International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol.4 Issue 7, July – 2020, Pages: 107-112

Functionally Graded Plates's Thermal Analysis with FEM Method

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Abstract: *Functionally Graded Materials (FGMs) are the advanced materials in the field of composites, which can resist high temperatures and are proficient in reducing the thermal stresses. It was proposed in 1980s, to replace the use of pure materials and classic composites. In recent decades, significant investigations are reported in the predicting the response of FGM plates subjected to thermal loads. FGM are non-homogenous and it is difficult to evaluate characteristics analytically. This paper presents an attempt to simulate the temperature profile of a rather complex and non-homogeneous material by using finite element method. An effort has been made to focus the discussion on the various research studies carried out till recently for the thermal analysis of FGM plates. Finally, some important conclusions and the suggestions for future directions of research in this area are presented. It is felt that this review paper will serve the interests of all the academicians, researchers and engineers involved in the analysis and design of FGM plates.*

Keywords—Thermal analysis; Temperature distribution; FGM; FEM.

1. INTRODUCTION

A numerical technique for finite element analysis of the thermal characteristics of functionally graded materials was developed [1]. and investigated the effect of significant governing parameters such as variation function of material composition and relative thickness of FGM layer inserted between metal and ceramic layers. Isoparametric bilinear two-dimensional quadrilateral element was chosen for finite element mesh in time domain and then Galerkin variational formulation in space coordinates was done. It was concluded that considerable improvement is possible by inserting FGM layer between metal and ceramic layers in classical biomaterial layered composites.

The solutions of temperature, displacements was presented [2], and thermal/mechanical stresses in a functionally graded circular hollow cylinder by using a multi-layered approach in which it was assumed that the hollow cylinder is composed of 10 fictitious layers. It was shown that due to non homogeneity of the material properties the variation of temperature is not linear through the thickness direction.

A solution method for the one-dimensional (1D) transient temperature and thermal stress fields in FGMs was established by [20]. Finite-element method is used for space discretization which results in a system of first-order differential equations. Transient solutions of these equations were obtained using either finite-difference method or mode superposition.

A multiscale modeling method to derive effective thermal conductivity in two-phase graded particulate composites was proposed [3]. In the particle-matrix zone, a graded representative volume element is constructed to represent the random microstructure at the neighborhood of a material point. At the steady state, the particle's averaged heat flux is solved by integrating the pair wise thermal interactions from all other particles. The homogenized heat flux and temperature gradient are further derived, through which the effective thermal conductivity of the graded medium is calculated. In the transition zone, a transition function is introduced to make the homogenized thermal fields continuous and differentiable. By means of temperature boundary conditions, the temperature profile in the gradation direction was solved. Parametric analyses and comparisons with other models and available experimental data were presented and validated.

An efficient meshless method for transient heat transfer and thermoelastic analysis of FGMs was proposed [4]. The analog equation method is used to obtain an equivalent homogenous system to the original non-homogenous governing equation, after which radial basis functions and fundamental solutions are used to construct the related approximated solutions of particular part and complementary part, respectively. Finally, all unknowns are determined by satisfying the governing equations at interior points and boundary conditions at boundary points. Numerical experiments showed that a good agreement was achieved between the results obtained from the proposed meshless method and available analytical solutions. The appropriate graded parameter can lead to different temperature distribution, low stress concentration and little change in the distribution of stress fields in the domain under consideration.

[5] discussed the steady heat conduction problem of a Ti-6Al-4V/ZrO2 composite FGM plate under heating boundary by the FEM. They showed that the temperature distribution of the three-layered composite FGM plate is very gentle and smooth Compared with the nongraded two-layered composite plate. Also, the variation of temperature with the change in

International Journal of Academic and Applied Research (IJAAR)
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FGM layer thickness, composition and porosity were shown with the help of FEM model.

[6] presented an improved finite element approach in which a node-based strain smoothing is merged into shear-locking-free triangular plate elements. The formulation used only linear approximations and its implementation into finite element programs is quite simple and efficient. The method was then applied for static, free vibration and mechanical/thermal buckling problems of functionally graded material (FGM) plates. In the FGM plates, the material properties were assumed to vary across the thickness direction by a simple power rule of the volume fractions of the constituents. The behavior of FGM plates under mechanical and thermal loads was numerically analyzed in detail through a list of benchmark problems. The numerical results showed high reliability and accuracy of the present method compared with other published solutions in the literature.

[7] focused on the finite element simulation on thermal stress for W/Cu FGM with different graded layers, composition and thicknesses. In addition, the variance of stresses for functionally graded coatings with the steady state heat flux were simulated by finite element analysis (ANSYS Workbench). The results showed that the W/Cu FGM was effectively beneficial for the stress relief of W coating. Meanwhile, the maximum von mises stress decreased approximately by 52.8 % compared to monolithic W plasma facing material. And the four-layer FGM with a compositional exponent of 2 was optimum for 1.5 mm W coating.

[8] studied the thermal conduction behavior of the three-dimensional axisymmetric functionally graded circular plate under thermal loads on its top and bottom surfaces. A temperature function that satisfies thermal boundary conditions at the edges and the variable separation method were used to reduce equation governing the steady state heat conduction to an ordinary differential equation (ODE) in the thickness coordinate which was solved analytically. Next, resulting variable coefficients ODE due to arbitrary distribution of material properties along thickness coordinate was also solved by the Peano-Baker series. The numerical results confirm that the influence of different material distributions, gradient indices and thickness of plate to temperature field in plate cannot be ignored.

2. FUNCTIONALLY GRADED MATERIALS (FGMS)

2.1 Introduction

FGM contains 2-phase composites with continuously varying volume fractions. The Material properties vary with location, also the matrix alloy (the metal), the reinforcement material (the ceramic), the volume, shape, and location of the reinforcement, and the fabrication method can all be tailored to achieve particular desired properties. We note some naturally occurring FGM: bamboo, bone.

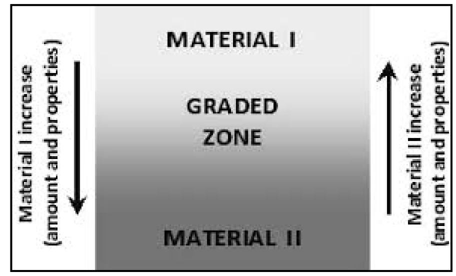


Fig. 1. *Functionally graded material*

The advantages of FGM are that the thermal stresses can be reduced, and it can be reduced at critical locations. Stress jumps at the interface can be avoided, and the driving force for crack extension, the stress intensity factor, can be reduced. Also, the strength of the interfacial bond can be increased. They are several applications of FGMs, as thermal Barrier coatings for turbine blades, or thermal protection systems for spacecraft, prosthesis joint increasing adhesive strength and reducing pain, polyester-calcium phosphate materials for bone replacement, etc…

2.2 Structure

Composed of a ceramic and a metal, with a material transition from 0% at 1 end to 100% to the other end. The smooth transition of material provides thermal protection as well as structural integrity.

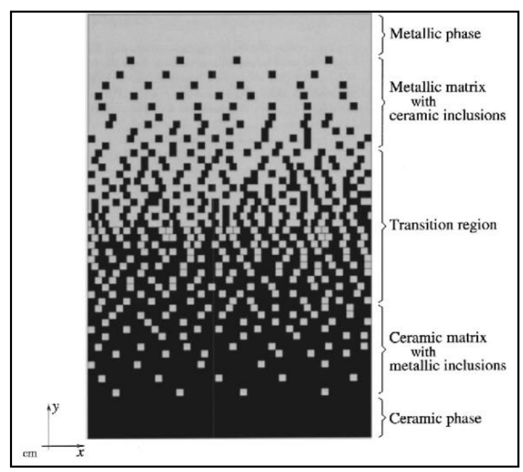


Fig. 2. *Continuously graded microstructure of FGMs*

Unlike conventional composites where thermo mechanicals properties remain continuously constant.

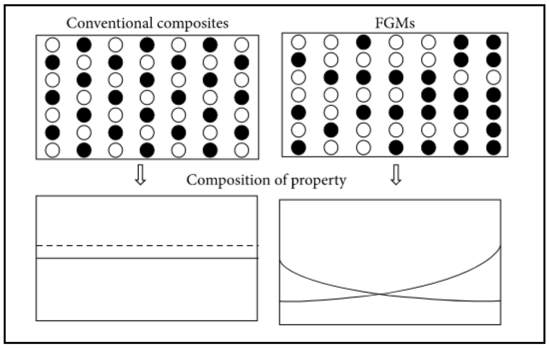
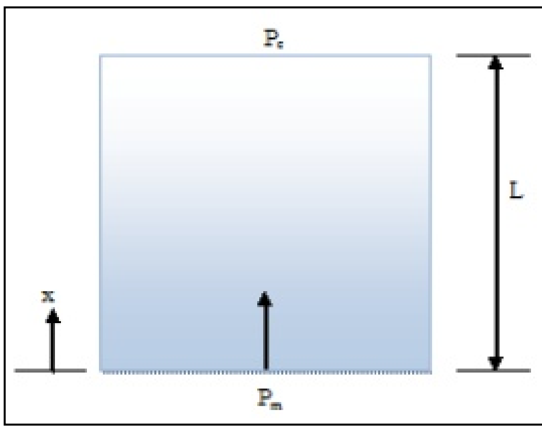


Fig. 3. *How FGM differs from conventional composite Schematic structure, elastic modulus (-) and thermal conductivity (---) of an FGM and a homogeneous materials*



The FGM structure is regarded by two types of distributions as shown in followed:

Fig. 4. *Illustration of FGM structure*

Power law type (P-type):

$V_c(x)=(\frac{x}{L})^n$

$P(x)=(P_c-P_m)V_c(x)+P_m$

$V_c(x)$: volume of ceramic at any point x throughout the thickness L.

P_c : Property of ceramic.

P_m: Property of metal.

n: power law index.

Exponential type(E-type):

$V_c(x)=e^{nx/L}$

$P(x)=V_c(x)P_c=P_c.\,e^{nx/L}$

$n=\ln(\frac{Pm}{Pc})$.

3. FEM STUDY AND METHODOLOGY

3.1 DIRECT APPROACH

The question is How to model a material with continuously varying properties?. The simplest approach is to use homogeneous elements each with different properties, giving a stepwise change in properties in the direction of the material gradient. We use some assumptions as: There are no heat sources within the plate, and the material's properties for each same ordinate x are homogenous and isotropic. Creeps are neglected and perfect bonding, and Temperature independent material constants, initially stress free state, and the width of the plate is assumed to be infinite.

The element chosen in discretization is 1D with 2 nodes, and the type of FGM chosen is P Type, with 1-D General heat conduction equation. After the set of the boundary conditions, we will then, solve for the unknowns i.e, the temperature at each node point

3.2 ELEMENT EQUATIONS

The main of this study is to determine the temperature distribution of the given plate subject to constant temperatures at both ends.

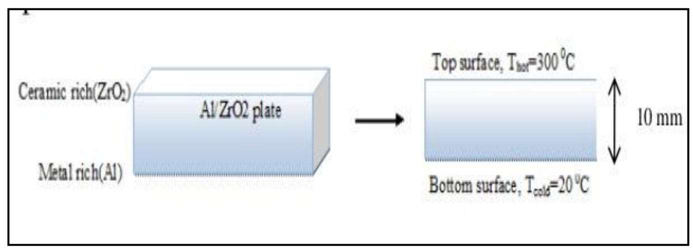


Fig. 5. *Material sample and boundrary conditions*

The mechanical characteristics are given in followed table.

Table 1: Material properties

Properties	Aluminium	Zirconium oxide(ZrO_2)
E (Gpa)	70	200
k (W/mK)	204	2.09
ρ (Kg/m^3)	2707	5700
α	23×10^{-6}/°C	10×10^{-6}/°C

We will use a direct approach, with 1-Dheat flow under steady conditions. The discretization occurred is given by followed figure.

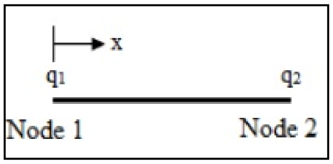


Fig. 6. *Discretization Occured*

We use the Fourier's low:

$$q=-K_x.A.\frac{dT}{dx}$$

q=heat flux (W).

k_x= thermal conductivity of the material that varies along the thickness direction, x (W/mK^{-1}).

A = area normal to the heat flow (m^2).

$V_c(x)=(\frac{x}{h})^n$

$K(x)=(K_c-K_m)V_c(x)+K_m$

K_c & K_m = thermal conductivity of ceramic and metal respectively.

$V_c(x)$ = ceramic volume fraction along the thickness direction, and n= power law index.

Nodal heat flow entering a typical node:

$$Q^e{}_1=\frac{k^eA^e(T^e{}_1-T^e{}_2)}{L^e} \qquad Q^e{}_2=-\frac{k^eA^e(T^e{}_1-T^e{}_2)}{L^e}$$

The conservation of energy requires: Q_1 $=Q_2$

We obtain the followed matrix notation:

$$\left(\frac{k^eA^e}{x^e}\right)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}T^e{}_1\\ T^e{}_2\end{Bmatrix}=\begin{Bmatrix}Q^e{}_1\\ Q^e{}_2\end{Bmatrix}$$

Or, $[K^e]\{T^e\}=\{Q^e\}$

Where [K^e]: element thermal conduction stiffness matrix.

[T^e]: element column vector of nodal temperatures.

[Q^e]: element column vector of nodal heat fluxes.

4. RESULT AND DISCUSSION

4.1 Result

All the results obtained are grouped together and presented in the following curves.

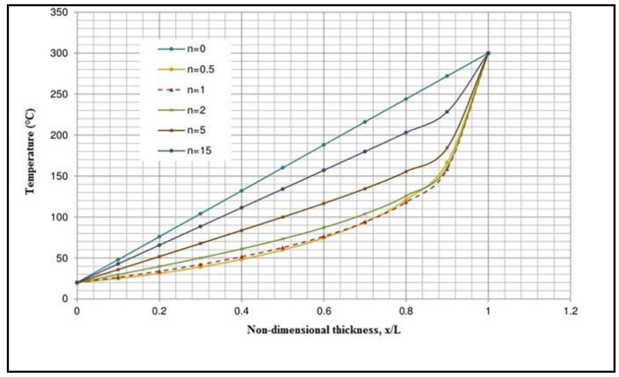


Fig. 7. *Temperature distrubition with FEM method*

The next step is to compare and validate our results with the study (Nguyen et al) that exists in the literature.

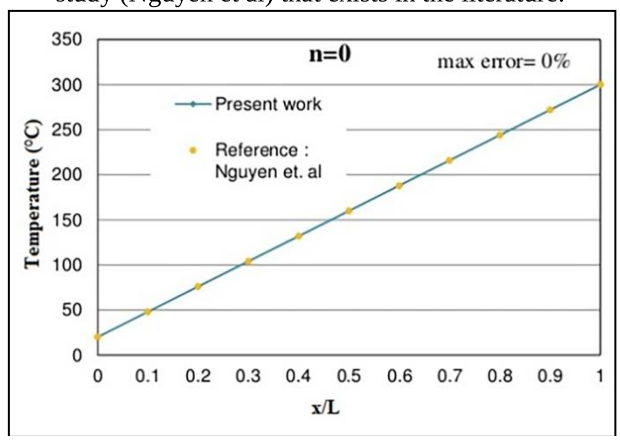


Fig. 8.*Temperature vs Non-dimensional thiknes for n=0*

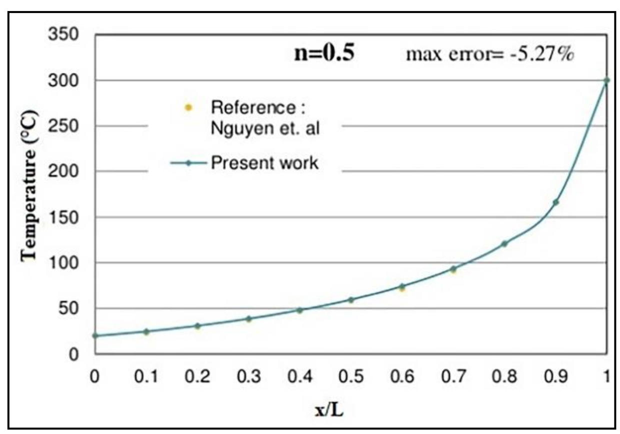


Fig. 9.*Temperature vs Non-dimensional thiknes for n=0,5*

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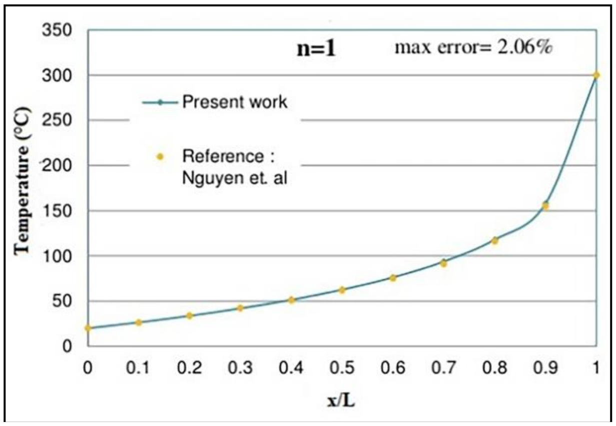

Fig.10. *Temperature vs Non-dimensional thickness for n=1*

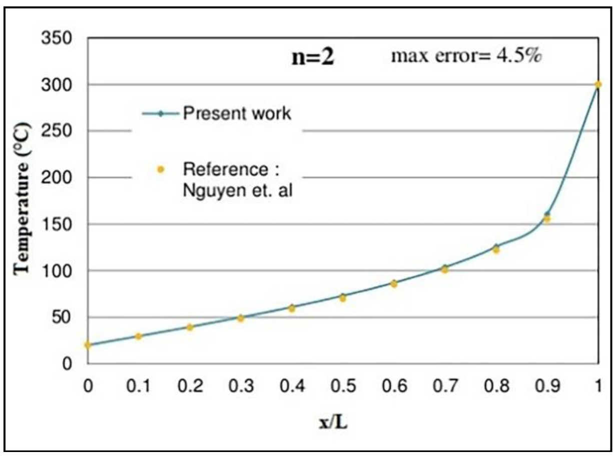

Fig.11. *Temperature vs Non-dimensional thickness for n=2*

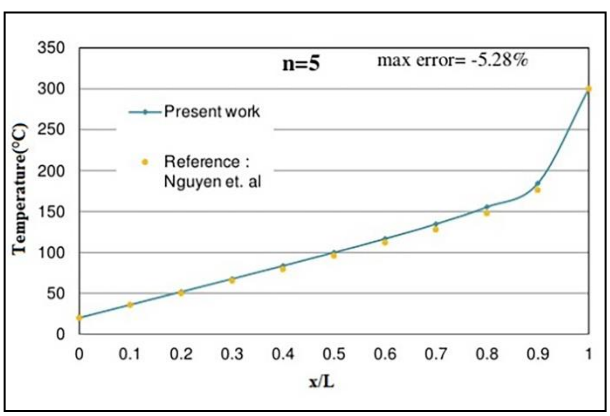

Fig.12. *Temperature vs Non-dimensional thickness for n=5*

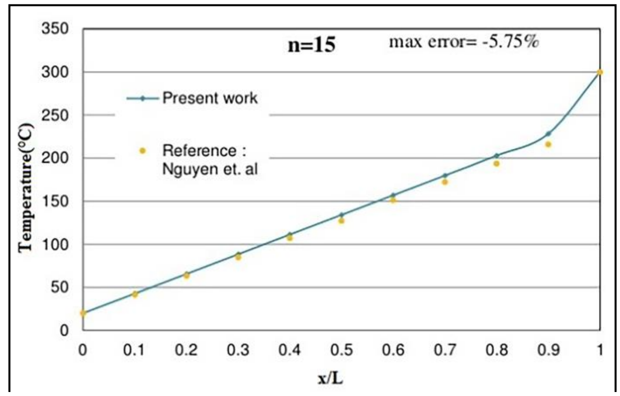

Fig.13. *Temperature vs Non-dimensional thickness for n=15*

The obtained results in the different curves show that the temperature of the material follows different paths each time the parameter n varies. it is linear for n = 0. Then it has more and more complicated shapes depending on the variation of the variable n. The influence of the gradient index n is also shown in different figures above. It is seen that with increasing gradient index, n, we leave the linear shape of temperature distribution, to become a nonlinear one. The difference between the linear and the nonlinear temperature distribution through the material thickness, can be explain the time path to reach the final temperature.

Then, the comparison with Nguyen et al allowed to validate the studied model. Indeed, our results match perfectly with the work in literature, and this, for the different values of n ranging from 0 to 15. Also we can conclude that as the ratio of k_m/k_c increases the temperature distribution becomes smoother and gradual, thus showing good characteristics of FGM under high thermal loading where thermal stresses are likely to occur. Hence greater the ratio of k_m/k_c, better will be its thermo-mechanical properties and response to loadings

We conclude that the FEM method remains a powerful tool to give an estimate of the temperature distribution of FGM material, if we take into account the good index n.

5. Conclusion

In this paper, we applied the finite element method with to study the temperature distribution of the FGM plate. It appears that the temperature distribution of the FGM plate is always less and gradual as compared to the homogenous materials. On simulation of temperature for each FGM, it was observed that Al/ZrO_2 shows the most smooth and gradual variation of temperature along its thickness as compared to all other FGMs. The efficiency and accuracy of the present approach is demonstrated with few numerical examples. This improved finite element technique shows insensitivity to shear locking and produces excellent results in temperature distribution of functionally graded plates.

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