Using Nuclear Shell Model to Calculate Energy Levels and Probability of Reduced Electric Quadruple Transition for ³⁵S Isotope

Ali K. Hasan¹, Zaid A. Mohamed²

^{1,2}Department of Physics, College of Education for Girls, University of Kufa, Iraq e-mail: <u>zaidalawadi87@gmail.com</u>

Abstract : In this paper, the OXBASH computer software has been used for apply the nuclear shell model to the sulfur isotope ${}^{35}S$ in order to calculate the excitation energies and the probability for electric quadrupole transition B (E2) and by depending each of effective interactions w and hbusd that describe the interaction between a nucleon-nucleon in an sd space for the nucleus under study, which contains 19 nucleons in the model space and by considering the isotope of oxygen ${}^{16}O$ as the closed inert core for it because it is a double magic nucleus. Our theoretical calculations have been compared with the experimental data available for this isotope and we have noted a good match with it.

Keywords: ³⁵S sulfur isotope, Nuclear shell model, excitation energies, the probability for electric quadrupole transition B (E2), model sd space, effective interactions w and hbusd, OXBASH program.

1. Introduction

The study of the structure of unstable and rich-neutrons nuclei is the most important aims of modern physics because of its importance in many applications in the field of astronomy in the study of the great explosions called (Supernova) and in other fields [1], and the information available on this region is very little due to the difficulty of generating these nuclei [2, 3]. There are several nuclear models to explain the nuclear structure, all of them had a major effect for determine the path that has been followed for explain the problem of nuclear structure [4]. And from the most important basic models proposed to describe the interaction between nucleons is the nuclear shell model, also called the independent particle model, and is based on several suppositions through which it succeeds in explaining some nuclear phenomena and fails in others when compared with experimental data [5]. The first to introduce the idea of closed nuclear shells is the scientist (W. Elasser) in 1934, as some studies on the average of binding energy as well as the properties of the nuclei led to that the nucleons inside the nucleus move within orbits like those in which electrons move in the atom, it has been called shell structure or levels structure in which the nucleus is stable [6]. The importance of the shell model is mainly shape in its ability to give a correct approximation of the energies of the levels in which nucleons can be found with different values of orbital angular momentum. It has been observed that many nuclear properties showed discontinuities at certain even values of numbers of neutrons or protons. Experiments have shown that stable nuclei are characterized by the number of their protons Z or the number of their neutrons N equal to one of the following numbers 2, 8, 20, 28, 50, 82, 126 [5,4], which was called (Magic Numbers). Magic numbers have been explained as closed shells or covers of neutrons or protons, and that the covers of neutrons and protons appear to be independent of each other according to the nuclear shell model, and for this reason, this model is considered the cornerstone of many nuclear studies [7].

2. Theoretical part

The nuclear properties of different states can be calculated by the binding energies, the excitation energies of the nuclei [8]. The binding energy of the nuclei is defined as the negative value of the energy needed to divide the nucleus into its component's neutrons and free protons [8, 9], and the indication that there is a direct relationship between the binding energy and the excitation energy with the of expectation value of Hamilton for nuclear system. The binding energy of the ground state is largest that possible while the excitation energy is defined as the difference between the binding energy of the state (n) and the binding energy of the ground state (0) according to the following relationship [8, 10]:

$$E_x(n) = E^b(n) - E^b(0)$$
 (1)

Where $E_x(n)$ the excitation energy of state, $E^b(n)$ the binding energy of state and $E^b(0)$ the binding energy of ground state. We assume that the nucleus has a closed core, and two additional particles, outside the closed core within the orbit ρ [8], if the two particles are located within the specified orbit ρ , it is called a pure configuration. But if the two particles fill more than one orbit, they share the values of the total momentum (J) and the isospin (T), however, it gets from connected them a state of a mixing and admixture for energy levels, this is called a mixed configuration [4, 5]. The distributions of the different limits of the total binding energy are given as [9, 10]:

$$E_n^b(core + \rho^2) = E^b(core) + 2e_p + E_n^{(1)}(\rho^2)$$
(2)

Each of the terms of the equation has a physical explanation, and it is:

 $E_n^b(core)$ It represents the binding energy of the particles in closed core and this term is a constant amount.

 $E_{\Gamma}^{(1)}(\rho)$ It represents the contribution to the binding energy from the actual nuclear interaction of the two particles outside the closed core; this term depends not only on the orbit but also on the total momentum (J) and the isospin (T) for two particles system. $2e_{\rho}$ It represents the negative value of the energy needed to remove two particles from the low voltage and make them move independently in orbit (ρ) and assumes that the voltage does not depend on the number of particles outside the core.

However, if we have (n) particles in orbit (ρ) and (m) particles in orbit (λ) outside the closed core, then the total binding energy equation is given by the following relationship [8]:

$$E_{\Gamma}^{b}(core + \rho^{\lambda} + \lambda^{m}) = E_{c} + E^{b}(core) + ne_{\rho} + me_{\lambda} + E_{\Gamma}^{(1)}(\rho^{n}\lambda^{m})$$
(3)

The term E_c represents Coulomb's energy, e_{ρ} and e_{λ} it represents the energy of a single particle in orbits (ρ) and (λ), and the last term it represents the residual reaction energy (matrix of the interacting elements) and is given by the following relationship [8]:

$$E_{\Gamma}^{(1)}(\rho^n \lambda^m) = \langle \rho^n \lambda^m | V(1,2) | \rho^n \lambda^m \rangle_{\Gamma}$$
(4)

3. Calculations and discussion

Theoretical calculations in this study include the following:

3.1 Energy levels: depending on the energy equations included in the shell model that were mentioned in the previous item and programmed using an OXBASH computer software. The excitation energies and the probability for electric quadrupole transition B (E2) for ³⁵S nucleus containing 19 nucleons outside the closed core ¹⁶O and in the model space sd of orbits

 $(1d_{5/2}, 2s_{1/2}, 1d_{3/2})$ and by depending all of the effective interactions w, hbusd, and the results were obtained listed in Tables (1) and (2):

Table (1): A	A comparison of the theoretical val interaction	ues of the excitation energies in the ³⁵ S nucleus computed using the W with available experiment results [12]	e effective
	Theoretical values for E	Experimental values	

\mathbf{J}^+	(MeV)	Experimental	Experimental values		
	(w) results	E (MeV)	\mathbf{J}^{π}		
3/21	0	0	3/2+		
$1/2_1$	1.558	1.572378	1/2+		
$5/2_1$	2.68	2.717	5/2+		
3/22	2.8	2.93864	3/2+		
7/21	3.471	3.594	(1/2,7/2)+		
3/23	4.054	4.022	(3/2-:11/2-)		
2/2	4.159	4.106	(1/2, 3/2, 5/2)+		
5/24		4.18	(1/2,3/2,5/2+)		
1/2	4.585	4.106	(1/2,3/2,5/2)+		
1/22		4.18	(1/2, 3/2, 5/2+)		
5/23	4.727	4.99	(1/2:5/2)+		
1./0	4.783	4.839	(1/2:9/2)+		
1/23		4.99	(1/2:5/2)+		
7/22	5.879	5.841	(1/2:7/2-)		
1/2	6.173	6.018	(1/2:9/2-)		
$1/2_4$		6.129	(1/2, 3/2, 5/2)+		
5/25	6.274	6.129	(1/2,3/2,5/2)+		
1/25	7.22	7.44221	1/2+		

1/27	9.241	
9/23	9.331	
11/22	9.437	
1/28	9.615	
7/2 ₈	9.71	
7/29	9.853	
9/24	9.898	
7/210	10.156	
9/25	10.188	
9/26	10.312	
13/21	10.539	
1/29	10.685	
11/23	10.827	
9/27	10.915	
1/210	11.402	
9/2 ₈	11.667	
11/24	11.851	
9/29	11.866	
11/25	12.102	
11/26	12.683	
9/210	12.717	
13/22	13.529	
11/27	14.017	
15/21	14.092	
11/28	14.137	
11/29	14.782	
13/23	15.022	
11/210	15.093	
13/24	15.612	
13/25	16.37	
13/26	17.059	
15/22	17.184	
13/27	17.702	

18.293		
18.687		
19.009		
19.227		
20.817		
21.328		
21.761		
22.934		
23.943		
24.639		
26.482		
28.315		
28.709		
29.087		
	18.293 18.687 19.009 19.227 20.817 21.328 21.761 22.934 23.943 24.639 26.482 28.315 28.709 29.087	18.293 18.687 19.009 19.227 20.817 21.328 21.761 22.934 23.943 26.482 28.315 29.087

By comparing our theoretical results using the interaction w with the experiment results of this isotope in the above table, the following was found:

1. The total angular momentum and the symmetry had been confirmed for ground state $3/2_1^+$ level when it compared with the available experiment values.

2. A good agreement of experiment energy values was obtained, (1.572, 2.717, 2.938, 7.442) MeV Corresponding to the angular momentums and symmetry $(1/2_1^+, 5/2_1^+, 3/2_2^+, 1/2_5^+)$ when compared with the available experiment values.

3. The total angular momentum has been confirmed only for experiment energies values (3.594, 4.106, 4.106, 4.99, 4.839, 4.99, 6.129, 6.129) MeV, experimentally not confirmed that corresponds to the angular momentums $(7/2_1^+, 3/2_4^+, 1/2_2^+, 5/2_3^+, 1/2_3^+, 1/2_4^+, 5/2_5^+)$ when compared with the available experiment values.

4. The total angular momentum has been confirmed for experiment energies values for which do not determined her symmetry (4.18, 4.18, 6.018) MeV, that corresponds to the angular momentums $(3/2_4^+, 1/2_2^+, 1/2_4^+)$ when compared with the available experiment values.

5. The total angular momentum has been confirmed for experiment energies values (4.022, 5.841) MeV, that corresponds to the angular momentums $(3/2_3^+, 7/2_2^+)$, but with a different symmetry when compared with the available experiment values.

6. We noticed that there are experiment energies values with angular momentum and symmetry that were not compared with our theoretical calculations because they do not converge with them.

7. We noticed through our calculations that there are (forty seven) levels with total angular momentum and symmetry that were not matched by any available experiment value as well. We also noted that the highest value of the calculated energy theoretically is (29.087) MeV, while the highest experiment value of energy is (9.155) MeV, meaning that we have obtained (forty seven) new energy levels above the experiment value.

Table (2): A comparison of the theoretical values of the excitation energies	es in the ³	⁵ S nucleus	computed	using	the l	nbusd
effective interaction with available experiment results [12]						

\mathbf{J}^+	Theoretical values for	Experimental values		
	E(MeV)			
	(hbusd)results	E(MeV)	\mathbf{J}^{π}	
3/21	0	0	3/2+	
1/21	1.726	1.572378	1/2+	
5/21	2.324	2.717	5/2+	

3/22	2.732	2.93864	3/2+
5/22	3.44	3.4211	5/2+
3/23	3.488	3.675	(1/2-,3/2-)
5		3.818	(3/2,9/2-)
3/24	3.971	3.885	(3/2-,5/2)
		3.89	(3/2-,5/2)
		4.106	(1/2,3/2,5/2)+
		4.18	(1/2,3/2,5/2+)
5/23	4.235	4.302	(1/2:5/2,7/2-)
		4.477	(1/2,3/2,5/2)+
		4.576	(1/2:5/2)+
1/2	1 677	4.839	(1/2:9/2)+
1/23	4.077	4.99	(1/2:5/2)+
		5.771	(1/2:9/2)+
1/24	5.986	5.841	(1/2:7/2-)
		5.752	(1/2:9/2-)
		6.354	(1/2:9/2-)
1/25	6.564	6.629	(1/2:9/2-)
		6.761	(1/2:9/2-)
3/28	7.395	7.712	(3/2:13/2)+
7/26	8.494	8.16	(1/2:7/2)(-)
1/26	8.583	8.16	(1/2:7/2)(-)
9/24	9.453		
7/28	9.498		
7/29	9.591		
1/28	9.618		
9/25	9.67		
9/26	10.053		
7/210	10.11		
13/21	10.228		
11/23	10.351		
1/29	10.473		
9/27	10.615		
9/28	11.107		
1/210	11.139		
11/24	11.357		
9/29	11.608		
11/25	11.677		
11/26	12.316		
9/210	12.53		
13/25	13.746		
11/27	13.763		
15/21	13.845		
11/28	14.075		
11/29	14.425		
13/23	14.716		
11/210	15.079		
13/24	15.29		
13/25	15.794		
13/26	16.557		
13/27	16.925		
15/22	17.186		
13/28	17.904		

15/23	18.036	
13/29	18.339	
13/210	18.952	
17/21	20.891	
15/24	21.083	
15/25	21.799	
15/26	22.695	
15/27	23.557	
17/22	24.15	
15/28	26.167	
15/29	27.687	
17/23	28.413	
15/210	29.008	

By comparing our theoretical results using the hbusd interaction with the experiment results of this isotope in the above table, the following was found:

1. The total angular momentum and the symmetry had been compared for ground state of $3/2_1^+$ level when compared with the available experiment values.

2. A good agreement of experiment energy values was obtained (1.572, 2.717, 2.938, 3.42) MeV Corresponding to the angular momentums and symmetry $(1/2_1^+, 5/2_1^+, 3/2_2^+, 5/2_2^+)$ when compared with the available experiment values.

3. The total angular momentum and symmetry of the experiment energies (4.235) MeV had been confirmed, experimentally not confirmed that corresponds to the angular momentums ($5/2_3^+$) when compared with the available experiment values.

4. The total angular momentum has been confirmed only for experiment energies values (4.106, 4.477, 4.576, 4.839, 4.99, 5.771, 7.712) MeV, experimentally not confirmed that corresponds to the angular momentums $(5/2_3^+, 5/2_3^+, 5/2_3^+, 1/2_3^+, 1/2_3^+, 1/2_4^+, 3/2_8^+)$, when compared with the available experiment values.

5. The total angular momentum has been confirmed for experiment energies values for which do not determined her symmetry (3.818, 4.302, 5.841,5.752, 6.354,6.629, 6.761, 8.16, 8.16) MeV, that corresponds to the angular momentums $(3/2_4^+, 5/2_3^+, 1/2_4^+, 1/2_5^+, 1/2_5^+, 1/2_5^+, 1/2_6^+, 1/2_6^+)$ when compared with the available experiment values.

6. The total angular momentum has been confirmed for experiment energies values for which did not determined her symmetry (3.675, 3.885, 3.89) MeV, that corresponds to the angular momentums $(3/2_3^+, 3/2_4^+, 3/2_4^+)$ when compared with the available experiment values.

7. We noticed that there are experiment energies values with angular momentum and symmetry that were not compared with our theoretical calculations because they do not converge with them.

8. We noticed through our calculations that there are (forty four) levels with total angular momentum and symmetry that were not matched by any available experiment value as well. We also noted that the highest value of the calculated energy theoretically is (29.008) MeV, while the highest experiment value of energy is (9.155) MeV meaning that we have obtained (forty four) new energy levels above the experiment value.

3.2 The probability of reduced electric quadrupole transition B (E2):

Information about the nuclei can be obtained from the study of electromagnetic transitions using the harmonic oscillator voltage b, HO (b>0) for each beam transmission. Where b (parameter) is the size of the harmonic oscillator and its value is equal to [5]

$$\boldsymbol{b_2} = \frac{(\hbar c)^2}{(\boldsymbol{m}c^2)(\hbar \boldsymbol{w})} = \frac{41.4 MeV f m^2}{\hbar \boldsymbol{w}}$$

Where, $\hbar c = 197.33 \text{ MeV}$. fm, $mc^2 = 940 \text{ MeV}$, $\hbar w = 41 \text{A}^{-1/3}$ b = 1.005 $A^{1/6}$.

The probability of transition is from very important quantities in nuclear studies after the excitation energies, therefore The probability of reduced electric quadrupole transition B (E2) was calculated in the nucleus of the sulfur isotope 35 S for the interactions w and hbusd, which corresponds to the experiment values available for this isotope as shown in the table (3):

 Table (3): A comparison between of the probability of reduced electric quadrupole transition B (E2) in the ³⁵S nucleus computed using the two-effective interaction hbusd and w with available experiment results [12]

We observe through the above table the values of B (E2) for the two reactions (w, hbusd) and we found that through our calculations we have obtained new transitions that have no experiment values yet.

4. Conclusions:

By application nuclear shell model and using an OXBASH computer software

For the two interactions W and HBUSD, the energy levels and the probability of reduced electric quadrupole transition B (E2) for the 35 S isotope had been studied. From the values we obtained for the energy levels, we conclude the following:

- The angular momentum and the symmetry for the ground state had been obtained agreement for both HBUSD and W interactions.
- An acceptable agreement was reached between the theoretical and experiment values for both HBUSD and W interactions
- Total angular momentum and symmetry levels have been confirmed for some uncertain energy angular momentum and symmetry experimentally for both HBUSD and W interactions.
- Total angular momentum is confirmed only for some uncertain energy levels by angular momentum experimentally for both HBUSD and W interactions.
- The total angular momentum is confirmed only for some values of experiment energies for which do not determined her symmetry for both HBUSD and W interactions.
- The total angular momentum has been confirmed for some experiment energies values but with a different symmetry for the HBUSD interaction.
- Higher energy levels were obtained from the higher values for the experiment levels of both HBUSD and W interactions.

from studying the probability of reduced electric quadrupole transition B (E2) for the ³⁵S isotope using HBUSD and W interactions, we found acceptable agreement compatibility between theoretical and experiment results.

Through the results, we observed that the two HBUSD and W interactions were appropriate for calculating energy levels and the probability of reduced electric quadrupole transition B(E2) for the studied isotopes, and we did not notice significant differences between them.

5. References

[**1**] A.

	Theoretical values B(E2) e ² fm ⁴			
$J^+_j \longrightarrow J^+_i$	w interaction $e_p = 1.36 e$ $e_n = 0.45 e$	hbusd interaction $e_p = 1.50 e$ $e_n = 0.50 e$	Experimental data B(E2) e ² fm ⁴ [12]	
$1/2_1 \longrightarrow 3/2_1$	77.000	44.8500		
$5/2_1 \longrightarrow 3/2_1$	44.440	37.5500		
$5/2_1 \longrightarrow 1/2_1$	3.954	0.0524		
$3/2_2 \longrightarrow 3/2_1$	12.590	8.5790		
$3/2_2 \longrightarrow 1/2_1$	13.270	12.0500		
$3/2_2 \longrightarrow 5/2_1$	60.840	29.8100		
$5/2_2 \longrightarrow 3/2_1$	0.000	1.3560		
$5/2_2 \longrightarrow 1/2_1$	41.710	27.5600		
5/2 ₂ → 5/2 ₁	22.390	6.3140		
$5/2_2 \longrightarrow 3/2_1$	4.733	21.5100		
$3/2_3 \longrightarrow 3/2_1$	7.554	4.2680		
$3/2_3 \longrightarrow 1/2_1$	8.474	0.0086		
$3/2_3 \longrightarrow 5/2_1$	3.476	29.3800		
$3/2_3 \longrightarrow 3/2_2$	3.331	0.3164		

Klimkiewicz, N. Paar, P. Adrich, M. Fallot, K. Boretzky, T. Aumann, D. Cortina-Gil, U. Datta Pramanik, Th. W. Elze, H. Emling, H. Geissel, M. Hellstrom, K. L. Jones, J. V. Kratz, R. Kulessa, C. Nociforo, R. Palit, H. Simon, G.

Surowka, K. Summerer, D. Vretenar, and W. Walus (2007). Nuclear symmetry energy and neutron skins derived from pygmy dipole resonances. Physical Review C, 76(5), 051603.

- [2] B. Fornal, R. Broda, K. H. Maier, J. Wrzesin'ski, G. J. Lane, M. Cromaz, A. O. Macchiavelli, R. M. Clark, K. Vetter, A. P. Byrne, G. D. Dracoulis, M. P. Carpenter, R. V. F. Janssens, I. Wiedenhoever, M. Rejmund, and J. Blomqvist (2001). Effective Charge of the π h_{11/2} Orbital and the Electric Field Gradient of Hg from the Yrast Structure of H 206 g. Physical review letters, 87(21), 212501.
- [3] D. Eccleshall, and M. J. L. Yates (1965). The $h_{11/2}$ single proton hole state in ²⁰⁷Tl. Physics Letters, 19(4), 301-303.
- [4] O. Sorlin, and M. G. Porquet, (2008). Nuclear magic numbers: New features far from stability. Progress in Particle and Nuclear Physics, 61(2), 602-673.
- [5] T. Otsuka, R. Fujimoto, Y. Utsuno, B. A. Brown, M. Honma, and T. Mizusaki, (2001). Magic numbers in exotic nuclei and spin-isospin properties of the NN interaction. Physical Review Letters, 87(8), 082502.
- [6] A. deShalit and H. Feshback,"Theoretical Nuclear Physics, Volume 1: Nuclear Structure", John and Sons, New York, (1974).
- [7] K. Krane,"Introductory Nuclear Physics", John Wiley and Sons, (1988).
- [8] P.J.Brussaard and P.W.M.Glaudemans, "Shell-Model Applications in Nuclear Spectroscopy", North –Holland , Amsterdam, (1977).
- [9] B. A. Brown, "Lecture Notes in Nuclear Structure Physics", Astronomy Michigan State University, (2005).
- [10] G. A. Lalazissis, and C. P. Panos, (1992). The harmonic oscillator energy level spacing for neutrons and protons in nuclei. Zeitschrift für Physik A Hadrons and Nuclei, 344(1), 17-19.
- [11] M. Honma, T. Otsuka, B. A. Brown, and T. Mizusaki, (2005). Shell-model description of neutron-rich pf-shell nuclei with a new effective interaction GXPF1. The European Physical Journal A-Hadrons and Nuclei, 25(1), 499-502.
- [12] D. R. Tilley, H. R. Weller, C. M. Cheves, and R. M. Chasteler (1995). Energy levels of light nuclei A= 18–19. Nuclear Physics A, 595(1), 1-170.