}$ where g.c.d(p_i ,p_j)=1, if i≠j and p_i's are prime numbers, and α_i any positive integers for all 1≤ $i \le n$, then the rational valued characters table of the cyclic group C_n can be given by:

$$\equiv^*(C_n) = \equiv^*(C_{p_1^{\alpha_1}}) \otimes \equiv^*(C_{p_2^{\alpha_2}}) \otimes .... \otimes \equiv^*(C_{p_n^{\alpha_n}})$$

Proposition(2.3.9):[9]
The rational valued characters table of Quaternion group Q_{2n} when m is an odd number is given by:

$\equiv^*(Q_{2n})$ =

	Γ-classes of C_{2n}								*[y]*
	X^{2r}				*X^{2r+1}*				
θ_1	$\equiv^*(n)$				$\equiv^*(C_n)$				0
⋮									⋮
$\theta_{(l/2)-1}$									0
$\theta_{(l/2)}$	1	1	…	1	1	1	…	1	1
$\theta_{(l/2)+1}$	$\equiv^*(C_n)$				*H*				0
⋮									⋮
θ_{l-1}									0
θ_l	1	1	…	1	1	1	…	1	-1
θ_{l+1}	2	2	…	2	-2	-2	…	-2	0

Table(4)

Where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2n}, θ_j such that $1 \le j \le l+1$ are the rational valued characters table of Quaternion group Q_{2m} if we denote C_{ij} the elements of $\equiv^*(C_n)$ and h_{ij} the elements of H it as defined by:

$$h_{ij}=\begin{cases} C_{ij} & if \quad i = 1 \\ -C_{ij} & if \quad i \neq 1 \end{cases}$$

Lemma(2.3.10):[1]

The rational valued characters table of $\mathbf{D_n}$ when n is an odd number is given by:

$\equiv^*(D_n)$ =

	Γ-classes of C_n					
θ_1	$\equiv^*(C_n)$					0
θ_2						0
⋮						⋮
θ_{l-1}						0
θ_l	1	1	1	…	1	1
θ_{l+1}	1	1	1	…	1	-1

Table(5)

Where l is the number of Γ-classes.

$\equiv^*(D_4)$ =

CL_α	$[I^*]$	$[t^2]$	[t]	[f]	[ft]
$\lvert CL_\alpha \rvert$	1	1	2	2	2
$\lvert C_{D_4}(CL_\alpha) \rvert$	8	8	4	4	4
ρ_1'	1	1	1	1	1
ρ_2'	1	1	-1	1	-1
ρ_3'	1	1	1	-1	-1
ρ_4'	1	1	-1	-1	1
ρ_5'	2	-2	0	0	0

Table(6)

3. The Main Results

3.1 Characters Table of the Group ($Q_{2n} \times D_4$) when n is an odd number:

The group ($Q_{2n} \times D_4$) is the direct product of the Quaternion group Q_{2n} of order 4n and the dihedral group D_4 of order 8 then the order of the group ($Q_{2n} \times D_4$) is 32 , the irreducible representations of the group ($Q_{2n} \times D_4$) are the tensor product Q_{2n} and D_4 and the irreducible characters of the group ($Q_{2n} \times D_4$) are the tensor product Q_{2n} and D_4. According to proposition(2.3.7), each irreducible character λ_i of Q_{2n}, defines four characters $\lambda_{(i,1)}$, $\lambda_{(i,2)}$, $\lambda_{(i,3)}$, $\lambda_{(i,4)}$ and $\lambda_{(i,5)}$. such that $\lambda_{(i,1)} = \lambda_i \lambda_1'$, $\lambda_{(i,2)} = \lambda_i \lambda_2'$, $\lambda_{(i,3)} = \lambda_i \lambda_3'$, $\lambda_{(i,4)} = \lambda_i \lambda_4'$ $\lambda_{(i,5)} = \lambda_i \lambda_5'$ of ($Q_{2m} \times D_4$) .
Then $\equiv(Q_{2n} \times D_4) = \equiv Q_{2n} \otimes \equiv D_4$.

Then , the general form of the characters table of ($Q_{2n} \times D_4$) when n is an odd number is given in the following table:

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CL_α	$[I,I']$	$[I,t^2]$	$[I,t]$	$[I,f]$	$[I,ft]$	$[a^2, I']$	$[a^2, t^2]$	$[a^2,t]$	$[a^2,f]$	$[a^2,ft]$	⋯	$[a^{p-1}, I']$	$[a^{p-1},t^2]$	$[a^{p-1},t]$	$[a^{p-1},f]$	
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	4	4	4	⋯	2	2	4	4	
$\lvert C_{Q_{2n}\times D_4}(CL\alpha) \rvert$	32n	32n	16n	16n	16n	16n	16n	8n	8n	8n	⋯	16n	16n	8n	8n	
$\lambda_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	⋯	1	1	1	1	
$\lambda_{(1,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	⋯	1	1	-1	1	
$\lambda_{(1,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	⋯	1	1	1	-1	
$\lambda_{(1,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	⋯	1	1	-1	-1	
$\lambda_{(1,5)}$	2	-2	0	0	0	2	-2	0	0	0	⋯	2	-2	0	0	
$\mu_{(2,1)}$	2	2	2	2	2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	⋯	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	
$\mu_{(2,2)}$	2	2	-2	2	-2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	⋯	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	$\upsilon^{2n-2}+\upsilon^2$	
$\mu_{(2,3)}$	2	2	2	-2	-2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$-(\upsilon^4+\upsilon^{2n-4})$	⋯	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	
$\mu_{(2,4)}$	2	2	-2	-2	2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^4+\upsilon^{2n-4}$	⋯	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	$-(\upsilon^{2n-1})+\upsilon^2)$	
$\mu_{(2,5)}$	4	-4	0	0	0	$2(\upsilon^4+\upsilon^{2n-4})$	$-2(\upsilon^4+\upsilon^{2n-4})$	0	0	0	⋯	$2(\upsilon^{2n-2}+\upsilon^2)$	$-2(\upsilon^{2n-2}+\upsilon^2)$	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	
$\mu_{((n-1),1)}$	2	2	2	2	2	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	
$\mu_{((n-1),2)}$	2	2	-2	2	-2	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$\upsilon^{n+1}+\upsilon^{n-1}$	
$\mu_{((n-1),3)}$	2	2	2	-2	-2	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	$-(\upsilon^{2n-2}+\upsilon^2)$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	
$\mu_{((n-1),4)}$	2	2	-2	-2	2	$\upsilon^{2n-2}+\upsilon^2$	$\upsilon^{2n-2}+\upsilon^2$	$-(\upsilon^{2n-2}+\upsilon^2)$	$-(\upsilon^{2n-1})+\upsilon^2)$	$\upsilon^{2n-2}+\upsilon^2$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	
$\mu_{((n-1),5)}$	4	-4	0	0	0	$2(\upsilon^{2n-2}+\upsilon^2)$	$-2(\upsilon^{2n-2}+\upsilon^2)$	0	0	0	⋯	$2(\upsilon^{n+1}+\upsilon^{n-1})$	$-2(\upsilon^{n+1}+\upsilon^{n-1})$	0	0	
$\lambda_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	⋯	1	1	1	1	1
$\lambda_{(2,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	⋯	1	1	-1	1	-1
$\lambda_{(2,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	⋯	1	1	1	-1	-1

$\lambda_{(2,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	⋯	1	1	-1	-1	1
$\lambda_{(2,5)}$	2	-2	0	0	0	2	-2	0	0	0	⋯	2	-2	0	0	0
$\mu_{(1,1)}$	2	2	2	2	2	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	
$\mu_{(1,2)}$	2	2	-2	2	-2	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$-(\upsilon^2+\upsilon^{2n-4})$	$\upsilon^2+\upsilon^{2n-4}$	$-(\upsilon^2+\upsilon^{2n-4})$	⋯	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$\upsilon^{n+1}+\upsilon^{n-1}$	
$\mu_{(1,3)}$	2	2	2	-2	-2	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$-(\upsilon^2+\upsilon^{2n-4})$	$-(\upsilon^2+\upsilon^{2n-4})$		$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	
$\mu_{(1,4)}$	2	2	-2	-2	2	$\upsilon^2+\upsilon^{2n-4}$	$\upsilon^2+\upsilon^{2n-4}$	$-(\upsilon^2+\upsilon^{2n-4})$	$-(\upsilon^2+\upsilon^{2n-4})$	$\upsilon^2+\upsilon^{2n-4}$		$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	
$\mu_{(1,5)}$	4	-4	0	0	0	$2(\upsilon^2+\upsilon^{2n-4})$	$-2(\upsilon^2+\upsilon^{2n-4})$	0	0	0		$2(\upsilon^{n+1}+\upsilon^{n-1})$	$-2(\upsilon^{n+1}+\upsilon^{n-1})$	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	

Table(7)

The characters table of matrix fron degree 4(n+3)×4(n+3) where $\upsilon = e^{2\pi i/2n}$, υ^n = -1

$\equiv(Q_{2n}\times D_4)$ =

$[a^{n-1},ft]$	$[a^n, I']$	$[a^n, t^2]$	$[a^n, t]$	$[a^p, f]$	$[a^n, ft]$	$[a, I']$	$[a,t^2]$	$[a,t]$	$[a , f]$	$[a , ft]$	⋯	$[a^{n-2}, I']$	$[a^{n-2},t^2]$	$[a^{n-2},t]$
4	1	1	2	2	2	2	2	4	4	4	⋯	2	2	4
8n	32n	32n	16n	16n	16n	16n	16n	8n	8n	8n	⋯	16n	16n	8n
1	1	1	1	1	1	1	1	1	1	1	⋯	1	1	1
-1	1	1	-1	1	-1	1	1	-1	1	-1	⋯	1	1	-1
-1	1	1	1	-1	-1	1	1	1	-1	-1	⋯	1	1	1
1	1	1	-1	-1	1	1	1	-1	-1	1	⋯	1	1	-1
0	2	-2	0	0	0	2	-2	0	0	0	⋯	2	-2	0
$\upsilon^{2n-2}+\upsilon^2$	2	2	2	2	2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	⋯	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$
$\upsilon^{2n-2}+\upsilon^2$	2	2	-2	2	-2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$-(\upsilon^2+\upsilon^{2(n-1)})$	$\upsilon^2+\upsilon^{2(n-1)}$	$-(\upsilon^2+\upsilon^{2(n-1)})$	⋯	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$
$-(\upsilon^{2n-2}+\upsilon^2)$	2	2	2	-2	-2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$-(\upsilon^2+\upsilon^{2(n-1)})$	$-(\upsilon^2+\upsilon^{2(n-1)})$	⋯	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$
$-(\upsilon^{2n-1)}+\upsilon^2)$	2	2	-2	-2	2	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$-(\upsilon^2+\upsilon^{2(n-1)})$	$-(\upsilon^2+\upsilon^{2(n-1)})$	$\upsilon^2+\upsilon^{2(n-1)}$	⋯	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$
0	4	-4	0	0	0	$2(\upsilon^2+\upsilon^{2(n-1)})$	$-2(\upsilon^2+\upsilon^{2(n-1)})$	0	0	0	⋯	$2(\upsilon^4+\upsilon^{2n-4})$	$-2(\upsilon^4+\upsilon^{2n-4})$	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\upsilon^{n+1}+\upsilon^{n-1}$	2	2	2	2	2	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	⋯	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$
$-(\upsilon^{n+1}+\upsilon^{n-1})$	2	2	-2	2	-2	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	-	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	⋯	$\upsilon^2+\upsilon^{2(n-1)}$	$\upsilon^2+\upsilon^{2(n-1)}$	$-(\upsilon^2+\upsilon^{2(n-1)})$

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								$(\upsilon^{n+1}+\upsilon^{n-1})$						
-$(\upsilon^{n+1}+\upsilon^{n-1})$	2	2	2	-2	-2	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	-$(\upsilon^{n+1}+\upsilon^{n-1})$	-$(\upsilon^{n+1}+\upsilon^{n-1})$	⋯	$\upsilon^{2}+\upsilon^{2(n-1)}$	$\upsilon^{2}+\upsilon^{2(n-1)}$	$\upsilon^{2}+\upsilon^{2(n-1)}$
$\upsilon^{n+1}+\upsilon^{n-1}$	2	2	-2	-2	2	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	- $(\upsilon^{n+1}+\upsilon^{n-1})$	-$(\upsilon^{n+1}+\upsilon^{n-1})$	$\upsilon^{n+1}+\upsilon^{n-1}$	⋯	$\upsilon^{2}+\upsilon^{2(n-1)}$	$\upsilon^{2}+\upsilon^{2(n-1)}$	-$(\upsilon^{2}+\upsilon^{2(n-1)})$
0	4	-4	0	0	0	$2(\upsilon^{n+1}+\upsilon^{n-1})$	-$2(\upsilon^{n+1}+\upsilon^{n-1})$	0	0	0	⋯	$2(\upsilon^{2}+\upsilon^{2(n-1)})$	-$2(\upsilon^{2}+\upsilon^{2(n-1)})$	0
1	1	1	1	1	1	1	1	1	1	1	⋯	1	1	1
-1	1	1	-1	1	-1	1	1	-1	1	-1	⋯	1	1	-1
-1	1	1	1	-1	-1	1	1	1	-1	-1	⋯	1	1	1
1	1	1	-1	-1	1	1	1	-1	-1	1	⋯	1	1	-1
0	2	-2	0	0	0	2	-2	0	0	0	⋯	2	-2	0
$\upsilon^{n+1}+\upsilon^{n-1}$	$2\upsilon^{n}$	$2\upsilon^{n}$	$2\upsilon^{n}$	$2\upsilon^{n}$	$2\upsilon^{n}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	⋯	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$
-$(\upsilon^{n+1}+\upsilon^{n-1}$	$2\upsilon^{n}$	$2\upsilon^{n}$	-$2\upsilon^{n}$	$2\upsilon^{n}$	-$2\upsilon^{n}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	-$(\upsilon+\upsilon^{2(n-1)})$	$\upsilon+\upsilon^{2(n-1)}$	-$(\upsilon+\upsilon^{2(n-1)})$	⋯	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	-$(\upsilon^{n+2}+\upsilon^{n-2})$
-$(\upsilon^{n+1}+\upsilon^{n-1})$	$2\upsilon^{n}$	$2\upsilon^{n}$	$2\upsilon^{n}$	-$2\upsilon^{n}$	-$2\upsilon^{n}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	-$(\upsilon+\upsilon^{2(n-1)})$	-$(\upsilon+\upsilon^{2(n-1)})$	⋯	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$
$\upsilon^{n+1}+\upsilon^{n-1}$	$2\upsilon^{n}$	$2\upsilon^{n}$	-$2\upsilon^{n}$	-$2\upsilon^{n}$	$2\upsilon^{n}$	$\upsilon+\upsilon^{2(n-1)}$	$\upsilon+\upsilon^{2(n-1)}$	-$(\upsilon+\upsilon^{2(n-1)})$	-$(\upsilon+\upsilon^{2(n-1)})$	$\upsilon+\upsilon^{2(n-1)}$	⋯	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	-$(\upsilon^{n+2}+\upsilon^{n-2})$
0	$4\upsilon^{n}$	$4\upsilon^{n}$	0	0	0	$2(\upsilon+\upsilon^{2(n-1)})$	-$2(\upsilon+\upsilon^{2(n-1)})$	0	0	0	⋯	$2(\upsilon^{n+2}+\upsilon^{n-2})$	-$2(\upsilon^{n+2}+\upsilon^{n-2})$	0

Table(7)

The characters table of matrix from degree 4(n+3)×4(n+3) where $\upsilon = e^{2\pi i/2n}$, υ^{n} = -1

≡$(Q_{2n}\times D_4)$ =

[a^{n-2},f]	[a^{n-2},tf]	[b , I']	[b,t^2]	[b,t]	[b , f]	[b , ft]	[ab, , I']	[ab,t^2]	[ab,t]	[ab ,f]	[ab , ft]
4	4	n	n	2n	2n	2n	n	n	2n	2n	2n
8n	8n	32	32	16	16	16	32	32	16	16	16
1	1	1	1	1	1	1	1	1	1	1	1
1	-1	1	1	-1	1	-1	1	1	-1	1	-1
-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
-1	1	1	1	-1	-1	1	1	1	-1	-1	1
0	0	2	-2	0	0	0	2	-2	0	0	0
$\upsilon^{4}+\upsilon^{2n-4}$	$\upsilon^{4}+\upsilon^{2n-4}$	0	0	0	0	0	0	0	0	0	0
$\upsilon^{4}+\upsilon^{2n-4}$	-$(\upsilon^{4}+\upsilon^{2n-4})$	0	0	0	0	0	0	0	0	0	0
-$(\upsilon^{4}+\upsilon^{2n-4})$	-$(\upsilon^{4}+\upsilon^{2n-4})$	0	0	0	0	0	0	0	0	0	0
-$(\upsilon^{4}+\upsilon^{2n-4})$	$\upsilon^{4}+\upsilon^{2n-4}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\upsilon^{2}+\upsilon^{2(n-1)}$	$\upsilon^{2}+\upsilon^{2(n-1)}$	0	0	0	0	0	0	0	0	0	0
$\upsilon^{2}+\upsilon^{2(n-1)}$	-$(\upsilon^{2}+\upsilon^{2(n-1)})$	0	0	0	0	0	0	0	0	0	0
-$(\upsilon^{2}+\upsilon^{2(n-1)})$	-$(\upsilon^{2}+\upsilon^{2(n-1)})$	0	0	0	0	0	0	0	0	0	0
-$(\upsilon^{2}+\upsilon^{2(n-1)})$	$\upsilon^{2}+\upsilon^{2(n-1)}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

1	-1	-1	-1	1	-1	1	-1	-1	1	-1	1
-1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1
-1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
0	0	-2	2	0	0	0	-2	2	0	0	0
$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	0	0	0	0	0	0	0	0	0	0
$\upsilon^{n+2}+\upsilon^{n-2}$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	0	0	0	0	0	0	0	0	0	0
$-(\upsilon^{n+2}+\upsilon^{n-2})$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	0	0	0	0	0	0	0	0	0	0
$-(\upsilon^{n+2}+\upsilon^{n-2})$	$\upsilon^{n+2}+\upsilon^{n-2}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Table(7)

The characters table of matrix from degree 4(n+3)×4(n+3) where $\upsilon = e^{2\pi i/2n}$, υ^n = -1

$\mu_{((n-2),1)}$	2	2	2	2	2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$
$\mu_{((n-2),2)}$	2	2	-2	2	-2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1}$
$\mu_{((n-2),3)}$	2	2	2	-2	-2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$-(\upsilon^{n+1}+\upsilon^{n-1})$
$\mu_{((n-2),4)}$	2	2	-2	-2	2	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^4+\upsilon^{2n-4}$	$-(\upsilon^4+\upsilon^{2n-4})$	$-(\upsilon^4+\upsilon^{2n-4})$	$\upsilon^4+\upsilon^{2n-4}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$\upsilon^{n+1}+\upsilon^{n-1}$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$-(\upsilon^{n+1}+\upsilon^{n-1})$	$\upsilon^{n+1}+\upsilon^{n-1}$
$\mu_{((n-2),5)}$	4	-4	0	0	0	$2(\upsilon^4+\upsilon^{2n-4})$	$-2(\upsilon^4+\upsilon^{2n-4})$	0	0	0	$2(\upsilon^{n+1}+\upsilon^{n-1})$	$-2(\upsilon^{n+1}+\upsilon^{n-1})$	0	0	0
$\lambda_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\lambda_{(3,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
$\lambda_{(3,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
$\lambda_{(3,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
$\lambda_{(3,5)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0
$\lambda_{(4,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\lambda_{(4,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
$\lambda_{(4,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
$\lambda_{(4,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
$\lambda_{(4,5)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0

$2\upsilon^n$	$2\upsilon^n$	$2\upsilon^n$	$2\upsilon^n$	$2\upsilon^n$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$
$2\upsilon^n$	$2\upsilon^n$	$-2\upsilon^n$	$2\upsilon^n$	$-2\upsilon^n$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$\upsilon^{n+2}+\upsilon^{n-2}$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$
$2\upsilon^n$	$2\upsilon^n$	$2\upsilon^n$	$-2\upsilon^n$	$-2\upsilon^n$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$
$2\upsilon^n$	$2\upsilon^n$	$-2\upsilon^n$	$-2\upsilon^n$	$2\upsilon^n$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{n+2}+\upsilon^{n-2}$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$-(\upsilon^{n+2}+\upsilon^{n-2})$	$\upsilon^{n+2}+\upsilon^{n-2}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$
$4\upsilon^n$	$4\upsilon^n$	0	0	0	$2(\upsilon^{n+2}+\upsilon^{n-2})$	$-2(\upsilon^{n+2}+\upsilon^{n-2})$	0	0	0	$2(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	$-2(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	0	0
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	-1
-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1
-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1
-2	2	0	0	0	-2	2	0	0	0	-2	2	0	0
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	-1
-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1
-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1
-2	2	0	0	0	-2	2	0	0	0	-2	2	0	0

$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	0	0	0	0	0	0	0	0	0	0
$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	0	0	0	0	0	0	0	0	0	0
$-(\upsilon^{(n-2)^2}+\upsilon^{n^2-4})$	0	0	0	0	0	0	0	0	0	0
$\upsilon^{(n-2)^2}+\upsilon^{n^2-4}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
-1	i	i	i	i	i	-i	-i	-i	-i	-i
1	i	i	-i	i	-i	-i	-i	i	-i	i
1	i	i	i	-i	-i	-i	-i	-i	i	i
-1	i	i	-i	-i	i	-i	-i	i	i	-i
0	2i	-2i	0	0	0	-2i	2i	0	0	0
-1	-i	-i	-i	-i	-i	i	i	i	i	i
1	-i	-i	i	-i	i	i	i	-i	i	-i
1	-i	-i	-i	i	i	i	i	i	-i	-i
-1	-i	-i	i	i	-i	i	i	-i	-i	i
0	-2i	2i	0	0	0	2i	-2i	0	0	0

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3.2 Theorem:

The rational valuer characters table of the group $Q_{2n}\times D_4$ when n is an odd number is given as follows:

$$\equiv^*(Q_{2n}\times D_4) = \equiv^*(Q_{2n}) \otimes \equiv^*(D_4)$$

Proof :-

Since $D_4 = \{I^*, t, t^2, t^3, f, ft, ft^2, ft^3\}$

Each element in $Q_{2n}\times D_4$ are $g_{pr} = g_p \cdot g_r \ \forall\ g_r \in D_4$, $r \in \{I^*, t, t^2, t^3, f, ft, ft^2, ft^3\}$

And each irrerucible character $\lambda_{(i,j)}$ of $Q_{2n}\times D_4$ is can be written as follows

$$\lambda_{(i,j)} = \lambda_i \cdot \lambda'_j$$

Where λ_i is an irrerucible character of Q_{2n} anr λ'_j is the irrerucible character of D_4 ,then

$$\lambda_{(i,j)}(p,r) = \begin{cases} \lambda i(g_p) \text{if } j = 1 \text{ and } r \in D_4 \\ \lambda i(g_p) \text{if } j = 2 \text{ and } r \in \{I^*, t^2, f, ft^2\} \\ -\lambda i(g_p) \text{if } j = 2 \text{ and } r \in \{t, t^3, ft, ft^3\} \\ \lambda i(g_p) \text{if } j = 3 \text{ and } r \in \{I^*, t, t^2, t^3\} \\ -\lambda i(g_p) \text{if } j = 3 \text{ and } r \in \{f, ft, ft^2, ft^3\} \\ \lambda i(g_p) \text{if } j = 4 \text{ and } r \in \{I^*, t^2, ft, ft^3\} \\ -\lambda i(g_p) \text{if } j = 4 \text{ and } r \in \{t, t^3, f, ft^2\} \\ 2\lambda i(g_p) \text{if } j = 5 \text{ and } r \in \{I^*\} \\ -2\lambda i(g_p) \text{if } j = 5 \text{ and } r \in \{t^2\} \\ 0 \quad \text{if } j = 1 \text{ and } r \in \{t, t^3, f, ft, ft^2\} \end{cases}$$

From proposition (2.3.4)

$$\rho_{(i,j)} = \sum_{\sigma\epsilon Gal(Q(\lambda_{(i,j)})/Q)} \sigma(\lambda_{(i,j)})$$

Where $\rho_{(i,j)}$ is the rational valuer character of $Q_{2n}\times D_4$

Then , $\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_{(i,j)}(g_{pr}))/Q)} \sigma(\lambda_{(i,j)}(g_{pr}))$

(I) If j=1 and $r \in D_4$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)) = \rho_i(g_p).1 = \rho_i(g_p).\rho'_j(g'_r)$

Where ρ_i is the rational valuer character of Q_{2n} .

(II) (a) if $j = 2$ anr $r \in \{I^*, t^2, f, ft^2\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)) = \rho_i(g_p).1 = \rho_i(g_p).\rho'_j(g'_r)$.

(b) if $j = 2$ anr $r \in \{t, t^3, ft, ft^3\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(-\lambda_i(g_p)) = -\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).-1 = \rho_i(g_p).-1 = \rho_i(g_p).\rho'_j(g'_r)$.

(III) (a) if $j = 3$ anr $r \in \{I^*, t, t^2, t^3\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)) = \rho_i(g_p).1 = \rho_i(g_p).\rho'_j(g'_r)$.

(b) if $j = 3$ anr $r \in \{f, ft, ft^2, ft^3\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(-\lambda_i(g_p)) = -\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).-1 = \rho_i(g_p).-1 = \rho_i(g_p).\rho'_j(g'_r)$.

(IV) (a) if $j = 4$ anr $r \in \{I^*, t^2, ft, ft^3\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)) = \rho_i(g_p).1 = \rho_i(g_p).\rho'_j(g'_r)$.

(b) if $j = 4$ anr $r \in \{t, t^3, f, ft^2\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(-\lambda_i(g_p)) = -\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).-1 = \rho_i(g_p).-1 = \rho_i(g_p).\rho'_j(g'_r)$.

(V) (a) if $j = 5$ anr $r \in \{I^*\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(2\lambda_i(g_p)) = 2\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).2 = \rho_i(g_p).2 = \rho_i(g_p).\rho'_j(g'_r)$.

(b) if $j = 5$ anr $r \in \{f^2\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(-2\lambda_i(g_p)) = -2\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).-2 = \rho_i(g_p).-2 = \rho_i(g_p).\rho'_j(g'_r)$.

(c) $j = 1$ anr $r \in \{t, t^3, f, ft, ft^2\}$

$\rho_{(i,j)}(g_{pr}) = \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(0.\lambda_i(g_p)) = 0.\sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p))$

$= \sum_{\sigma\epsilon Gal(Q(\lambda_i(g_p))/Q)} \sigma(\lambda_i(g_p)).0 = 0 = \rho_i(g_p).\rho'_j(g'_r)$.

From [I],[II],[III],[IV] anr [V] we have

$$\rho_{(i,j)} = \rho_i \cdot \rho'_j$$

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Then $\equiv^*(Q_{2n}\times D_4) = \equiv^*(Q_{2n}) \otimes \equiv^*(D_4)$. 8

Example(3.3):-

To find the rational valued characters table of $Q_{26}\times D_4$, we can use theorem (3.2) .

By proposition(2.3.9) and proposition (2.3.10), we have

$\equiv^*(Q_{26})$ =

Γ-classes	[1]	[x^2]	[x^7]	[x]	[y]
ρ_1	1	1	1	1	1
ρ_2	12	-1	12	-1	0
ρ_3	1	1	1	1	-1
ρ_4	12	-1	-12	1	0
ρ_5	2	2	-2	-2	0

Table(8)

and

$\equiv^* D_4$ =

CL_α	[I^*]	[r^2]	[r]	[s]	[sr]
$\lvert CL_\alpha\rvert$	1	1	2	2	2
$\lvert C_{D_4}(CL_\alpha)\rvert$	8	8	4	4	4
ρ_1'	1	1	1	1	1
ρ_2'	1	1	-1	1	-1
ρ_3'	1	1	1	-1	-1
ρ_4'	1	1	-1	-1	1
ρ_5'	2	-2	0	0	0

Then **,** by theorem (3.2)

$$\equiv^*(Q_{26}\times D_4) = \equiv^*(Q_{26}) \otimes \equiv^*(D_4) .$$

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Γ-classes	[I, I*]	[a², I*]	[a⁷,I*]	[a,I*]	[b,I*]	[I, t²]	[a², t²]	[a⁷, t²]	[a, t²]	[b, t²]	,t]	[a²,t]	[a⁷, t]	[a, t]	[b, t]	[I, f]	[a², f]	[a⁷, f]	[a, f]	[b, f]	[I, ft]	[a², ft]	[a⁷ ,ft]	[a , ft]	[b, ft]
$\lvert CL_\alpha \rvert$	1	2	1	2	2n	1	2	1	2	2p	2	4	2	4	4n	2	4	2	4	4n	2	4	2	4	4n
$C_{Q_{26}\times D4}(CL_\alpha)\rvert$	224	112	224	112	16	224	112	224	112	16	112	56	112	56	8	112	56	112	56	8	112	56	112	56	8
$\rho_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\rho_{(2,1)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
$\rho_{(3,1)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
$\rho_{(4,1)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
$\rho_{(5,1)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0
$\rho_{(1,2)}$	12	12	12	12	12	-1	-1	-1	-1	-1	12	12	12	12	12	-1	-1	-1	-1	-1	0	0	0	0	0
$\rho_{(2,2)}$	12	12	-12	12	-12	-1	-1	1	-1	1	12	12	-12	12	-12	-1	-1	1	-1	1	0	0	0	0	0
$\rho_{(3,2)}$	12	12	12	-12	-12	-1	-1	-1	1	1	12	12	12	-12	-12	-1	-1	-1	1	1	0	0	0	0	0
$\rho_{(4,2)}$	12	12	-12	-12	12	-1	-1	1	1	-1	12	12	-12	-12	12	-1	-1	1	1	-1	0	0	0	0	0
$\rho_{(5,2)}$	24	-24	0	0	0	-2	2	0	0	0	24	-24	0	0	0	-2	2	0	0	0	0	0	0	0	0
$\rho_{(1,3)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
$\rho_{(2,3)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	-1	1
$\rho_{(3,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	-1	1	1
$\rho_{(4,3)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	-1	-1	1	1	-1
$\rho_{(5,3)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	-2	2	0	0	0
$\rho_{(1,4)}$	12	12	12	12	12	-1	-1	-1	-1	-1	-12	-12	-12	-12	-12	1	1	1	1	1	0	0	0	0	0
$\rho_{(2,4)}$	12	12	-12	12	-12	-1	-1	1	-1	1	-12	-12	12	-12	12	1	1	-1	1	-1	0	0	0	0	0
$\rho_{(3,4)}$	12	12	12	-12	-12	-1	-1	-1	1	1	-12	-12	-12	12	12	1	1	1	-1	-1	0	0	0	0	0
$\rho_{(4,4)}$	12	12	-12	-12	12	-1	-1	1	1	-1	-12	-12	12	12	-12	1	1	-1	-1	1	0	0	0	0	0
$\rho_{(5,4)}$	24	-24	0	0	0	-2	2	0	0	0	-24	24	0	0	0	2	-2	0	0	0	0	0	0	0	0
$\rho_{(1,5)}$	2	2	2	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0
$\rho_{(2,5)}$	2	2	-2	2	-2	2	2	-2	2	-2	-2	-2	2	-2	2	-2	-2	2	-2	2	0	0	0	0	0
$\rho_{(3,5)}$	2	2	2	-2	-2	2	2	2	-2	-2	-2	-2	-2	2	2	-2	-2	-2	2	2	0	0	0	0	0
$\rho_{(4,5)}$	2	2	-2	-2	2	2	2	-2	-2	2	-2	-2	2	2	-2	-2	-2	2	2	-2	0	0	0	0	0
$\rho_{(5,5)}$	4	-4	0	0	0	4	-4	0	0	0	-4	4	0	0	0	-4	4	0	0	0	0	0	0	0	0

Table(9) 10

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