International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 7, July – 2020, Pages: 48-63

Spectrum of Intuitionistic Fuzzy Prime Q-Filters of Q-Algebra

Rakzan Shaker Naji, Habeeb Kareem Abdullah,

Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq

Corresponding Author Email: shakrakzan192@gmail.com

Abstract: *The purpose of this study is investigate the Zariski topology of an involutory Q-algebra. We investigated new concepts of a Q-algebra, fuzzy prime Q-filter, intuitionistic fuzzy prime Q-filter, some properties of them, some basic properties of these concepts, we clarify the relationship with prime Q-filter. In addition, we will study Zariski's topology on intuitionistic fuzzy of a bounded Q-algebra. Moreover, we study some topological properties of a spectrum of a bounded Q-algebra.*

Keywords: Q-algebra, Q-filter, prime Q-filter, fuzzy prime Q-filter, intuitionistic fuzzy prime Q-filter, Zariski topology

1-Introduction

In 2001, the class of Q-algebra, which is a generalisation of (BCK- BCH- BCI)-algebras, was introduced by Neggers. J, Ahn, S.S and Kim, H.S[5]. In 2019 Salman,H.SH [4]."Some new of fuzzy filter in Q-algebra and Pseudo Q-algebra" the nation of Q-filter in Q-algebra. In 2020 Abdullah, H.K and Naji, R.S[3]"Spectrum of prime Q-filter of Q-algebra" the nation of prime Q-filter in Q-algebra. In 1965, Zadeh .L.[9] introduced in the real physical world the notion of fuzzy sub set of the set as a tool for verbal doubt. Atanassov K.T. [6,7] further described The generalization of Intuitionistic fuzzy, Takeuti .G and Titanti S.[14] have also intuitionistic fuzzy sets , but Titanti S. intuitionistic fuzzy mysterious logic in the narrow sense and they derive from the set theory of logic which they said to by (Intuitionistic fuzzy set theory). In 2020 Abdullah, H.K and Naji, R.S[2]"intuitionistic fuzzy Q-filter of Q-algebra " the nation of intuitionistic fuzzy Q-filter in Q-algebra.

In 1944, the Zariski topology was first introduced by O. Zariski[11].The Zariski topology on the spectrum of prime ideals of a commutative ring is one of the main tools in algebraic geometry. Atiyah and Macdonald [10] introduced the spectrum *Spec*(*R*) of a ring *R* as the following: for each ideal *I* of *R*,$V(I)=\{P\in Spec(R) : I\subseteq P\}$,then the set $V(I)$ satisfy the axioms for the closed sets of a topology on *Spec*(*R*), called the Zariski topology. In this study, we introduced a new explanation of (fuzzy prime Q-filter, intuitionistic fuzzy prime Q-filter) and explained a very important features of it, then moved to explain (The Zariski topology) putting forward some examples and some features topological.

(2)Background:

We will explain here some of previous and current features that related to our study.

2.1.Definition:[5]

A set W is known Q-algebra with a " $*$ " binary operation and "0" constant, if $\forall w,v,p\in W$,then

Q_1- $w*w=0$

Q_2- $w*0=w$

Q_3- $(w*v)*p=(w*p)*v$

We re going to define a binary relation denoted $\preccurlyeq$ on W, then $w\preccurlyeq v \Leftrightarrow w*v=0, \forall w,v\in W$.

2.2.Definition :[1]

In Q-algebra W, if $e\in W$ and $w\preccurlyeq e$,$\forall w\in W$, then e is called unit of W. If Q-algebra content unit is called a bounded Q-algebra. In a bounded Q-algebra W, we denoted $e*w$ by w^* $for\ each$ $w\in W$.

2.3.Remark:[1]

In a bounded Q-algebra W, if $w,v\in W$, then

1- $w^*\ast v=v^*\ast w$

2- $0\ast w=0$

2.4.Remark:

From now on, all of the Q − algebra is bounded by unity is unique, the sets W and V are also bounded Q-algebras.

2.5.Definition:[5]

If $\mathtt{f}: (W,*,0) \longrightarrow (V,\grave{*},\grave{0})$ mapping, $\mathtt{f}$ *is said to be*
(1) *Homomorphism* , *if* $\mathtt{f}(w*v) = \mathtt{f}(w) \grave{*} \mathtt{f}(v), \forall w,v \in W$.
(2)*Monomorphism, if* $\mathtt{f}$ *is an injective homomorphism* .
(3)*Epimorphism* , *if* $\mathtt{f}$ *is a surjective homomorphism* .
(4)*Isomorphisom* , *if* $\mathtt{f}$ *is a surjective and injective homomorphism.*

2.6.Proposition :[5]

Let $\mathtt{f}: (W,*,0) \longrightarrow (V,\grave{*},\grave{0})$ be an epimorphism mapping. Then
(1) $\mathtt{f}(w^*) = \left(\mathtt{f}(w)\right)^*, \ \forall w \in W$
(2) if e *is* a unit of W and e' is a units of V , then $\mathtt{f}(e) = e'$.
(3) if $\mathtt{f}$ is an isomorphism, then $\mathtt{f}^{-1}(v^*) = (\mathtt{f}^{-1}(v))^*, \forall v \in V$.

2.7.Definition: [1]

If the w is an element of W satisfies $w^{**} = w$, then w is called an involution, and W is said to be an involutory if for all w of W is an involution.

2.8.Proposition:[2]

If W is an involutory $\mathtt{Q}$-algebra, then $v^* \leq w^*$ iff $w \leq v, \forall w,v \in W$.

2.9.Proposition:[4]

Let $(W,*,0)$ be $\mathtt{Q}$-algebra . If $w,v \in W$, then the following condition are equivalent:
(1) W is involutory.
(2) $w*v = v^* * w^*$
(3)$w*v^* = v*w^*$
(4) if $w \leq v^*$ then $v \leq w^*$

2.10.Notation:[3]

Let $(W,*,0)$ be $\mathtt{Q}$-algebra. For any $w,v \in W$, then :
1- $w \wedge v = (v*(v*w))$
2- $w \vee v = (w^* \wedge v^*)^*$

2.11.Proposition:[3]

If W is an involutory $\mathtt{Q}$-algebra, for all $w,v \in W$, then :
1- $w \leq w \vee v$ *and* $v \leq w \vee v$
2-if $w \leq v$, *then* $w \vee v = v, \forall w,v \in W$.

2.12.Proposition:[3]

Let W be a $\mathtt{Q}$-algebra. If $w^* = v^*$, *then* $w \vee r = v \vee r, \forall r \in W$

2.13.Definition:[4]

If a subset $\mathtt{M}$ of a $\mathtt{Q}$-algebra W, then $\mathtt{M}$ is called a $\mathtt{Q}$-filter of W, $\forall w,v \in W$, if it satisfies
(1) $\mathrm{e} \in \mathtt{M}$,
(2) $(w^* * v^*)^* \in \mathtt{M}$, and $v \in \mathtt{M}$ implies $w \in \mathtt{M}$.

2.14.Definition:[3]

A proper $\mathtt{Q}$-filter $\mathtt{P}$ of W is said to be prime $\mathtt{Q}$-filter, denoted [$\mathtt{P}$-filter] if $w \vee v \in \mathtt{P}$ implies $w \in \mathtt{P}$ *or* $v \in \mathtt{P}$ for any $w,v \in W$

2.15.Definition:[9]

Let W be a nonempty set . A fuzzy set μ in W is a function $\mu: W \to [0,1]$.

2.16.Definition:[9]

Let μ *and* λ be two fuzzy sets in W, then

1-If μ *and* λ are two fuzzy subset of W, then by $\mu \preccurlyeq \lambda$, we mean $\mu(w) \preccurlyeq \lambda(w), \forall w \in W$.

2-The complement of μ [symbolize it, $\overline{\mu}$] is the fuzzy set in $W$by : $\overline{\mu}(w) = 1 - \mu(w), \forall w \in W$.

3-$(\mu \cap \lambda)(w) = min\{\mu(w), \lambda(w)\}$,

4- $(\mu \cup \lambda)(w) = max\{\mu(w), \lambda(w)\}$,

In general, if $\{\mu_i : i \in I\}$ is a family of fuzzy sets in W, then

$\bigcap_{i\in I} \mu_i(w) = inf\{\mu_i(w), i \in I\}$, and

$\bigcup_{i\in I} \mu_i(w) = sup\{\mu_i(w), i \in I\}$,

Which are also fuzzy sets in W.

2.17.Definition:[13]

If $W \neq \emptyset$ and a fuzzy set μ in W, *for any* $s \in [0,1]$,the sets

I) $L(\lambda; s) = \{w: \lambda(w) \preccurlyeq s\}$,it's said to be lower s-level cut of W.

II) $U(\mu; s) = \{w: \mu(w) \succcurlyeq s\}$,it's said to be upper s-level cut of W.

2.18.Definition: [4]

The fuzzy set μ in Q-algebra W it's called a fuzzy Q-filter,(shortly, F-Q-filter) if :

Q_{f1}) $\mu(e) \geq \mu(w), \forall w \in W$

Q_{f2}) $\mu(w) \geq \min\{\mu((w^* * v^*)^*), \mu(v)\}, \forall w, v \in W$

2.19. Proposition :

If μ is a F-Q-filter of W, then the set $W_\mu = \{w \in W, \mu(w) = \mu(e)\}$ is a Q-filter of W.

Proof

Let $(w^* * v^*)^*, v \in W_\mu$. Then $\mu((w^* * v^*)^*) = \mu(e), \mu(v) = \mu(e)$, since μ is F-Q-filter of W,then $\mu(w) \geq \min\{\mu((w^* * v^*)^*), \mu(v)\} = \mu(e)$, but $\mu(e) \geq \mu(w), then\ \mu(w) = \mu(e)$,

thus $w \in W_\mu$, hence W_μ is a Q-filter of W.

2.20.Proposition:

Let μ be a F-Q-filter of an involutory Q-algebra W. If $w \leq v$, then $\mu(w) \leq \mu(v), \forall w, v \in W$.

Proof

If $w \leq v$, then $w * v = 0$, so by Proposition(2.10), $v^* * w^* = 0$. Now

$\mu(v) \geq min\{\mu((v^* * w^*)^*), \mu(w)\}$.

$\mu(v) \geq min\{(0)^*, \mu(w)\}$.

$\mu(v) \geq \mu(w), \forall w, v \in W$

2.21.Corollary:

Let μ be a F-Q-filter of an involutory Q-algebra W. Then $\mu(w) \leq \mu(w \vee v)$, $\mu(v) \leq \mu(w \vee v)$ $\forall w, v \in W$.

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 7, July – 2020, Pages: 48-63

Proof

By Proposition (2.12),1 and Proposition (2.20)

2.22.Remark:[4]

1- Let $\mathtt{M}$ be a Q-filter of W. If $\{\mu_{\mathtt{M}_i}: i \in I\}$ is a family of F-Q-filter, then $\bigcap_{i\in\Delta} \mu_{F_i}$ is a F-Q-filter.

2- The union of two F-Q-filters, it is not necessarily F-Q-filter, in general.

2.23.Proposition:[4]

Let μ be a fuzzy subset of a Q-algebra W. Then μ is a F-Q-filter in W if and only if, μ_s is a Q-filter, $\forall s \in [0, \mu(e)]$.

2.24.Definition:[12]

Let $W \neq \emptyset$, the set $\mathcal{A}$ is an intuitionistic fuzzy set (shortly, IFS) in W ,such that $\mathcal{A} = \{(w, \mu_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(w)) : w \in W\}$, so that is the functions $\mu_{\mathcal{A}} : W \to [0,1]$, $\lambda_{\mathcal{A}} : W \to [0,1]$ mean the degree of membership and mean the degree of nonmember ship correspondingly, such that $0 \preccurlyeq \mu_{\mathcal{A}}(w) + \lambda_{\mathcal{A}}(w) \preccurlyeq 1$. For ease the form is used $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$.

2.25 Definition: [8]

An IFS $\mathcal{A} = (\mu_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(w))$ of a non-empty set W, then

(1)$\Diamond \mathcal{A} = \{(w, 1 - \lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(w)) : w \in\} = \{(w, \overline{\lambda_{\mathcal{A}}}(w), \lambda_{\mathcal{A}}(w)) : w \in W\}$

(2) $\Box\mathcal{A} = \{(w, \mu_{\mathcal{A}}(w), 1 - \mu_{\mathcal{A}}(w)) : w \in W\} = \{(w, \mu_{\mathcal{A}}(w), \overline{\mu_{\mathcal{A}}}(w)) : w \in W\}$

2.26.Definition:[12]

Considerer IFS $\mathcal{A} = (\mu_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(w))$ and $\beta = (\mu_{\beta}(w), \lambda_{\beta}(w))$, $W \neq \emptyset$,then

(1)$\mathcal{A} \cup \beta = \{(w, \mu_{\mathcal{A}}(w) \vee \mu_{\beta}(w), \lambda_{\mathcal{A}}(w) \wedge \lambda_{\beta}(w)) : w \in W\}$

$=\{ (w, max(\mu_{\mathcal{A}}(w), \mu_{\beta}(w)), min(\lambda_{\mathcal{A}}(w), \lambda_{\beta}(w)) : w \in W\}$.

(2)$\mathcal{A} \cap \beta = \{(w, \mu_{\mathcal{A}}(w) \wedge \mu_{\beta}(w), \lambda_{\mathcal{A}}(w) \vee \lambda_{\beta}(w)) : w \in W\}$.

$=\{(w, min(\mu_{\mathcal{A}}(w), \mu_{\mathcal{K}}(w)), max(\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{K}}(w))) : w \in W\}$

(3) $\mathcal{A} \sqsubseteq \beta \; iff \; \mu_{\mathcal{A}}(w) \leq \mu_{\mathcal{B}}(w) \; and \; \lambda_{\mathcal{A}}(w) \geq \lambda_{\mathcal{B}}(w), \; \forall w \in W$.

2.27.Definition:[12]

Let $\{\mathcal{A}_i, i \in \Delta\}$ by a family of IFS in a set W. Then

1- $\cup \mathcal{A}_i = \{(w, \wedge \mu_{\mathcal{A}i}(w), \vee \lambda_{\mathcal{A}i}(w)) : w \in W\}$.

2-$\cap \mathcal{A}_i = \{(w, \vee \mu_{\mathcal{A}i}(w), \wedge \lambda_{\mathcal{A}i}(w)) : w \in W\}$.

Where $(\wedge \mathcal{A}_i)(w) = inf \{\mu_{\mathcal{A}_i}(w), i \in \mathrm{I}\}$, and $(\vee \mu_{\mathcal{A}_i})(w) = sup\{\mu_{\mathcal{A}_i}(w), i \in \mathrm{I}\}$

2.28.Definition:[6]

Let $\mathtt{f}$ be a mapping from a set W to a set V. If $\mathcal{B} = \{(v, \mu_{\mathcal{B}}(v), \lambda_{\mathcal{B}}(v)) | v \in V\}$ is an IFS in V, then the pre-image of $\mathcal{B}$ under $\mathtt{f}$ denoted by $\mathtt{f}^{-1}(\mathcal{A})$ is the IFS in W, defined by :

$f^{-1}(\mathcal{B}) = \{(w, f^{-1}(\mu_{\mathcal{B}}(w)), f^{-1}(\lambda_{\mathcal{B}}(w)) | w \in W\}$, such that : $f^{-1}(\mu_{\mathcal{B}}(w)) = \mu_{\mathcal{B}}(f(w))$, and also $f^{-1}(\lambda_{\mathcal{B}}(w)) = \lambda_{\mathcal{B}}(f(w))$. And if $\mathcal{A} = \{(w, \mu_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(w)) | w \in W\}$ is an IFS in W, then the image of $\mathcal{A}$ under f denoted by : $f(\mathcal{A}) = \{(v, f_{Sup}(\mu_{\mathcal{B}}(v)), f_{Inf}(\lambda_{\mathcal{B}}(v)) | v \in V\}$, where

$$f_{Sup}(\mu_{\mathcal{B}}(v)) = \begin{cases} \sup\{\mu_{\mathcal{A}}(w) : w \in f^{-1}(v)\} & \text{if } f^{-1}(v) \neq \emptyset \\ 0 & otherwise \end{cases}$$

and

$$f_{Inf}(\lambda_{\mathcal{B}}(v)) = \begin{cases} \inf\{\lambda_{\mathcal{A}}(w) : w \in f^{-1}(v)\} & \text{if } f^{-1}(v) \neq \emptyset \\ 0 & otherwise \end{cases}$$, for each $\in V$.

2.29.Definition:[12]

The IFS $\tilde{0}$ *and* $\tilde{1}$ in W are define as $\tilde{0} = \{< w, 0,1 >, w \in W\}$ and $\tilde{1} = \{< w, 1,0 >, w \in W\}$, where 1 and 0 represent the constant maps sending every element of w to 1 and 0, respectively.

2.30.Definition: [2]

An IFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is called intuitionistic fuzzy Q-filter of W,(briefly IFS-Q-filter), if:

I_1- $\mu_{\mathcal{A}}(e) \geqslant \mu_{\mathcal{A}}(w)$, and $\lambda_{\mathcal{A}}(e) \leqslant \lambda_{\mathcal{A}}(w), \forall w \in W$

I_2- $\mu_{\mathcal{A}}(w) \geqslant min\{\mu_{\mathcal{A}}((w^* * v^*)^*), \mu_{\mathcal{A}}(v)\}$,

I_3- $\lambda_{\mathcal{A}}(w) \leqslant max\{\lambda_{\mathcal{A}}((w^* * v^*)^*), \lambda_{\mathcal{A}}(v)\}, \forall w, v \in W$.

2.31.Proposition:[2]

An intuitionistic fuzzy set $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of W is IFS- Q -filter if and only if $\mu_{\mathcal{A}}$ and $\overline{\lambda_{\mathcal{A}}}$ are F-Q-filters of W.

2.32.Corollary [2]

If $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS in W. Than $\square\mathcal{A} = (\mu_{\mathcal{A}}, \overline{\mu_{\mathcal{A}}})$ and $\Diamond\mathcal{A} = (\overline{\lambda_{\mathcal{A}}}, \lambda_{\mathcal{A}})$ are IFS- Q- filters if and only if $\mathcal{A}$ is IFS- Q -filter of W.

2.33.Remark:[2]

1- in W , if $\{\mathcal{A}_i : i \in I\}$ is a family of IFS-Q-filters, then $\bigcap_{i \in I} \mathcal{A}_i$ is IFS-Q-filter.

2- The union of two IFS-Q-filters, it is not necessarily IFS-Q-filter, in general.

2.34.Remark:[2]

Let f be epimorphosim mapping from$(W, *, 0)$into $(V, \grave{*}, \grave{0})$,

1- if $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS in V . such that $f^{-1}(\mathcal{A}) =< \mu_{f^{-1}(\mathcal{A})}, \lambda_{f^{-1}(\mathcal{A})} >$ is an IFS- Q -filter of W, then $\mathcal{A}$ is an IFS- Q -filter of V

2- if $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS- Q-filter of V. Then $f^{-1}(\mathcal{A})$ is an IFS-Q-filter of W.

2.34.Proposition:[2]

An IFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of a Q-algebra W is an IFS- Q-filter if and only if the sets $U(\mu_{\mathcal{A}}; t)$ and $L(\lambda_{\mathcal{A}}; s), \forall t, s \in [0,1]$ are empty or Q-filters of W.

3.Fuzzy prime Q-filter

In this section, we define a fuzzy prime Q-filter of bounded Q-algebra and investigate its properties.

3.1.Definition

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 7, July – 2020, Pages: 48-63

A fuzzy Q-filter μ in a Q-algebra $(W,*,0)$ is said to be fuzzy prime Q-filter,(shortly, F-P-filter),if $\mu\,(w \vee v) \preccurlyeq max\{\,\mu(w), \mu(v)\}, \forall\, w, v \in W$

3.2.Example

Let $W = \{0, \mathrm{a}, \mathrm{b}, \mathrm{c}, e\}$ and a binary operation $*$ is defined by:

*	0	a	b	c	e
0	0	0	0	0	0
a	a	0	a	a	0
b	b	b	0	0	0
c	c	c	0	0	0
e	e	c	a	a	0

Then $(W,*,0)$ is a bounded Q-algebra with unit e. If ,

$$\mu(w) = \begin{cases} 0.8 & \text{if } w = \mathrm{a}, e \\ 0.3 & \text{if } w = 0, \mathrm{c}, \mathrm{b} \end{cases}$$

Then the fuzzy set μ is F-P-filter of $\mathcal{X}$, since

$\mu\,(w \vee v) \preccurlyeq max\{\,\mu(w), \mu(v)\}, \forall\, w \text{ or } v \in \{\mathrm{a}, e\}$

$\mu\,(0 \vee \mathrm{b}) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\mu(0), \mu(\mathrm{b})\} = 0.3$

$\mu\,(\mathrm{b} \vee 0) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\,\mu(\mathrm{b}), \mu(0)\} = 0.3$

$\mu\,(\mathrm{b} \vee \mathrm{c}) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\mu(\mathrm{b}), \mu(\mathrm{c})\} = 0.3$

$\mu\,(\mathrm{c} \vee \mathrm{b}) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\mu(\mathrm{c}), \mu(\mathrm{b})\} = 0.3$

$\mu\,(\mathrm{c} \vee 0) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\mu(\mathrm{c}), \mu(0)\} = 0.3$

$\mu\,(0 \vee \mathrm{c}) = \mu(\mathrm{c}) = 0.3 \preccurlyeq max\{\mu(0), \mu(\mathrm{c})\} = 0.3$

If,

$$\mu(w) = \begin{cases} 0.6 & \text{if } w = \mathrm{c}, e \\ 0.2 & \text{if } w = 0, \mathrm{a}, \mathrm{b} \end{cases}$$

Then the fuzzy set μ is not F-P-filter of W, since

$\mu\,(\mathrm{a} \vee \mathrm{b}) = \mu(e) = 0.6 \not\preccurlyeq max\{\mu(\mathrm{a}), \mu(\mathrm{b})\} = 0.2.$

3.3.Proposition:

Let $(W,*,0)$ be an involutory Q-algebra. Then μ is F-P-filter of W if and only if $\mu(w \vee v) = \mu(v)$ or $\mu(w \vee v) = \mu(v)$.

Proof

By Proposition (2.12),1, Proposition (2.21), and Definition (3.1)

3.4.Proposition:

Let μ be a fuzzy subset of W. Then μ is a F-P-filter in W if and only if μ_s is a P-filter $\forall s \in [0,1]$, such that $\mu_s \neq \emptyset$

Proof

Let μ be a F-P-filter in W and $s \in [0,1]$ such that $\mu_s \neq \emptyset$. Then μ_s is Q-filter by Proposition (2.24), let $w \vee y \in \mu_s$ so $\mu(w \vee y) \geq s$. Then $\mu(w \vee y) \preccurlyeq \max\{\mu(w), \mu(y)\}$,

[since μ is F-P-filter] then $s \preccurlyeq max\{\mu(w), \mu(y)\}, so\ s \preccurlyeq \mu(w)$, or $s \preccurlyeq \mu(y)$, thus $w \in \mu_s\ or$ $y \in \mu_s$, hence μ_s is a P-filter of W .

Conversely, first μ is F-Q-filter by Proposition(2.24). Let $w, y \in W$ and $\mu(w \vee y) = s$. Thus $w \vee y \in \mu_s$ since μ_s is $\mathcal{P}$-filter of W then $w \in \mu_s$ or $y \in \mu_s$, so
$max\ \{\mu(w), \mu(y)\} \geq s = \mu(w \vee y), \forall w, y \in W$. Hence μ is F-P-filter of W.

3.5.Proposition:

If μ is a F-P-filter of W, then the set $W_\mu = \{w \in W, \mu(w) = \mu(e)\}$, is a P-filter of W.

Proof

The fact that W_μ is Q-filter of W (by Proposition (2.20)), let $w, y \in W$ such that $w \vee y \in W_\mu$. Then $\mu(e) = \mu(w \vee y) \preccurlyeq max\{\mu(w), \mu(y)\} = \mu(w)\ or\ \mu(y)$ it follows from that
$\mu(w) = \mu(e)\ or\ \mu(y) = \mu(e)$. Hence $w \in W_\mu\ or\ y \in W_\mu$ so W_μ is P-filter of W.

3.6.Proposition

Let $(W, *, 0)$ be a Q-algebra. If $P \subseteq W$ and $s \in [0,1)$. The P is a P-filter iff μ is an F-P-filter in W, where

$$\mu(w) = \begin{cases} 1 & if\ w \in P \\ s & otherwise \end{cases}$$

Proof

$\Longrightarrow$ Clear

$\Longleftarrow$ 1- Since $\mu(e) \geq \mu(w), \forall w \in W$, so $\mu(e) = 1$, thus $e \in P$

2- Let $(w^* * y^*)^*, y \in P$. Then $\mu((w^* * y^*)^*) = 1\ and\ \mu(y) = 1$
since $\mu(w) \geq \min\{\mu(((w^* * y^*)^*), \mu(y)\} = 1$, then $\mu(w) = 1$,
thus $w \in P$.

3- Let $w, y \in W$ such that $w \vee y \in P$, then
$\mu(w \vee y) = 1 \preccurlyeq max\{\mu(w), \mu(y)\}$
$= \mu(w)\ or\ \mu(y)$, so $w \in P$ or $y \in P$

Then P is P-filter of W.

3.7.Proposition:

In W, if $\{\mu_i, i \in \Delta\}$ is an arbitrary family of F-P-filters of W, then $\cap\ \mu_i$ is a F-P-filter of W.

Proof

since $\mu_i, i \in$ I is a F-P-filter, then μ_i is F-Q-filter of W, so $\cap\ \mu_i$ is a F-Q-filter of W (by (Remark(2.23). Now
$\mu_i(w \vee y) \preccurlyeq max\{\mu_i(w), \mu_i(y)\}$,
$inf\ \mu_i\ (w \vee y) \preccurlyeq inf\ \{max\{\mu_i(w), \mu_i(y)\}\}$,
$\preccurlyeq max\{inf\ \mu_i(w), inf\ \mu_i(y)\}$

Then $\cap\ \mu_i$ is a F-P-filter of W.

3.8.Remark:

Note that, the union of two F-P-filters it is not necessarily F-P-filter, since the union of two F-Q-filters in general are not F-Q-filter by Remark(2.23).

4.Intuitionistic fuzzy prime Q-filter

In this section, we define intuitionistic fuzzy prime Q-filter of a bounded Q-algebra and study relationship with prime Q-filter.

4.1.Definition:

An IFS-Q-filter $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of a Q-algebra $(W, *, 0)$ is said to be intuitionistic fuzzy prime Q-filter of W,(briefly IFS-P-filter), if

1- $\mu_{\mathcal{A}}\ (w \vee y) \preccurlyeq max\{\mu_{\mathcal{A}}\ (w), \mu_{\mathcal{A}}\ (y)\}$.

2- $\lambda_{\mathcal{A}}\ (w \vee y) \succcurlyeq min\{\lambda_{\mathcal{A}}\ (w), \lambda_{\mathcal{A}}\ (y)\}, \forall\ w, y \in W$.

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 7, July – 2020, Pages: 48-63

4.2.Example:

Let $W = \{0, \mathrm{a}, \mathrm{b}, \mathrm{c}, e\}$ and a binary operation * is defined by

*	0	a	b	e	c
0	0	0	0	0	0
a	a	0	a	0	c
b	b	b	0	0	b
e	e	b	0	0	b
c	c	0	c	0	0

Then $(W,*,0)$ is a bounded Q-algebra with unit e.
If,

$$\mu_{\mathcal{A}}(x) = \begin{cases} 0.8 & if\ w = e, b \\ 0.4 & if\ w = 0, a, \mathrm{c} \end{cases} \quad \lambda_{\mathcal{A}}(w) = \begin{cases} 0.2 & if\ w = e, b \\ 0.6 & if\ w = 0, a, \mathrm{c} \end{cases}$$

Then $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is IFS-Q-filter of W, and IFS- P-filter, since

$\mu_{\mathcal{A}}(0 \vee \mathrm{a}) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(0), \mu_{\mathcal{A}}(a)\} = 0.4$
$\mu_{\mathcal{A}}(a \vee 0) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(a), \mu_{\mathcal{A}}(0)\} = 0.4$
$\mu_{\mathcal{A}}(a \vee \mathrm{a}) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(a), \mu_{\mathcal{A}}(a)\} = 0.4$
$\mu_{\mathcal{A}}(a \vee \mathrm{c}) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(a), \mu_{\mathcal{A}}(\mathrm{c})\} = 0.4$
$\mu_{\mathcal{A}}(\mathrm{c} \vee \mathrm{a}) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(\mathrm{c}), \mu_{\mathcal{A}}(a)\} = 0.4$
$\mu_{\mathcal{A}}(\mathrm{c} \vee \mathrm{c}) = \mu_{\mathcal{A}}(0) = 0.4 \preccurlyeq max\{\mu_{\mathcal{A}}(\mathrm{c}), \mu_{\mathcal{A}}(\mathrm{c})\} = 0.4$
$\lambda_{\mathcal{A}}(0 \vee \mathrm{a}) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(0), \lambda_{\mathcal{A}}(a)\} = 0.6$
$\lambda_{\mathcal{A}}(a \vee 0) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(a), \lambda_{\mathcal{A}}(0)\} = 0.6$
$\lambda_{\mathcal{A}}(a \vee a) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(a), \lambda_{\mathcal{A}}(a)\} = 0.6$
$\lambda_{\mathcal{A}}(a \vee \mathrm{c}) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(a), \lambda_{\mathcal{A}}(\mathrm{c}) = 0.6$
$\lambda_{\mathcal{A}}(\mathrm{c} \vee \mathrm{a}) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(\mathrm{c}), \lambda_{\mathcal{A}}(a) = 0.6$
$\lambda_{\mathcal{A}}(\mathrm{c} \vee \mathrm{c}) = \lambda_{\mathcal{A}}(0) = 0.6 \succcurlyeq \min\{\lambda_{\mathcal{A}}(\mathrm{c}), \lambda_{\mathcal{A}}(\mathrm{c})\} = 0.6$

If,

$$\alpha_{\mathcal{B}}(w) = \begin{cases} 0.6 & if\ w = 0, e \\ 0.4 & if\ w = a, b, \mathrm{c} \end{cases} \quad \beta_{\mathcal{B}}(w) = \begin{cases} 0.3 & if\ w = 0, e \\ 0.5 & if\ w = a, b, \mathrm{c} \end{cases}$$

Then $\mathcal{B} = (\alpha_{\mathcal{B}}, \beta_{\mathcal{B}})$ is IFS-Q-filter of W, but is not IFS-P-filter, since

$\alpha_{\mathcal{B}}(a \vee \mathrm{c}) = \alpha_{\mathcal{B}}(0) = 0.6 \not\preccurlyeq max\{\alpha_{\mathcal{B}}(a), \alpha_{\mathcal{B}}(\mathrm{c})\} = 0.4$

4.3.Proposition:

In Q-algebra W, if $\{\mathcal{A}_i, i \in \Delta\}$ is an arbitrary family of IFS- P-filter of W, the $\cap\, \mathcal{A}_i$ is an IFS-P -filter of W.

Proof

since $\mathcal{A}_i, \mathrm{i} \in \Delta$ are IFS- P-filters, then $\mathcal{A}_i$ are IFS-Q-filter, so $\cap\, \mathcal{A}_i$ is an IFS-Q-filter (by Remark(2.33). Now, since

$$\mu_{\mathcal{A}_i}(w \vee y) \preccurlyeq \max\{\mu_{\mathcal{A}_i}(w), \mu_{\mathcal{A}_i}(y)\},$$
$$\wedge\mu_{\mathcal{A}_i}(w \vee y) \preccurlyeq \wedge\{\max\{\mu_{\mathcal{A}_i}(w), \mu_{\mathcal{A}_i}(y)\}\},$$
$$\preccurlyeq \max\{\wedge\, \mu_{\mathcal{A}_i}(w), \wedge\mu_{\mathcal{A}_i}(y)\}$$

And,

$$\lambda_{\mathcal{A}_i}(w \vee y) \succcurlyeq \min\{\lambda_{\mathcal{A}_i}(w), \lambda_{\mathcal{A}_i}(y)\}$$
$$\vee\lambda_{\mathcal{A}_i}(w \vee y) \succcurlyeq \vee\{\min\{\lambda_{\mathcal{A}_i}(w), \lambda_{\mathcal{A}_i}(y)\}\}$$

$\geqslant \min\{\vee\lambda_{\mathcal{A}_i}(w), \vee\lambda_{\mathcal{A}_i}(y)\}$

Then $\cap\, \mathcal{A}_i$ is an IFS- P-filter of W .

4.4.Remark:

Note that, the union of two IFS- P-filter, it is not necessarily IFS- P-filter in general, since the union of two IFS-Q-filter is not IFS-Q-filter in general.

4.5.Proposition:

An IFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of $(W,*,0)$ is an IFS- P-filter if and only if $\mu_{\mathcal{A}}$ and $\overline{\lambda_{\mathcal{A}}}$ are F-P-filter of W.

Proof

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be an IFS-P-filter of W. Since $\mathcal{A}$ is an IFS- Q-filter, then by Proposition (2.32), $\mu_{\mathcal{A}}$ *and* $\overline{\lambda_{\mathcal{A}}}$ are F-Q-filters of W. Clearly, $\mu_{\mathcal{A}}$ is F-P-filter of W.

$\forall\, w, y \in W$, we have $\overline{\lambda_{\mathcal{A}}}(x) = 1 - \lambda_{\mathcal{A}}(w)$ [by Definition (2.17)]

$$\begin{aligned}\overline{\lambda_{\mathcal{A}}}(w \vee y) = 1 - \lambda_{\mathcal{A}}(w \vee y) &\leq 1 - min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\}\\ &= max\{1 - \lambda_{\mathcal{A}}(w), 1 - \lambda_{\mathcal{A}}(y)\}\\ &= max\{\overline{\lambda_{\mathcal{A}}}(w), \overline{\lambda_{\mathcal{A}}}(y)\}\end{aligned}$$

Hence $\overline{\lambda_{\mathcal{A}}}$ is a F-P-filter of .

Conversely, let $\mu_{\mathcal{A}}$ *and* $\overline{\lambda_{\mathcal{A}}}$ are F-P-filters of W. So $\mu_{\mathcal{A}}$ *and* $\overline{\lambda_{\mathcal{A}}}$ are F -Q-filters of W, then $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is IFS- Q-filter (by Proposition 2.32)). Also,

$\mu_{\mathcal{A}}(w \vee y) \leqslant max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$, and

$$\begin{aligned}1 - \lambda_{\mathcal{A}}(w \vee y) = \overline{\lambda_{\mathcal{A}}}(w \vee y) &\leqslant max\{\overline{\lambda_{\mathcal{A}}}(w), \overline{\lambda_{\mathcal{A}}}(y)\}\\ &= max\{1 - \lambda_{\mathcal{A}}(w), 1 - \lambda_{\mathcal{A}}(y)\}\\ &= 1 - min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\},\end{aligned}$$

So, $\lambda_{\mathcal{A}}(w \vee y) \geqslant \min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\}$, thus $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is IFS-P-filter

4.6.Corollary:

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be an IFS in a Q-algebra . Than $\Box\mathcal{A} = (\mu_{\mathcal{A}}, \overline{\mu_{\mathcal{A}}})$ and $\Diamond\, \mathcal{A} = (\overline{\lambda_{\mathcal{A}}}, \lambda_{\mathcal{A}})$ are IFS- P-filters if and only if $\mathcal{A}$ is IFS- P-filter of W.

Proof

Since $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS-P-filter, then $\Box\mathcal{A} = (\mu_{\mathcal{A}}, \overline{\mu_{\mathcal{A}}})$ and $\Diamond\, \mathcal{A} = (\overline{\lambda_{\mathcal{A}}}, \lambda_{\mathcal{A}})$ are IFS-Q-filters (by Proposition (2.33)). Now, since $\mu_{\mathcal{A}}(w \vee y) \leqslant max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$, then

$$\begin{aligned}1 - \mu_{\mathcal{A}}(w \vee y) &\geq 1 - max\,\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}\\ &\geqslant min\,\{1 - \mu_{\mathcal{A}}(w), 1 - \mu_{\mathcal{A}}(y)\}\\ &\geqslant min\{\overline{\mu_{\mathcal{A}}}(w), \overline{\mu_{\mathcal{A}}}(y)\},\ \forall\, w, y \in W\end{aligned}$$

Hence $\Box\mathcal{A} = (\mu_{\mathcal{A}}, \overline{\mu_{\mathcal{A}}})$ is IFS- P-filter of W. Also, $\lambda_{\mathcal{A}}(w \vee y) \geq min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\}$

Now,

$$\begin{aligned}\overline{\lambda_{\mathcal{A}}}(w \vee y) = 1 - \lambda_{\mathcal{A}}(w \vee y) &\leqslant 1 - min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\}\\ &\leqslant max\{1 - \lambda_{\mathcal{A}}(w), 1 - \lambda_{\mathcal{A}}(y)\}\\ &\leqslant max\{\overline{\lambda_{\mathcal{A}}}(w), \overline{\lambda_{\mathcal{A}}}(y)\},\ \forall\, w, y \in \mathcal{X}\end{aligned}$$

Hence $\Diamond\, \mathcal{A} = (\overline{\lambda_{\mathcal{A}}}, \lambda_{\mathcal{A}})$ is an IFS- P-filter of W.

4.7.Proposition:

An IFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of a Q-algebra W is an IFS- P-filter if and only if the sets $U(\mu_{\mathcal{A}}; t)$ and $L(\lambda_{\mathcal{A}}; s)$, $\forall t, s \in [0,1]$ are empty or P-filters of W.

Proof

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be an IFS- P-filter of W. Then $\mathcal{A}$ is IFS-Q-filter of W, so $U(\mu_{\mathcal{A}}; t) \neq \emptyset \neq L(\lambda_{\mathcal{A}}; s)$ are Q-filters $\forall t, s \in [0,1]$ (by Proposition (2.35)).
Let $w, y \in W$ such that $w \vee y \in U(\mu_{\mathcal{A}}; t)$. So $\alpha_{\mathcal{A}}(w \vee y) \geqslant t$. Since $\mu_{\mathcal{A}}(w \vee y) \leqslant max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$, then $\mu_{\mathcal{A}}(w) \geqslant t$ or $\mu_{\mathcal{A}}(y) \geqslant t$, so $w \in U(\mu_{\mathcal{A}}; t)$ or $y \in U(\mu_{\mathcal{A}}; t)$. Hence $U(\mu_{\mathcal{A}}; t)$ is a P-filter of .
Let $w, y \in W$, such that $w \vee y \in L(\lambda_{\mathcal{A}}; s)$. Then $\lambda_{\mathcal{A}}(w \vee y) \leqslant s$. Since $\lambda_{\mathcal{A}}(w \vee y) \geqslant min\{\lambda_{\mathcal{A}}(w), \beta_{\mathcal{A}}(y)\}$, so $\lambda_{\mathcal{A}}(w) \leqslant s$ or $\lambda_{\mathcal{A}}(y) \leqslant s$, then $w \in L(\lambda_{\mathcal{A}}; s)$ or $y \in L(\lambda_{\mathcal{A}}; s)$.Hence $L(\lambda_{\mathcal{A}}; s)$ is a P-filter .
Conversely, suppose that $U(\mu_{\mathcal{A}}; t)$ *and* $L(\lambda_{\mathcal{A}}; s)$ are P-filters and $\forall t, s \in [0,1]$ such that $U(\mu_{\mathcal{A}}; t) \neq \emptyset \neq L(\lambda_{\mathcal{A}}; s)$, then $U(\mu_{\mathcal{A}}; t)$ *and* $L(\lambda_{\mathcal{A}}; s)$ are Q-filters so by Proposition(2.35), $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is IFS- Q-filter.
Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be no IFS- P-filter. Then there exist $w, y \in W$,such that $\mu_{\mathcal{A}}(w \vee y) > max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$ or $\lambda_{\mathcal{A}}(w \vee y) < min\{\lambda_{\mathcal{A}}(w), \lambda_{\mathcal{A}}(y)\}$.
If we put $t = \mu_{\mathcal{A}}(w \vee y)$. So $w \vee y \in U(\mu_{\mathcal{A}}; t)$,
but $w, y \notin U(\mu_{\mathcal{A}}; t)$, which is a contradiction.
similarly if we put $s = \lambda_{\mathcal{A}}(w \vee y)$. So $w \vee y \in L(\lambda_{\mathcal{A}}; s)$, but $w, y \notin L(\lambda_{\mathcal{A}}; s)$, which is a contradiction, hence $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS- P-filter of W.

4.7.Proposition:

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be an IFS- P-filter of a Q-algebra W. Then $W_\mu = \{w \in W; \mu_{\mathcal{A}}(w) = \mu_{\mathcal{A}}(e)\}$ and $W_\lambda = \{w \in W: \lambda_{\mathcal{A}}(w) = \beta_{\mathcal{A}}(e)\}$ are P-filters of W.

Proof

Let $w, y \in W$ such that $w \vee y \in W_\mu$. So $\mu_{\mathcal{A}}(w \vee y) = \mu_{\mathcal{A}}(e)$, since $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ IFS-P-filter of W, then $\mu_{\mathcal{A}}(w \vee y) \leqslant max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$, but $\mu_{\mathcal{A}}(e) \geqslant \mu_{\mathcal{A}}(w)$
so $\mu_{\mathcal{A}}(e) = max\{\mu_{\mathcal{A}}(w), \mu_{\mathcal{A}}(y)\}$, now either $\mu_{\mathcal{A}}(e) = \mu_{\mathcal{A}}(w)$, thus $w \in W_\mu$, or $\mu_{\mathcal{A}}(e) = \mu_{\mathcal{A}}(y)$, thus $y \in W_\mu$ hence W_μ is a P-filter.
And, let $w, y \in W$ such that $w \vee y \in W_\lambda$ so $\lambda_{\mathcal{A}}(w \vee y) = \lambda_{\mathcal{A}}(e)$. Since $\lambda_{\mathcal{A}}(w \vee y) \geqslant min\{\lambda_{\mathcal{A}}(w), \beta_{\mathcal{A}}(y)\}$ [since $\mathcal{A}$ is an IFS-P-filter], then $\lambda_{\mathcal{A}}(e) \leqslant \lambda_{\mathcal{A}}(w)$, so $\lambda_{\mathcal{A}}(e) = \lambda_{\mathcal{A}}(w)$, thus $w \in W_\lambda$, or $\lambda_{\mathcal{A}}(e) = \lambda_{\mathcal{A}}(y)$, thus $y \in W_\lambda$. Hence W_λ is P-filter of W.

4.8.Proposition:

Let $(W, *, 0)$ be an involutory Q-algebra. Then $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is IFS- P-filter of W if and only if $\mu_{\mathcal{A}}(w \vee y) = \mu_{\mathcal{A}}(w)$ or $\mu_{\mathcal{A}}(w \vee y) = \mu_{\mathcal{A}}(y)$ and $\lambda_{\mathcal{A}}(w \vee y) = \lambda_{\mathcal{A}}(w)$ or $\lambda_{\mathcal{A}}(w \vee y) = \lambda_{\mathcal{A}}(y)$.

Proof

Let $w, y \in W$. Then by Proposition(2.12),1, $w \leq w \vee y$ *and* $y \leq w \vee y$ so by corollary (2.22), $\mu_{\mathcal{A}}(w) \leq \mu_{\mathcal{A}}(w \vee y)$, $\mu_{\mathcal{A}}(y) \leq \mu_{\mathcal{A}}(w \vee y)$, $\lambda_{\mathcal{A}}(w \vee y) \geq \lambda_{\mathcal{A}}(w)$ *and* $\lambda_{\mathcal{A}}(w \vee y) \geq \lambda_{\mathcal{A}}(y)$, then by Definition (4.1), $\mu_{\mathcal{A}}(w \vee y) = \mu_{\mathcal{A}}(w)$ or $\mu_{\mathcal{A}}(w \vee y) = \mu_{\mathcal{A}}(y)$ and $\lambda_{\mathcal{A}}(w \vee y) = \lambda_{\mathcal{A}}(w)$ or $\lambda_{\mathcal{A}}(w \vee y) = \lambda_{\mathcal{A}}(y)$

4.9.Proposition:

If $(W, *, 0)$and $(V, *, 0)$ are bounded Q-algebra and f is an epimorphosim mapping from W into V, then $f^{-1}(\mathcal{A})$ is an IFS- P-filter of W if $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS- P-filter of V.

Proof

Let $w, y \in W$ and $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be an IFS- P-filter of V. Now

$$\begin{aligned}\mu_{f^{-1}(\mathcal{A})}(w \vee y) = \mu_{\mathcal{A}}(f(w \vee y)) &= \mu_{\mathcal{A}}(f((y^* * (y^* * w^*)^*) \\ &= \mu_{\mathcal{A}}(f(y^*) * (f(y^*) * f(w^*))^*) \\ &= \mu_{\mathcal{A}}\big(f(w) \vee f(y)\big)\end{aligned}$$

$$\preccurlyeq max\{\mu_{\mathcal{A}}(\mathtt{f}(w),\mu_{\mathcal{A}}(\mathtt{f}(y)\}$$
$$= max\ \{\mu_{\mathtt{f}^{-1}(\mathcal{A})}(w),\mu_{\mathtt{f}^{-1}(\mathcal{A})}(y)\}$$

Also,

$$\lambda_{\mathtt{f}^{-1}(\mathcal{A})}(w \vee y) = \lambda_{\mathcal{A}}(\mathtt{f}(w \vee y)) = \lambda_{\mathcal{A}}(\mathtt{f}((y^* * (y^* * w^*)^*)$$
$$= \lambda_{\mathcal{A}}((\mathtt{f}(y^*) * (\mathtt{f}(y^*) * \mathtt{f}(w^*))^*)$$
$$= \lambda_{\mathcal{A}}\big(\mathtt{f}(w) \vee \mathtt{f}(y)\big)$$
$$\succcurlyeq min\{\lambda_{\mathcal{A}}(\mathtt{f}(w),\lambda_{\mathcal{A}}(\mathtt{f}(y)\}$$
$$= min\{\lambda_{\mathtt{f}^{-1}(\mathcal{A})}(w),\lambda_{\mathtt{f}^{-1}(\mathcal{A})}(y)\}$$

And since $\mathtt{f}^{-1}(\mathcal{A})$ is an IFS- Q-filter of W (by Remark(2.34),2, then $\mathtt{f}^{-1}(\mathcal{A})$ is an IFS- P-filter of W.

4.10.Proposition:

If $\mathtt{f}$ is an epimorphosim mapping from $(W,*,0)$into $(V,\dot{*},\dot{0})$, and $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS of V, such that $\mathtt{f}^{-1}(\mathcal{A}) =< \mu_{\mathtt{f}^{-1}(\mathcal{A})}, \lambda_{\mathtt{f}^{-1}(\mathcal{A})} >$ is an IFS- P-filter of W, then $\mathcal{A}$ is an IFS- P-filter of V.

Proof

$\forall c, d \in V, \exists w, y \in W$, such that $\mathtt{f}(w) = c, \mathtt{f}(y) = d$,then

$$\mu_{\mathcal{A}}(c \vee d) = \mu_{\mathcal{A}}\big(\mathtt{f}(w) \vee \mathtt{f}(y)\big) = \mu_{\mathcal{A}}\big(\mathtt{f}(w \vee y)\big)$$
$$= \mu_{\mathtt{f}^{-1}(\mathcal{A})}(w \vee y) \preccurlyeq max\big\{\mu_{\mathtt{f}^{-1}(\mathcal{A})}(w), \mu_{\mathtt{f}^{-1}(\mathcal{A})}(y)\big\}$$
$$= max\big\{\mu_{\mathcal{A}}\big(\mathtt{f}(w)\big), \mu_{\mathcal{A}}\big(\mathtt{f}(y)\big)\big\}$$
$$= max\{\mu_{\mathcal{A}}(c), \mu_{\mathcal{A}}(d)\}$$

And

$$\lambda_{\mathcal{A}}(c \vee d) = \lambda_{\mathcal{A}}\big(\mathtt{f}(w) \vee \mathtt{f}(y)\big) = \lambda_{\mathcal{A}}\big(\mathtt{f}(w \vee y)\big)$$
$$= \lambda_{\mathtt{f}^{-1}(\mathcal{A})}(w \vee y) \succcurlyeq min\big\{\lambda_{\mathtt{f}^{-1}(\mathcal{A})}(w), \lambda_{\mathtt{f}^{-1}(\mathcal{A})}(y)\big\}$$
$$= min\big\{\lambda_{\mathcal{A}}\big(\mathtt{f}(w)\big), \lambda_{\mathcal{A}}\big(\mathtt{f}(y)\big)\big\}$$
$$= min\{\lambda_{\mathcal{A}}(c), \lambda_{\mathcal{A}}(d)\}$$

Also $\mathcal{A}$ is an IFS-Q-filter of V (by Remark(2.34),)), then $\mathcal{A}$ is an IFS-P-filter of V.

5. The Spectrum of intuitionistic fuzzy Q-algebra.

In this section, we provide the notion of spectrum of intuitionistic in a bounded Q-algebra and we introduce some of its properties.

5.1.Definition:

Let $(W,*,0)$ be a bounded -algebra and $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is an IFS of W, we define .

1- $Spec(W) = \{\mathrm{P}: \mathrm{P}\ is\ IFS - \mathrm{P} - \text{filter}\}$

2-$\mathcal{V}(\mathcal{A}) = \{\mathrm{P} \in Spec(W): \mathcal{A} \sqsubseteq \mathrm{P}\}$

3- $X(\mathcal{A}) = \{\mathrm{P} \in Spec(W): \mathcal{A} \not\sqsubseteq \mathrm{P}\} = {}^{Spec(W)}\big/_{\mathcal{V}(\mathcal{A})}$

Note that, $X(\mathcal{A})$ is the complement of $\mathcal{V}(\mathcal{A})$ in $Spec(W)$.

5.2.Proposition:

Let $(W,*,0)$ be Q-algebra. And $\mathcal{A}, \mathcal{B}$ are IFS of W. If $\mathcal{A} \sqsubseteq \mathcal{B}$ implies that, $\mathcal{V}(\mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{A})$ ($X(\mathcal{A}) \sqsubseteq X(\mathcal{B})$).

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 7, July – 2020, Pages: 48-63

Proof

Let $\mathcal{P} \in \mathcal{V}(\mathcal{B})$. Then $\mathcal{B} \sqsubseteq \mathrm{P}$. Since $\mathcal{A} \sqsubseteq \mathcal{B}$, then $\mathcal{A} \sqsubseteq \mathrm{P}, so\ \mathcal{P} \in \mathcal{V}(\mathcal{A})$, hence$\mathcal{V}(\mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{A})$.

5.3.Proposition:

If P is a smallest IFS- P-filter continuing $\mathcal{A}$, then $\mathcal{V}(\mathcal{A}) = \mathcal{V}(\mathrm{P})$.

Proof

It's clear that $\mathcal{V}(\mathrm{P}) \sqsubseteq \mathcal{V}(\mathcal{A})$ by Proposition (5.2). Now

Let $\mathrm{P}_1 \in \mathcal{V}(\mathcal{A})$. $So\ \mathcal{A} \sqsubseteq \mathrm{P}_1$, but P is as smallest IFS- P-filter continuing $\mathcal{A}$. Then, $\mathrm{P} \sqsubseteq \mathrm{P}_1$, so $\mathrm{P}_1 \in \mathcal{V}(\mathrm{P})$ thus $\mathcal{V}(\mathcal{A}) = \mathcal{V}(\mathcal{P})$

5.4.Proposition:

Let $\mathcal{A}\ and\ \mathcal{B}$ be two IFS- P-filters of W. Then $\mathcal{V}(\mathcal{A} \cup \mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{A}) \cup \mathcal{V}(\mathcal{B})$

Proof

Since $\mathcal{A} \sqsubseteq \mathcal{A} \cup \mathcal{B}\ and\ \mathcal{B} \sqsubseteq \mathcal{A} \cup \mathcal{B}$, so $\mathcal{V}(\mathcal{A} \cup \mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{A})\ and\ \mathcal{V}(\mathcal{A} \cup \mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{B})$, then $\mathcal{V}(\mathcal{A} \cup \mathcal{B}) \sqsubseteq \mathcal{V}(\mathcal{A}) \cup \mathcal{V}(\mathcal{B})$

5.5.Definition:

For an IFS-Q-filter $\mathcal{A}$ of a bounded Q-algebra $(W,*,0)$ the prime radical $rad(\mathcal{A})$ of $\mathcal{A}$ is the intersection of all IFS-P-filter of W containing $\mathcal{A}$. In case there is no such IFS- P-filter containing β, $rad(\beta) = \tilde{1}$

5.6.Proposition:

Let $\mathcal{A}, \mathcal{B}$ be two IFS-Q-filters of W. Then :

1- $\mathcal{A} \sqsubseteq rad(\mathcal{A})$

2- $rad(rad(\mathcal{A})) = rad(\mathcal{A})$

3-if $\mathcal{A}$ is IFS- P-filter, then $red(\mathcal{A}) = \mathcal{A}$

4-if $\mathcal{A} \sqsubseteq \mathcal{B}, then\ rad(\mathcal{A}) \sqsubseteq rad(\mathcal{B})$

Proof

1- By Definition (5.5).

2- $rad\big(rad(\mathcal{A})\big) \supseteq rad(\mathcal{A})$ (by 1). Now, if $rad(\mathcal{A}) \sqsubseteq \mathrm{P}$ where P is IFS- P-filter, then $\mathcal{A} \sqsubseteq \mathrm{P}$ (since $\mathcal{A} \sqsubseteq rad(\mathcal{A})$), then $rad(\mathcal{A}) \sqsubseteq rad\big(rad(\mathcal{A})\big)$. Thus $rad\big(rad(\mathcal{A})\big) = rad(\mathcal{A})$.

3- Since $\mathcal{A}$ is IFS- P-filter, then $rad(\mathcal{A}) \sqsubseteq \mathcal{A}$, since $\mathcal{A} \sqsubseteq red(\mathcal{A})$ then $rad(\mathcal{A}) = \mathcal{A}$

4- Since $\mathcal{B} \sqsubseteq rad(\mathcal{B})$ and $\mathcal{A} \sqsubseteq \mathcal{B}$, so $\mathcal{A} \sqsubseteq rad(\mathcal{B})$ then,

$rad(\mathcal{A}) \sqsubseteq rad\big(rad(\mathcal{B})\big) = rad(\mathcal{B})$, so $rad(\mathcal{A}) \sqsubseteq rad(\mathcal{B})$.

5.7.Proposition:

For any IFS-Q-filters $\mathcal{A}\ and\ \mathcal{B}$ of W, the following are hold :

1- $\mathcal{V}(\mathcal{A}) = \mathcal{V}(rad(\mathcal{A}))$

2- $\mathcal{V}(\mathcal{A}) = \mathcal{V}(\mathcal{B}) \leftrightarrow rad(\mathcal{A}) = rad(\mathcal{B})$

Proof

1-Since $\mathcal{A} \sqsubseteq rad(\mathcal{A})$, then $\mathcal{V}(rad(\mathcal{A})) \sqsubseteq \mathcal{V}(\mathcal{A})$ [by Proposition (5.2)].

And let $\mathrm{P} \in \mathcal{V}(\mathcal{A})$. Then $\mathcal{A} \sqsubseteq \mathrm{P}$,so $rad(\mathcal{A}) \sqsubseteq rad(\mathrm{P}) = \mathrm{P}$, thus $\mathcal{P} \in \mathcal{V}(rad(\mathcal{A}))$.

Hence $\mathcal{V}(\mathcal{A}) = \mathcal{V}(rad(\mathcal{A}))$.

2- $\Longrightarrow$ Since $rad(\mathcal{A}) = \cap_{\mathrm{P} \in \mathcal{V}(\mathcal{A})} \mathrm{P} = \cap_{\mathrm{P} \in \mathcal{V}(\mathcal{B})} \mathrm{P} = rad(\mathcal{B})$.

$\Leftarrow$ By (1)

5.8.Proposition:

Let $(W,*,0)$ be a bounded Q-algebra and $\mathrm{T}(W) = \{X(\mathcal{A}): \mathcal{A}\ \ is\ \mathrm{IFS}-\mathrm{Q}-\mathrm{filter}\ in\ W\}$, then

1- $Spec(W), \emptyset \in \mathrm{T}(W)$.

2-$\bigcup_{i\in I} X\ (\mathcal{A}_i) \in \mathrm{T}(W)$, for any family $\{\mathcal{A}_{i\in I}: \mathcal{A}_i\ is\ \mathrm{IFS}-\mathrm{Q}-\mathrm{filter}\ in\ W\}$.

Proof

1-Since $\mathcal{V}(\tilde{0}) = Spec(W)$ and $\mathcal{V}(\tilde{1}) = \emptyset$, then $X(\tilde{0}) = \emptyset, X(\tilde{1}) = Spec(W)$, so $Spec(W), \emptyset \in \mathrm{T}(W)$

2- For any family $\{X(\mathcal{A}_i)\}_{i\in I} \subseteq \mathrm{T}(W)$

let $\mathrm{P} \in \bigcup_{i\in I} X\ (\mathcal{A}_i)$. So $\mathcal{A}_i \nsubseteq \mathrm{P}$ for some $i \in I$, then $\bigcup_{i\in I} \mathcal{A}_i \nsubseteq \mathrm{P}$

$so\ \mathrm{P} \in X(< \bigcup_{i\in I} \mathcal{A}_i >)$, thus $\bigcup_{i\in I} X\ (\mathcal{A}_i) \subseteq X(\bigcup_{i\in I} \mathcal{A}_i)$.

And let $\mathrm{P} \in X(< \bigcup_{i\in I} \mathcal{A}_i >), so\ \bigcup_{i\in I} \mathcal{A}_i \nsubseteq \mathrm{P}, then\ \mathcal{A}_i \nsubseteq \mathrm{P}$ for some $i \in \mathrm{I}, \mathrm{P} \in \bigcup_{i\in I} X\ (\mathcal{A}_i)$

$X(< \bigcup_{i\in I} \mathcal{A}_i >) \subseteq \bigcup_{i\in I} X\ (\mathcal{A}_i)$, then $X(\bigcup_{i\in I} \mathcal{A}_i) = \bigcup_{i\in I} X\ (\mathcal{A}_i)$, Hence $\bigcup_{i\in I} X\ (\mathcal{A}_i) \in \mathrm{T}(W\ )$.

Thus from Proposition (5.8), $\mathrm{T}(W)$ contains the empty set and $Spec(W)$, and $\mathrm{T}(W)$ is closed under arbitrary unions. Then $\mathrm{T}(W)$ is a topology on $Spec(W)$ if $\mathrm{T}(W)$ is closed under finite intersection.

5.9.Definition:

Let $(W,*,0)$ be a bounded Q-algebra and $\mathrm{T}(W) = \{X(\mathcal{A}): \mathcal{A}\ \ is\ \mathrm{IFS}-\mathrm{Q}-\mathrm{filter}\ in\ W\}$. Then $\mathrm{T}(W)$ is called Zariski topology of W if $X(\mathcal{A}) \cap X(\mathcal{B}) \in \mathrm{T}(W)$, for every $\mathcal{A}\ and\ \mathcal{B}$ two IFS-Q-filters of W.

5.10.Proposition:

If W is a bounded Q-algebra, then the following statements are equivalent :

1) $\mathrm{T}(W)$ is a Zariski topology of W.

2) If $\mathrm{P} \in Spec(W)$ and $\mathcal{A}, \mathcal{B}$ are two IFS-Q-filters in W such that $rad(\mathcal{A}) \cap rad(\mathcal{B}) \subseteq \mathrm{P}$, then either $rad(\mathcal{A}) \subseteq \mathrm{P}$ or $rad(\mathcal{B}) \subseteq \mathrm{P}$.

3) $\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) = \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$.

Proof

1) $\Longrightarrow$ 2) Let $\mathrm{P} \in Spec(W)$ and $\mathcal{A}, \mathcal{B}$ be two IFS-Q-filters in W such that $rad(\mathcal{A}) \cap rad(\mathcal{B}) \subseteq \mathrm{P}$. By (1), there exists a IFS-Q-filter $\mathcal{C}$ in W, such that $\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) = \mathcal{V}(\mathcal{C})$. Since $rad(\mathcal{A}) = \cap_{i\in \mathrm{I}} \mathcal{A}_i$, $\mathcal{A}_i$ is a IFS- P-filter in W containing $\mathcal{A}$, then $rad(\mathcal{A}) \subseteq \mathcal{A}_i$, $i \in \mathrm{I}$, thus $\mathcal{A}_i \in \mathcal{V}(rad(\mathcal{A})) \subseteq \mathcal{V}(\mathcal{C})$. Thus $\mathcal{C} \subseteq \mathcal{A}_i$, $\forall i \in \mathrm{I}$.

Hence $\mathcal{C} \in rad(\mathcal{A})$. Similarly $\mathcal{C} \subseteq rad(\mathcal{B})$, so $\mathcal{C} \subseteq rad(\mathcal{A}) \cap rad(\mathcal{B})$. Now

$\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) \subseteq \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B})) \subseteq \mathcal{V}(\mathcal{C}) = \mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B}))$.

Hence $\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) = \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$. Since $\mathrm{P} \in \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$, then $rad(\mathcal{A}) \subseteq \mathrm{P}$ or $rad(\mathcal{B}) \subseteq \mathrm{P}$.

2) $\Longrightarrow$ 3) Let $\mathcal{A}$ and $\mathcal{B}$ be two IFS-Q-filters in W. It is clear

$\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) \subseteq \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$.

Let $\mathrm{P} \in \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$. Then $rad(\mathcal{A}) \cap rad(\mathcal{B}) \subseteq \mathrm{P}$, by (2) we have $rad(\mathcal{A}) \subseteq \mathrm{P}$ or $rad(\mathcal{B}) \subseteq \mathrm{P}$, so $\mathrm{P} \in \mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B}))$, and hence

$\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) = \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$.

3) $\Longrightarrow$ 1) Let $X(\mathcal{A}), X(\mathcal{B}) \in \mathrm{T}(W)$ where $\mathcal{A}$ and $\mathcal{B}$ are two IFS-Q-filters in W. By (3), we have $\mathcal{V}(rad(\mathcal{A})) \cup \mathcal{V}(rad(\mathcal{B})) = \mathcal{V}(rad(\mathcal{A}) \cap rad(\mathcal{B}))$ then

$X(rad(\mathcal{A})) \cup X(rad(\mathcal{B})) = X(rad(\mathcal{A}) \cap rad(\mathcal{B}))$, since $X(\mathcal{A}) = X(rad(\mathcal{A}))$,

$X(\mathcal{B}) = X(rad(\mathcal{B}))$(by Proposition (5.7),1), then $X(\mathcal{A}) \cap X(\mathcal{B}) = X(rad(\mathcal{A}) \cap rad(\mathcal{B})) \in \mathrm{T}(W)$.

This proves (1)

5.11Proposition:

Let $(W,*,0)$ be a Q-algebra and $\mathcal{A}$ *is an* IFS-Q-filter of W. Then $Spec(W)$ is a $\mathrm{T}_0 -$ space.

Proof

Let $\mathcal{A}, \beta \in Spec(W)$ and $\mathcal{A} \neq \beta$. Then $\mathcal{A} \nsubseteq \beta$ or $\beta \nsubseteq \mathcal{A}$, if $\mathcal{A} \nsubseteq \beta$, then $\beta \notin V(\mathcal{A})$, but $\mathcal{A} \in V(\mathcal{A})$, Then $\beta \in X(\mathcal{A})$, and $\mathcal{A} \notin X(\mathcal{A})$. If $\beta \nsubseteq \mathcal{A}$, similarly we have $\mathcal{A} \in X(\beta)$ but $\mathcal{A} \notin X(\beta_F)$. Then $Spec(W)$ is a $\mathrm{T}_0 - space$.

5.12.Proposition:

Let $(W,*,0)$ be a Q-algebra. Then topological Space(W) is T_1-space if and only if for every IFS- P-filter P of W, there exists an IFS-Q-filter $\mathcal{A}$ such that $\mathcal{A} \sqsubseteq \mathrm{P}$ and $\mathcal{A} \nsqsubseteq \mathrm{F}$, for every $\mathrm{F} \in Spec(W)$.

proof

Let $Spec(W)$ be T_1-space and $\mathrm{P} \in Spec(W)$. Then $\exists$ IFS-Q-filter $\mathcal{A}$ *of* W, such that $\{\mathrm{P}\} = \mathcal{V}(\mathcal{A})$ hence $\mathcal{A} \nsqsubseteq \mathrm{F}$, $\forall\, \mathrm{F} \in Spec(W)$.

Conversely, let $\mathrm{P} \in \mathrm{Spec}(W)$. Then there exists an IFS-Q-filter $\mathcal{A} \sqsubseteq \mathcal{P}$ and $\mathcal{A} \nsqsubseteq \mathrm{F}$, $\forall \mathrm{F} \in Spec(W)$ and $\mathcal{P} \nsqsubseteq \mathrm{F}$, hence $\mathcal{V}(\mathcal{A}) = \{\mathrm{P}\}$.Thus $Spec(W)$ is T_1-space.

5.13.Proposition:

Let $(W,*,0)$ be Q-algebra. Then $Spec(W)$ is a $T_3 -$ space if and only if for every IFS-P-filter P in W and any IFS-Q-filter $\mathcal{A}$ in W such that $\mathcal{A} \nsubseteq \mathrm{P}$, there exist two IFS-Q-filter $\beta, \mathcal{H}$ such that $\beta \nsubseteq \mathrm{P}$and $rad\ (\beta \cap \mathcal{H}) = rad\ (\tilde{0})$, $rad(\mathrm{P} \cup \mathcal{H}) = \tilde{1}$.

Proof

Let $Spec(W)$ be $\mathrm{T}_3 -$ space and $\mathrm{P} \in Spec(W)$, $\mathcal{A}$ be IFS-Q-filter in W such that $\mathcal{A} \nsubseteq \mathrm{P}$. Then $\mathrm{P} \notin V(\mathcal{A})$, since $Spec(W)$ is $\mathrm{T}_3 -$ space, then there exist two IFS-Q-filters β *and* $\mathcal{H}$ in $\mathcal{X}$ such that $\mathrm{P} \in X(\beta)$, $V(\mathcal{A}) \subseteq X(\mathcal{H})$ and $X(\beta) \cap X(\mathcal{H}) = \emptyset$, thus $V(\mathcal{A}) \cup V(\mathcal{H}) = \tilde{1}$, so $\beta \nsubseteq \mathrm{P}$, $rad\ (\beta \cap \mathcal{H}) = rad\ (\tilde{0})$ and $rad(\mathrm{P} \cup \mathcal{H}) = \tilde{1}$.

Conversely, let $\mathrm{P} \in Spec(W)$, such that $\mathrm{P} \notin V(\mathcal{A})$ for IFS-Q-filter $\mathcal{A}$ in W, then $\mathcal{A} \nsubseteq \mathrm{P}$, so there exist two IFS-Q-filters $\beta, \mathcal{H}$ such that $\beta \nsubseteq \mathrm{P}$, $rad\ (\beta \cap \mathcal{H}) = rad\ (\tilde{0})$ and

$rad(\mathcal{A} \cup \mathcal{H}) = \tilde{1}$. Then $\mathcal{A} \in X(\beta)$ and $X(\beta) \cap X(\mathcal{H}) = X(\tilde{0})$ and $X(\mathcal{A}) \subseteq X(\mathcal{H})$. Thus $Spec(W)$ is $T_3 -$ space.

5.14.Proposition:

Let $(W,*,0)$ be Q-algebra. Then $Spec(W)$ is a $\mathrm{T}_4 -$ space if and only if for every IFS-Q-filter $\mathcal{A}, \beta$ in $\mathcal{X}$, such that $rad\ (\mathcal{A} \cup \beta) = \tilde{1}$, there exist two IFS-Q-filters $\mathcal{H}, \mathcal{K}$ in W such that $rad\ (\mathcal{A} \cap \mathcal{H}) = \tilde{1}$, and $rad(\beta \cup \mathcal{K}) = \tilde{1}$.

Proof

Let $\mathcal{A}, \beta$ be two IFS-Q-filters in W such that $rad\ (\mathcal{A} \cup \beta) = \tilde{1}$ then $V\ (\mathcal{A}) \cap V(\beta) = \emptyset$. Since $Spec(W)$ is a $T_4 -$ space then there exist two IFS-Q-filters $\mathcal{H}, \mathcal{K}$ in W such that
$V(\mathcal{A}) \subseteq X(\mathcal{H}), V(\beta) \subseteq X(\mathcal{K})$ and $X(\mathcal{H}) \cap X(\mathcal{K}) = X(\tilde{1})$. Then $V(\mathcal{A} \cup \beta) = V(\tilde{1})$,
$V(\mathcal{A}) \cap V(\mathcal{H}) = \emptyset, V(\beta) \cap V(\mathcal{K}) = \emptyset$. So $rad\ (\mathcal{A} \cup \beta) = \tilde{1}$, $V(\mathcal{A} \cup \mathcal{H}) = V(\tilde{1})$ and $V(\beta \cup \mathcal{K}) = V(\tilde{1})$.
Thus $rad(\mathcal{H} \cap \mathcal{K}) = rad\ (\tilde{0})$, $rad(\mathcal{A} \cup \mathcal{H}) = \tilde{1}$ and $rad(\mathcal{A} \cup \mathcal{K}) = \tilde{1}$
Conversely, let $\mathcal{A}, \beta$ be two IFS-Q-filters in W such that $V(\mathcal{A}) \cap V(\beta) = \emptyset$, so there exist two IFS-Q-filters $\mathcal{H}, \mathcal{K}$ in W such that $rad(\mathcal{H} \cap \mathcal{K}) = rad\ (\tilde{0})$, $rad(\mathcal{A} \cup \mathcal{H}) = \tilde{1}$ and
$rad(\beta \cup \mathcal{K}) = \tilde{1}$, so we have $X(\mathcal{H} \cap \mathcal{K}) = X(\tilde{0})$, $V(\mathcal{H} \cup \mathcal{K}) = V(\tilde{1})$, and $V(\beta \cup \mathcal{K}) = V(\tilde{1})$
then $V(\mathcal{A}) \cap V(\mathcal{H}) = \emptyset$ and $V(\beta) \cap V(\mathcal{K}) = \emptyset$. Therefore $V(\mathcal{A}) \subseteq X(\mathcal{H})$ and $V(\beta) \subseteq X(\mathcal{K})$. Thus $Spec(W)$ is a $\mathrm{T}_4 -$ space.

REFERENCES

1. A.H. Kareem. and J. H. Kadhum, "New types of Ideals in Q-algebra", Journal university of kerbala, Vol.16, No.4 scientific (2018), pp.9-21
2. Abdulla H.K, and Naij .R.SH, Intuitionistic fuzzy Q-filters of Q-algebra, IOP Conf. Series: Materials Science and Engineering 871(2020)012047.
3. Abdulla H.K, and Naij .R.SH, Spectrum of prime Q-filter of Q-algebra"2nd International Scientific Conference of Al- Ayen University (ISCAU-2020)
4. H.SH Salman. Some new of fuzzy filter in Q-algebra and Pseudo Q-algebra ,thesis ,Department of mathematics, University of kufa , (2019),pp1-90
5. J. Neggers, S. S. Ahn and H. S. Kim, "On Q-algebras", *International Journal* of *Mathematics* and *Mathematical* Sciences, Vol. 27, No.12, (2001), pp: 749-757
6. K.T. Atanassov. ,"Intuitionistic fuzzy sets" , Fuzzy sets and Systems 35 87–96 . 1986
7. K.T. Atanassov. Review and new results on Intuitionistic fuzzy sets,preprint.IM-MFAISI-1-88,sofia,1988.
8. K. T. Atanossov, New operations de…ned over the intuitionistic fuzzy sets, Fuzzy Sets and Syst. 61(1994)137 172
9. L.A. Zadeh , "Fuzzy sets", Information And Control,Vol. 8, (1965), pp: 338-353
10. M. F.Atiyah. and Macdonald .I., Introduction to commutative algebra, Longman Higher Eduction, New York 1969.
11. O. Zariski, "The compactness of the Riemann manifold of an abstract field of algebraic functions" Bull. Amer. Math. Soc., 50 : 10 (1944) pp. 683–691
12. P.A. Ejegwa , S.O. Akowe, P.M. Otene, J.M. Ikyule,"An Overview On Intuitionistic Fuzzy Sets" International Journal of scientific & technology research , 3(2014), 2277-8616

13. S.M .Mostafa, M.A .Abdel Naby, and O.R. Elgendy Intuitionistic fuzzy Q-Ideals in Q-algebra .Global Journal of pure and Applied Mathematics (2012),8(2):135-144
14. Takeuti .G, and Titants.S,1984 Intuitionistic fuzzy Logic and Intuitionistic fuzzy set theory, *Journal of Symbolic Logic*,49,851-866