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Separation Axioms of IVIG-Induced Topological Space from Tritopological Space on Undirected Graphs

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Abstract: *In the present paper the definitions of separation axioms of $IVIG$-Induced Topology From Tritopological Space On Undirected Graph (locally finite graph) are introduced. and their basic properties are investigated with respect to three unique and well known topologies associated with graph, which are; The first is the newly we proposed (Independent Topology), the second is (Incident Topology) and the third (Graphic Topology). That is, the $VIG - T_i$; ($i = 0,1,2,3,4$) spaces and notions of $IVIG$ −normal and $IVIG$ −regular spaces are discussed in detail, also we introduce some theorems shows how one of the $IVIG$ −topological spaces implies the others with the help of many examples. Giving a fundamental step toward studying some properties of locally finite graphs by their corresponding $IVIG$ −separation axioms of tritopological spaces is our motivation.*

Keywords: Locally finite graph, tritopological spaces, *Independent Topology*, *Incident Topology*, *Graphic Topology*. $\mathrm{IVIG} - \mathrm{T}_i$; ($i = 0,1,2,3,4$) space.

1. Introduction

In Mathematics graph theory have a long history, one branch of graph theory is a topological graph theory. The relation between graph theory and topological theory existed before and used many times by researchers to deduce a topology from a given graph. Some of them makes models defined on the set of vertices V of the graph G only and others made it on the set of edges E . They study graphs as a topologies and have been applied in almost every scientific field. Many excellent basics on the mathematics of graph theory, topological graph theory and some applications may be found in the sources [1-7].

In general graphs divided in two types; directed and undirected graph. To an undirected graph some researchers associate a topological spaces as fellow;
In 2013 [8], Jafarian et al. associate a Graphic Topology with the vertex set of a locally finite graph without isolated vertex, and they defined a sub-basis family for a graphic topology as a sets of all vertices adjacent to the vertex v .

And in 2018 [9], Kilicman and Abdulkalek associate an Incidence Topology with a set of vertices for any simple graph without isolated vertex. where they defined a sub-basis family for an incident topology as a sets of all incident vertices with the edge e.

The previous works of topology on graphs was associated with a set of vertices without isolated vertex, these topologies are not appropriate to be associated with graphs that have an isolated vertex. Therefore, these reasons motivate us to associate a topology on the vertex set of any undirected graph (not only simple graph or locally finite graph) and which may contain one isolated vertex or more. By introducing a new Sub-basis family defined as a sets of all vertices non-adjacent to the vertex v to induce the new topology (which we named it *Independent Topology*) [10] in 2020, and we present a fundamental steps toward studying some main properties of undirected graphs by their corresponding topologies.

A tritopological space is simply a non-empty set X which is associated with three arbitrary topologies was initiated in 2000 by Kovar [11]. In 2004, Asmhan was deal with it in detail and introduced the definition of open set in tritopological spaces [12]. The author (Asmhan F. H.) provide the tritopological theory by all topological structures of that open set, like the base, connectedness, compactness, lindelofness, countability, separability, product space, quotient space. And introduced some relations, the reader can find that in [13 – 21]. Also the same author in 2017 [22] first initiated the soft tritopological theory, and in 2019, first initiated the fuzzy soft tritopological theory [23].

In 2020, the same authors associated a tritopological space with undirected graphs [24], i.e. a three different topologies induced from a one graph or three different graphs. These three different topologies are the *Independent Topology* $\mathcal{T}_{\mathrm{IV}}$ (which we proposed in 2020), the *Incident Topology* $\mathcal{T}_{\mathrm{I}}$, and the third is *Graphic Topology* $\mathcal{T}_{\mathrm{G}}$. And we introduced the IVIG-induced topology (i.e. $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$), also we give a fundamental step toward studying some properties of undirected graphs by their corresponding tritopological spaces.

In the present paper our motivation or target is to introduce a new kind of separation axioms of IVIG-induced topology (named IVIG-separation axioms) which induced from a tritopological space associated with undirected graphs (three different topologies induced from the same graph or three different graphs). These three different topologies are the unique three proposed to associate topological spaces with graphs, the *Independent Topology* (which we proposed in 2020), the *Incident Topology*, and the third is *Graphic Topology*. And studying some properties of IVIG-separation axioms with undirected graphs by their corresponding tritopological spaces.

In Section 2 of the article we give some fundamental definitions and preliminaries of graph theory, topology, tritopology, three topologies associated with graphs and IVIG-induced topology. In section 3 and 4 we define our new IVIG-separation axioms, IVIG -T_i ; (i = 0,1,2) Spaces, IVIG-regular, IVIG -normal and IVIG -T_i ; (i = 3,4) spaces with some examples. Section 5 is dedicated to main conclusions of this new IVIG-separation axioms are presented.

2. PRELIMINARIES

In this section we give some fundamental definitions and preliminaries of graph theory and topology. All this definition is standard, and can be found for example in sources [2] [3], [4], [8], [9], [10], [12], [24].

Usually the graph is a pair $G = (V, E)$, for more exactly A graph G consist of a non-empty set V of vertices (or nodes), and a set E of edges (or arcs). If e is an edge in G we can write $e = v\,u$ (e is join each vertex v and u),where v and u are vertices in V, then (v and u) are said adjacent vertices and incident with the edge e.If there is no vertex adjacent with a vertex v, then v is said isolated vertex. the degree of the vertex v denoted by $d(v)$ is the number of the edges where v incident with e, and $\Delta(G)$ is the maximum degree of vertices in G. A vertex of degree 0 is isolated. An independent set in a graph G is a set of pairwise non-adjacent vertices. The graph G is finite if the number of the vertices in G also the number of the edges in G is finite, then; otherwise it is an infinite graph. If any vertex can be reached from any other vertex in G by travelling along the edges, then G is called connected graph and is called disconnected otherwise.
We use notations K_n ,$K_{m,n}$,P_n and C_n for a complete graph with n vertices, the complete bipartite graph when partite sets have sizes m and n, the path on n vertices and the cycle on n vertices, respectively.

A topology $\mathcal{T}$ on a set $\mathcal{X}$ is a combination of subsets of $\mathcal{X}$, called open, such that the union of the members of any subset of $\mathcal{T}$ is a member of $\mathcal{T}$, the intersection of the members of any finite subset of $\mathcal{T}$ is a member of $\mathcal{T}$, and both empty set and $\mathcal{X}$ are in $\mathcal{T}$. The ordered pair $(\mathcal{X}, \mathcal{T})$ is called a topological space. When the topology $\mathcal{T} = P(\mathcal{X})$ on $\mathcal{X}$ is called discrete topology while the topology $\mathcal{T} = \{\mathcal{X}, \varphi\}$ on $\mathcal{X}$ is called indiscrete (or trivial) topology. A tritopological space is simply a non-empty set X which is associated with three arbitrary topologies $\mathcal{T}_1, \mathcal{T}_2$ and $\mathcal{T}_3$ on X as $(X, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$.

There are only three topological spaces associated with graphs, *Independent Topology, Graphic Topology and Incidence Topology*, which defined as fellows;
Let $G = (V, E)$ be an undirected graph, and Let $S_{Iv} = \{I_x : x \in V\}$ such that I_x is the set of all vertices not adjacent to x. G which may contain one isolated vertex or more, we have $V = \cup_{x \in V} I_x$. Hence S_{Iv} forms a sub-basis for a topology $\mathcal{T}_{IV}$ on V, and $\mathcal{T}_{IV}$ called *Independent Topology* of G.
Let $G = (V, E)$ be a locally finite graph, i.e. a graph in which every vertex has finitely many adjacent vertices (a simple graph without isolated vertex). Define S_G as follows: $S_G = \{A_u \mid u \in V\}$ such that A_u is the set of all vertices adjacent to u. Since G has no isolated vertex, we have $V = \cup_{u \in V} A_u$. Hence S_G forms a sub-basis for a topology $\mathcal{T}_G$ on V, and $\mathcal{T}_G$ called *Graphic Topology* of G.
Let $G = (V, E)$ be a locally finite graph, I_e be the incidence vertices with the edge e. Define S_{IG} as follows: $S_{IG} = \{I_e \mid e \in E\}$. G is a simple graph without isolated vertex, we have $V = \cup_{e \in E} I_e$. Hence S_{IG} forms a sub-basis for a topology $\mathcal{T}_I$ on V, and $\mathcal{T}_I$ called *Incidence Topology* of G.
The three topologies $\mathcal{T}_{IV}, \mathcal{T}_I$ and $\mathcal{T}_G$ on V give the tritopological space $(V, \mathcal{T}_{IV}, \mathcal{T}_I, \mathcal{T}_G)$.
, then $(V, \mathcal{T}_{IVIG})$ where $\mathcal{T}_{IVIG} = \mathcal{T}_{IV} \cap \mathcal{T}_I \cap \mathcal{T}_G$ is called the IVIG-induced topological space, and let $H \subseteq X$. H is called an IVIG-tri-open set in V if H is open in the IVIG-induced topology (i.e. $H \in \mathcal{T}_{IV} \cap \mathcal{T}_I \cap \mathcal{T}_G$), and is called an IVIG-tri-closed set in X if is closed in the IVIG-induced topology.

1. IVIG -T_i ; ($i = 0, 1, 2$) Spaces Of Tritopological Spaces On Graphs

Definition 3.1. A, Let $(V, \mathcal{T}_{IVIG})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{IV}, \mathcal{T}_I, \mathcal{T}_G)$ over V, then $(V, \mathcal{T}_{IVIG})$ is said to be IVIG -T_0-space if for every pair of distinct vertices $x, y \in V$, there is an IVIG -tri-open sets F_1, F_2 such that ‘ $x \in F_1$, $y \notin F_1$ ’ or ‘ $x \notin F_2$, $y \in F_2$ ’

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Example.3.2. Let$G = (V, E)$ be graph, such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ as show in Fig (1) . Then,

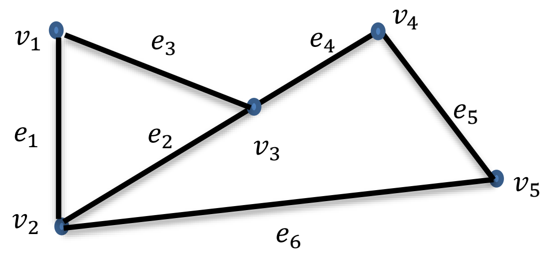

Fig (1)

The *independent topology*

$$S_{IV} = \{\varphi, \{v_4, v_5\}, \{v_1\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_1, v_3\}\}$$

$\mathcal{T}_{\mathrm{Iv}} = \{\varphi, V, \{v_4, v_5\}, \{v_1\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_2, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}\}$

$v_1, v_2 \in V$, $v_1 \neq v_2$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_2 \notin \{v_1\}$
$v_2, v_3 \in V$, $v_2 \neq v_3$, there is an open set $\{v_1, v_3\}$ such that $v_3 \in \{v_1, v_3\}$, $v_2 \notin \{v_1, v_3\}$ $v_3, v_4 \in V$, $v_3 \neq v_4$, there is an open set $\{v_4\}$ such that $v_4 \in \{v_4\}$, $v_3 \notin \{v_4\}$
$v_4, v_5 \in V$, $v_4 \neq v_5$, there is an open sets $\{v_4\}$ such that $v_4 \in \{v_4\}$, $v_5 \notin \{v_4\}$
$v_1, v_3 \in V$, $v_1 \neq v_3$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_3 \notin \{v_1\}$
$v_1, v_4 \in V$, $v_1 \neq v_4$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_4 \notin \{v_1\}$
$v_1, v_5 \in V$, $v_1 \neq v_5$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_5 \notin \{v_1\}$
$v_2, v_4 \in V$, $v_2 \neq v_4$, there is an open set $\{v_1, v_2\}$ such that $v_2 \in \{v_1, v_2\}$, $v_4 \notin \{v_1, v_2\}$
$v_2, v_5 \in V$, $v_2 \neq v_5$, there is an open set $\{v_1, v_2\}$such that $v_2 \in \{v_1, v_2\}$, $v_5 \notin \{v_1, v_2\}$
$v_3, v_5 \in V$, $v_3 \neq v_5$, there is an open set $\{v_1, v_3\}$such that $v_3 \in \{v_1, v_3\}$, $v_5 \notin \{v_1, v_3\}$
Then the *independent topology* is T_0-space

The *graphic topology*

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_5\}, \{v_2, v_3\}\}$$

$$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_5\}, \{v_2, v_3\} \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_5\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4\} \{v_1, v_2, v_3, v_5\}, \{v_2, v_3, v_4\}\}$$

$v_1, v_2 \in V$, $v_1 \neq v_2$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_2 \notin \{v_1\}$
$v_2, v_3 \in V$, $v_2 \neq v_3$, there is an open set $\{v_2\}$ such that $v_2 \in \{v_2\}$, $v_3 \notin \{v_2\}$
$v_3, v_4 \in V$, $v_3 \neq v_4$, there is an open set $\{v_2, v_4\}$ such that $v_4 \in \{v_2, v_4\}$, $v_3 \notin \{v_2, v_4\}$
$v_4, v_5 \in V$, $v_4 \neq v_5$, there is an open sets $\{v_2, v_4\}$ such that $v_4 \in \{v_2, v_4\}$, $v_5 \notin \{v_2, v_4\}$
$v_1, v_3 \in V$, $v_1 \neq v_3$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_3 \notin \{v_1\}$
$v_1, v_4 \in V$, $v_1 \neq v_4$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_4 \notin \{v_1\}$
$v_1, v_5 \in V$, $v_1 \neq v_5$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_5 \notin \{v_1\}$
$v_2, v_4 \in V$, $v_2 \neq v_4$, there is an open set $\{v_2\}$ such that $v_2 \in \{v_2\}$, $v_4 \notin \{v_2\}$
$v_2, v_5 \in V$, $v_2 \neq v_5$, there is an open set $\{v_2\}$such that $v_2 \in \{v_2\}$, $v_5 \notin \{v_2\}$
$v_3, v_5 \in V$, $v_3 \neq v_5$, there is an open set $\{v_3\}$such that $v_3 \in \{v_3\}$, $v_5 \notin \{v_3\}$
Then the *graphic topology* is a T_0-space

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Vol. 4, Issue 8, August – 2020, Pages: 5-19

The *Incidence Topology*

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_2, v_5\}, \{v_1, v_3\}, \{v_4, v_5\}, \{v_3, v_4\}\}$$

$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\},$
$\{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_3\}, \{v_2, v_4\},$
$\{v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3\}$
$\{v_1, v_3, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_3, v_4, v_5\}, \{v_1, v_2, v_5\}$
$\{v_2, v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_2, v_5, v_1, v_3\}, \{v_2, v_4, v_3, v_5\}, \{v_3, v_5, v_2\},$
$\{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_5, v_1, v_3\}, \{v_4, v_1, v_3\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}\}$

$v_1, v_2 \in V$, $v_1 \neq v_2$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_2 \notin \{v_1\}$
$v_2, v_3 \in V$, $v_2 \neq v_3$, there is an open set $\{v_2\}$ such that $v_2 \in \{v_2\}$, $v_3 \notin \{v_2\}$
$v_3, v_4 \in V$, $v_3 \neq v_4$, there is an open set $\{v_3\}$ such that $v_4 \in \{v_3\}$, $v_3 \notin \{v_3\}$
$v_4, v_5 \in V$, $v_4 \neq v_5$, there is an open sets $\{v_4\}$ such that $v_4 \in \{v_4\}$, $v_5 \notin \{v_4\}$
$v_1, v_3 \in V$, $v_1 \neq v_3$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_3 \notin \{v_1\}$
$v_1, v_4 \in V$, $v_1 \neq v_4$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_4 \notin \{v_1\}$
$v_1, v_5 \in V$, $v_1 \neq v_5$, there is an open set $\{v_1\}$ such that $v_1 \in \{v_1\}$, $v_5 \notin \{v_1\}$
$v_2, v_4 \in V$, $v_2 \neq v_4$, there is an open set $\{v_2\}$ such that $v_2 \in \{v_2\}$, $v_4 \notin \{v_2\}$
$v_2, v_5 \in V$, $v_2 \neq v_5$, there is an open set $\{v_2\}$ such that $v_2 \in \{v_2\}$, $v_5 \notin \{v_2\}$
$v_3, v_5 \in V$, $v_3 \neq v_5$, there is an open set $\{v_3\}$ such that $v_3 \in \{v_3\}$, $v_5 \notin \{v_3\}$
Then the *Incidence Topology* is a T_0-space

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:

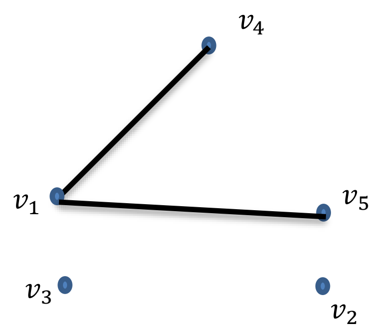
$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3, v_4\},$
$\{v_1, v_2, v_3, v_5\}\}$

It is clear that for every distinct vertices in IVIG-induced topology, there exist IVIG -tri-open set such that one of them contain one vertex but not the other.
Then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_0-space.

Now, in the following example the independent topology is the dominant and the most powerful because it is possible to take advantage of the isolated points (vertices) that control the form of the induced topology to suit the intersection of the other topologies.

Example.3.3. In this example we deal with a three topologies on a three different graphs. Let $V = \{v_1, v_2, v_3, v_4, v_5\}$, then:

The *independent topology*



International Journal of Academic and Applied Research (IJAAR)
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Fig (3)

$$S_{IV} = \{\varphi\ ,\{\ v_2\},\{\ v_3\},\{\ v_4\},\{\ v_5\},\{v_2,v_3\},\{v_1,v_3,v_4,v_5\},\{v_1,v_2,v_4,v_5\},\{v_2,v_3,v_5\} \\ ,\{v_2,v_3,v_4\},\{v_1,v_4,v_5\},\{v_3,v_5\},\{v_3,v_4\},\{v_2,v_5\},\{v_2,v_4\}\}$$

$$\mathcal{T}_{\mathrm{Iv}} =\{\ \varphi\ ,V,\{\ v_2\},\{\ v_3\},\{\ v_4\},\{\ v_5\},\{v_2,v_3\},\{v_1,v_3,v_4,v_5\},\{v_1,v_2,v_4,v_5\},\{v_2,v_3,v_5\} \\ ,\{v_2,v_3,v_4\},\{v_1,v_4,v_5\},\{v_3,v_5\},\{v_3,v_4\},\{v_2,v_5\},\{v_2,v_4\}\{v_4,v_5\},\{v_1,v_2,v_4,v_5\},\{v_2,v_3,v_4,v_5\} \\ ,\{v_2,v_4,v_5\},\{v_3,v_4,v_5\}\}$$

It is clear that, the *independent topology* is a T_0-space.

The *graphic topology*

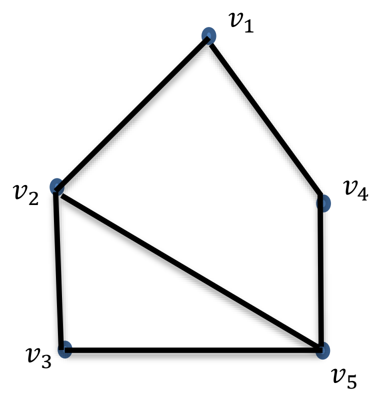

Fig (2)

$$S_G = \{\varphi\ ,\{\ v_2\},\{\ v_3\},\{\ v_5\},\{v_2,v_4\},\{v_1,v_3,v_5\},\{v_2,v_5\},\{v_1,v_5\},\{v_2,v_3,v_4\}\}$$

$$\mathcal{T}_{\mathrm{G}} = \{\varphi,V,\{\ v_2\},\{\ v_3\},\{\ v_5\},\{v_2,v_4\},\{v_1,v_3,v_5\},\{v_2,v_5\},\{v_1,v_5\},\{v_2,v_3,v_4\} \\ ,\{v_2,v_3\},\{v_3,v_5\},\{v_1,v_2,v_3,v_5\},\{v_1,v_2,v_5\},\{v_2,v_3,v_5\},\{v_2,v_4,v_5\} \\ \{v_2,v_3,v_4,v_5\},\{v_1,v_2,v_4,v_5\}$$

It is clear that, the *graphic topology* is a T_0-space

The *Incidence Topology*

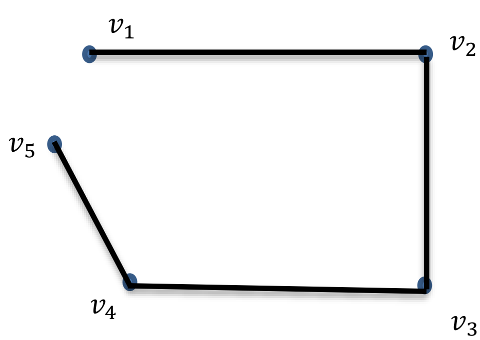

Fig (4)

$$S_I = \{\varphi,\{v_2\},\{v_3\},\{v_4\},\{v_1,v_2\},\{v_2,v_3\},\{v_3,v_4\},\{v_4,v_5\}\}$$

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Vol. 4, Issue 8, August – 2020, Pages: 5-19

$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_3\}, \{v_3, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_4, v_5\}\}$$

It is clear that, the *Incidence topology* is a T_0-space.

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:
$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_2\}, \{v_3\}, \{v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_2, v_3\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_4, v_5\}\}$. Also, it is clear that the $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_0-space.

Definition 3.4. A, Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be IVIG -T_1-space if for every pair of distinct vertices $x, y \in V$, there is an IVIG -tri-open sets F_1, F_2 such that ' $x \in F_1$, $y \notin F_1$ ' and ' $x \notin F_2$, $y \in F_2$ '.

Remark 3.5. Only when the three topologies are discrete topologies, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ will be IVIG -T_1 and IVIG -T_2 as shows in the following examples.

Example 3.6. Let $G = (V, E)$ be a graph, such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ as show in Fig (5). Then,

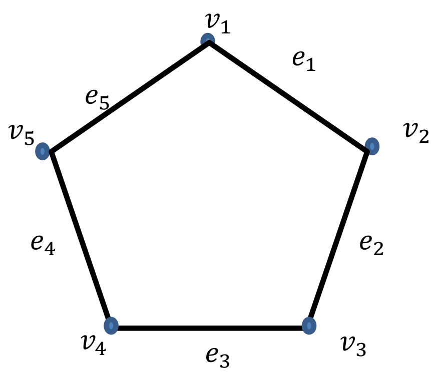

Fig (5)

The *Independent Topology*

$$S_{IV} = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}, \{v_1, v_2\}, \{v_2, v_3\}\}$$

$$\mathcal{T}_{\mathrm{IV}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_3, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_2, v_5, v_1, v_3\}, \{v_2, v_4, v_3, v_5\}, \{v_3, v_5, v_2\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_5, v_1, v_3\}\{v_4, v_1, v_3\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}\}$$

Since The *Independent Topology* is represent a discrete topology, then it a T_1-space.

The *Incidence Topology*

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}\}$$

$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3\}$$

$$\{v_1,v_3,v_5\},\{v_1,v_2,v_4\},\{v_1,v_4,v_5\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\},\{v_3,v_4,v_5\},\{v_1,v_2,v_5\}$$
$$\{v_2,v_3,v_5\},\{v_3,v_4,v_5\},\{v_1,v_3,v_4\},\{v_2,v_5,v_1,v_3\},\{v_2,v_4,v_3,v_5\},\{v_3,v_5,v_2\},$$
$$\{v_1,v_2,v_4,v_5\},\{v_1,v_2,v_3,v_4\},\{v_5,v_1,v_3\},\{v_4,v_1,v_3\},\{v_2,v_3,v_4,v_5\},\{v_1,v_3,v_4,v_5\}\}$$

The *Incidence Topology* is represent a discrete topology, then it is a T_1-space.

The *Graphic Topology*

$$S_G=\{\varphi,\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_5\},\{v_1,v_3\},\{v_2,v_5\},\{v_2,v_4\},\{v_3,v_5\},\{v_1,v_4\}\}$$
$$\mathcal{T}_{\mathrm{G}}=\{\varphi,V,\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_5\},\{v_1,v_3\},\{v_2,v_5\},\{v_2,v_4\},\{v_3,v_5\},\{v_1,v_4\},\{v_1,v_2\}$$
$$,\{v_1,v_5\},\{v_1,v_2,v_4\},\{v_1,v_2,v_4\},\{v_1,v_3,v_5\},\{v_2,v_3\},\{v_2,v_4\},\{v_2,v_5\}\{v_1,v_2,v_3\}$$
$$,\{v_2,v_3,v_5\},\{v_3,v_4\},\{v_3,v_5\},\{v_2,v_3,v_5\},\{v_1,v_3,v_4\},\{v_4,v_5\},\{v_2,v_4,v_5\},\{v_1,v_3,v_4\}$$
$$,\{v_3,v_4,v_5\},\{v_1,v_3,v_5\},\{v_1,v_4,v_5\},\{v_2,v_5,v_1,v_3\},\{v_2,v_4,v_3,v_5\},\{v_3,v_5,v_2\},\{v_1,v_2,v_4,v_5\}$$
$$,\{v_1,v_2,v_3,v_4\},\{v_5,v_1,v_3\},\{v_4,v_1,v_3\},\{v_2,v_3,v_4,v_5\},\{v_1,v_3,v_4,v_5\}\}$$

The *Graphic Topology* is represent a discrete topology, then it is a T_1-space.

Then, $\mathcal{T}_{\mathrm{IVIG}}=\mathcal{T}_{\mathrm{IV}}\cap\mathcal{T}_{\mathrm{I}}\cap\mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:

$$\mathcal{T}_{\mathrm{IVIG}}=\{\varphi,V,\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_5\},\{v_2,v_5\},\{v_1,v_3\},\{v_2,v_4\},\{v_3,v_5\},\{v_1,v_4\},$$
$$\{v_1,v_2\},\{v_1,v_3\},\{v_1,v_4\},\{v_1,v_5\},\{v_1,v_3,v_4\},\{v_1,v_4,v_5\},\{v_1,v_2,v_3\},\{v_2,v_4\},\{v_2,v_5\},$$
$$\{v_2,v_3,v_4\},\{v_2,v_4,v_5\},\{v_1,v_2,v_5\},\{v_3,v_5\},\{v_3,v_4,v_5\},\{v_1,v_3,v_5\},\{v_1,v_2,v_3\},$$
$$\{v_1,v_3,v_5\},\{v_1,v_2,v_4\}\{v_1,v_4,v_5\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\},\{v_3,v_4,v_5\},\{v_1,v_2,v_5\}\{v_2,v_3,v_5\},$$
$$\{v_3,v_4,v_5\},\{v_1,v_3,v_4\},\{v_2,v_5,v_1,v_3\},\{v_2,v_4,v_3,v_5\},\{v_3,v_5,v_2\},\{v_1,v_2,v_4,v_5\},$$
$$\{v_1,v_2,v_3,v_4\},\{v_5,v_1,v_3\},\{v_4,v_1,v_3\},\{v_2,v_3,v_4,v_5\},\{v_1,v_3,v_4,v_5\}\}$$

Also, it is clear that $\mathcal{T}_{\mathrm{IVIG}}$ is represent a discrete topology, and that for every distinct vertices in IVIG-induced topology, there exist IVIG -tri-open sets such that one of them contain the first vertex but not the other and the second set contain the second vertex but not the first. Thus $(V,\mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_1-space.

Example 3.7. In this example we deal with a three topologies on a three different graphs. Let $V=\{v_1,v_2,v_3\}$, then:

The *Independent Topology*

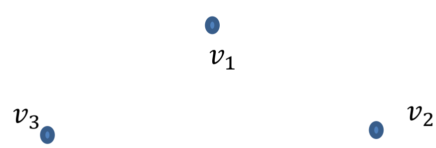

Fig (7)

$$S_{Iv}=\{\varphi,\{v_1\},\{v_2\},\{v_3\},\{v_1,v_2\},\{v_2,v_3\},\{v_1,v_3\}\}$$
$$\mathcal{T}_{\mathrm{Iv}}=\{\varphi,V,\{v_1\},\{v_2\},\{v_3\},\{v_1,v_2\},\{v_2,v_3\},\{v_1,v_3\}\}$$

Clearly The *Independent Topology* is T_1-space.

The *Incidence Topology*

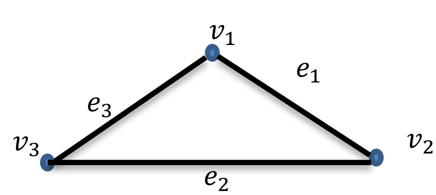

Fig (6)

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Clearly The *Incidence Topology* is discrete and then a T_1-space

The *Graphic Topology*

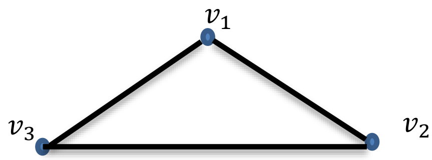

Fig (8)

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Also The *Graphic Topology* is clear a T_1-space.

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:

$$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Since the three topologies are discrete then the intersection among them ($\mathcal{T}_{\mathrm{IVIG}}$) is also discrete, thus $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_1-space.

Definition 3.8. Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V , then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be IVIG -T_2-space if for every pair of distinct vertices x , y $\in V$, there is an IVIG -tri-open sets F_1, F_2 such that **"** x $\in F_1$, y $\in F_2$ and $F_1 \cap F_2 = \varphi$ **".**

Example.3.9. In this example we deal with a three topologies on a three different graphs as shown in the figures below. Let $V = \{v_1, v_2, v_3, v_4\}$, then:
The *Independent Topology*

Fig (9)

$$S_{IV} = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\} \{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$$

$$\mathcal{T}_{\mathrm{IV}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\} \{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$$

$v_1, v_2 \in V$, there is an IVIG -tri-open sets $\{v_1\}, \{v_2\}$ such that ' $v_1 \in \{v_1\}$, $v_2 \in \{v_2\}$' and $\{v_1\} \cap \{v_2\} = \Phi$, $v_2, v_3 \in V$, there is an IVIG -tri-open sets $\{v_2\}, \{v_3\}$ such that ' $v_2 \in \{v_2\}$, $v_3 \in \{v_3\}$' and $\{v_2\} \cap \{v_3\} = \Phi$, $v_3, v_4 \in V$, there is an IVIG -tri-open sets $\{v_3\}, \{v_4\}$ such that ' $v_3 \in \{v_3\}$, $v_4 \in \{v_4\}$' and $\{v_3\} \cap \{v_4\} = \Phi$, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be independent topology T_2-space

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Vol. 4, Issue 8, August – 2020, Pages: 5-19

The *Graphic Topology*

Fig (10)

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}$$

$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}$
$\{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$

$v_1, v_2 \in V$, there is an IVIG -tri-open sets $\{v_1\}, \{v_2\}$ such that ' $v_1 \in \{v_1\}$, $v_2 \in \{v_2\}$' and $\{v_1\} \cap \{v_2\} = \varphi$, $v_2, v_3 \in V$, there is an IVIG -tri-open sets $\{v_2\}, \{v_3\}$ such that ' $v_2 \in \{v_2\}$, $v_3 \in \{v_3\}$' and $\{v_2\} \cap \{v_3\} = \varphi$, $v_3, v_4 \in V$, there is an IVIG -tri-open sets $\{v_3\}, \{v_4\}$ such that ' $v_3 \in \{v_3\}$, $v_4 \in \{v_4\}$' and $\{v_3\} \cap \{v_4\} = \varphi$, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be graphic topology T_2-space

The *Incidence Topology*

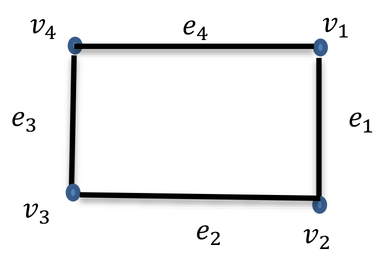

Fig (11)

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_1, v_4\}\}$$

$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\},$
$\{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$

Clearly, the *Incidence Topology* is a T_2-space.

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:
$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}$
$\{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$

Since, $v_1 \neq v_2 \in V$, there is an IVIG -tri-open sets $\{v_1\}, \{v_2\}$ such that ' $v_1 \in \{v_1\}$, $v_2 \in \{v_2\}$' and $\{v_1\} \cap \{v_2\} = \varphi$.

$v_2 \neq v_3 \in V$, there is an IVIG -tri-open sets $\{v_2\}, \{v_3\}$ such that ' $v_2 \in \{v_2\}$, $v_3 \in \{v_3\}$ and $\{v_2\} \cap \{v_3\} = \varphi$,

$v_3 \neq v_4 \in V$, there is an IVIG -tri-open sets $\{v_3\}, \{v_4\}$ such that ' $v_3 \in \{v_3\}$, $v_4 \in \{v_4\}$ and $\{v_3\} \cap \{v_4\} = \varphi$, similarly the other cases, $v_1 \neq v_3$, $v_1 \neq v_4$, $v_2 \neq v_4$.

Hence $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_2-space

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Vol. 4, Issue 8, August – 2020, Pages: 5-19

4. IVIG-regular, IVIG -normal and IVIG -$\mathrm{T_i}$; ($\mathbf{i = 3, 4}$) spaces

In this section, we define IVIG -T_3 and IVIG -T_4 spaces using ordinary points and characterize IVIG-regular and IVIG -normal spaces.

Definition 4.1. Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be IVIG -regular space if for every IVIG-closed set H in V, and every $x \notin H$, there is an IVIG -tri-open sets F_1 and F_2 such that **“** $x \in F_1$, $H \subseteq F_2$ and $F_1 \cap F_2 = \varphi$ **”.**

Example 4.2. Let $G = (V, E)$ be a simple graph as in **Fig (12)** such that
$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. Then,

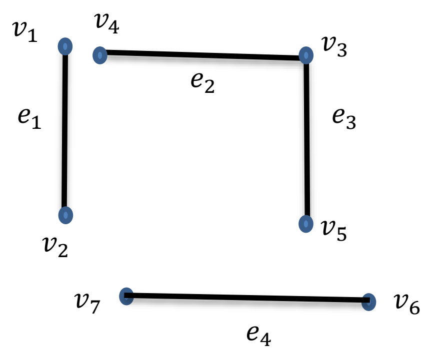

Fig (12)

The *Independent Topology*

$$S_{Iv} = \{\varphi, \{v_4\}, \{v_5\}, \{v_6, v_7\}, \{v_1, v_2\}, \{v_3, v_4, v_5\}, \{v_4, v_6, v_7\}, \{v_5, v_6, v_7\}, \{v_1, v_2, v_4\}, \{v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_6, v_7\}\{v_1, v_2, v_5\}, \{v_1, v_2, v_4, v_6, v_7\}, \{v_1, v_2, v_5, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5\}\}$$

$$\mathcal{T}_{\mathrm{Iv}} = \{\varphi, V, \{v_4\}, \{v_5\}, \{v_6, v_7\}, \{v_1, v_2\}, \{v_3, v_4, v_5\}, \{v_4, v_6, v_7\}, \{v_5, v_6, v_7\}, \{v_1, v_2, v_4\}, \{v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_6, v_7\}\{v_1, v_2, v_5\}, \{v_1, v_2, v_4, v_6, v_7\}, \{v_1, v_2, v_5, v_6, v_7\}, \{v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_4, v_5\}, \{v_4, v_5\}, \{v_1, v_2, v_3, v_4, v_5\}\}$$

$\{v_1, v_5\}$ is a closed set, and $v_6 \notin \{v_1, v_5\}$ there is an open sets $\{v_6, v_7\}$ and $\{v_1, v_2, v_5\}$, such that $v_6 \in \{v_6, v_7\}$, $\{v_1, v_5\} \subseteq \{v_1, v_2, v_5\}$ and $\{v_6, v_7\} \cap \{v_1, v_2, v_5\} = \varphi$. Similarly the other closed sets and points not belong to it, the all satisfy.
Then *the independent topology* is a regular space.

The *graphic topology*

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_6\}, \{v_7\}, \{v_4, v_5\}\}$$

$$\mathcal{T}_{\mathrm{G}} = \{\varphi, v, \{v_1\}, \{v_2\}, \{v_3\}, \{v_6\}, \{v_7\}, \{v_4, v_5\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4, v_5\}, \{v_1, v_6\}, \{v_1, v_7\}, \{v_2, v_3\}, \{v_2, v_6\}, \{v_2, v_7\}, \{v_2, v_4, v_5\}, \{v_3, v_6\}, \{v_3, v_7\}, \{v_3, v_4, v_5\}, \{v_6, v_7\}, \{v_4, v_5, v_6\}, \{v_4, v_5, v_7\}, \{v_1, v_2, v_3, v_7\}, \{v_1, v_2, v_4, v_5\}, \{v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_6, v_7\} \{v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_6, v_7\}, \{v_4, v_5, v_6, v_7\}, \{v_2, v_3, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5\}\}$$

$\{v_1, v_5\}$ is a closed set, and $v_3 \notin \{v_1, v_5\}$ there is an open sets $\{v_3\}$ and $\{v_1, v_4, v_5\}$ such that $v_3 \in \{v_3, v_7\}$,$\{v_1, v_5\} \subseteq \{v_1, v_2, v_4, v_5\}$ and $\{v_3\} \cap \{v_1, v_4, v_5\} = \varphi$.

$\{v_1, v_4\}$ is a closed set, and $v_3 \notin \{v_1, v_4\}$ there is an open sets $\{v_3\}$ and $\{v_1, v_4, v_5\}$ such that $v_3 \in \{v_3\}, \{v_1, v_4\} \subseteq \{v_1, v_4, v_5\}$ and $\{v_3\} \cap \{v_1, v_4, v_5\} = \varphi$ **.** And so on…

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Vol. 4, Issue 8, August – 2020, Pages: 5-19

Then the *graphic topology* is a regular space.

The *Incidence Topology*

$$S_I = \{\varphi, \{v_3\}, \{v_1, v_2\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_6, v_7\}\}$$

$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_3\}, \{v_1, v_2\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_6, v_7\}, \{v_1, v_2, v_3\}, \{v_3, v_6, v_7\},$
$\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_3, v_4, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5\}, \{v_3, v_5, v_6, v_7\}$
$, \{v_1, v_2, v_6, v_7\}, \{v_1, v_2, v_3, v_6, v_7\}, \{v_3, v_4, v_5, v_6, v_7\}\}$

$\{v_1, v_5\}$ is a closed set, and $v_6 \notin \{v_1, v_5\}$there is an open sets $\{v_6, v_7\}$ and $\{v_1, v_2, v_3, v_5\}$ such that $v_6 \in \{v_6, v_7\}$, $\{v_1, v_5\} \subseteq \{v_1, v_2, v_3, v_5\}$ and $\{v_6, v_7\} \cap \{v_1, v_2, v_3, v_5\} = \varphi$ **.** And so on…
Then the *Incidence Topology* is a regular space.

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:
$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1, v_2\}, \{v_3, v_4, v_5\}, \{v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_6, v_7\}$
$\{v_3, v_4, v_5, v_6, v_7\}\}$

$\{v_1, v_4\}$ is a closed set, and $v_7 \notin \{v_1, v_4\}$ there is an open sets $\{v_6, v_7\}$and $\{v_1, v_2, v_3, v_4, v_5\}$ such that $v_7 \in \{v_6, v_7\}, \{v_1, v_4\} \subseteq \{v_1, v_2, v_3, v_4, v_5\}$ and $\{v_6, v_7\} \cap \{v_1, v_2, v_3, v_4, v_5\} = \varphi$.
Similarly the other cases.
Thus $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -regular space.

Definition 4.3. Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V is said to be an IVIG -T_3-space iff it is an IVIG -regular and IVIG -T_1-space.

Example 4.4 . In this example we deal with a three topologies on a three different graphs as shown in the figures below. Let $V = \{v_1, v_2, v_3\}$, then:

The *Independent Topology*

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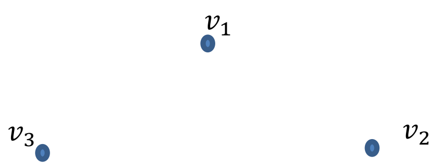

Fig (14)

$$S_{Iv} = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

$$\mathcal{T}_{\mathrm{Iv}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

It is clear that, the *Independent topological space* is a T_1-space and regular. Thus it is a T_3-space.

The *Incidence Topology*

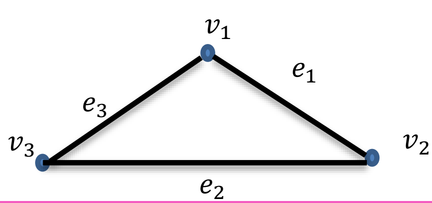

Fig (13)

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

It is clear that, the *Incidence topological space* is a T_1-space and regular. Thus it is a T_3-space.

The *Graphic Topology*

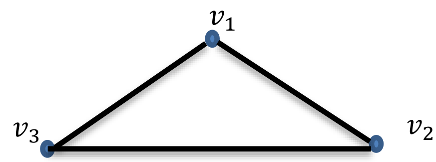

Fig (15)

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

It is clear that, the *Graphic topological space* is a T_1-space and regular. Thus it is a T_3-space.
Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:

$$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Also, it is clear that the $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_1-space and IVIG -regular space then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG_T_3-space.

Definition 4.5. Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be IVIG-normal space if for every pair of IVIG -tri-closed sets H and M in V such that $H \cap M = \varphi$, there is an IVIG –tri-open sets F_1 and F_2 such that " $H \subseteq F_1$, $M \subseteq F_2$ and $F_1 \cap F_2 = \varphi$ ".

Example 4.6. Let $G = (V, E)$ be a graph, such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, $E = \{e_1, e_2, e_3, e_4\}$ as show in Fig (16). Then,

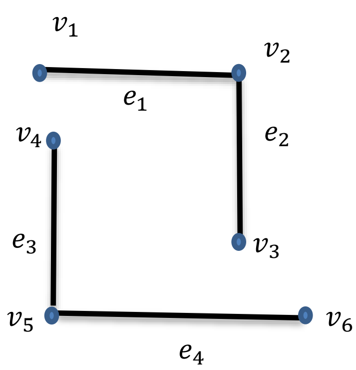

Fig (16)

The *Independent Topology*

$$S_{Iv} = \{\varphi, \{v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6\}, \{v_1, v_4, v_5, v_6\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_6\} \\ \{v_1, v_2, v_3\}\}$$

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ISSN: 2643-9603
Vol. 4, Issue 8, August – 2020, Pages: 5-19

$$\mathcal{T}_{\mathrm{Iv}} = \{\varphi, V, \{v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6\}, \{v_1, v_4, v_5, v_6\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_6\} \{v_1, v_2, v_3\}\}$$

$\{v_1\}$ *and* $\{v_6\}$ are closed sets such that $\{v_1\} \cap \{v_6\} = \varphi$, there is an open sets $\{v_1, v_2, v_3\}$ *and* $\{v_4, v_5, v_6\}$ such that $\{v_1\} \subseteq \{v_1, v_2, v_3\}$, $\{v_6\} \subseteq \{v_4, v_5, v_6\}$ and $\{v_1, v_2, v_3\} \cap \{v_4, v_5, v_6\} = \varphi$.

$\{v_1\}$ *and* $\{v_4\}$ are closed sets such that $\{v_1\} \cap \{v_4\} = \varphi$, there is an open sets $\{v_1, v_2, v_3\}$ *and* $\{v_4, v_5, v_6\}$ such that $\{v_1\} \subseteq \{v_1, v_2, v_3\}$, $\{v_4\} \subseteq \{v_4, v_5, v_6\}$ and $\{v_1, v_2, v_3\} \cap \{v_4, v_5, v_6\} = \varphi$. Similarly, the other closed sets.

Then the *Independent Topology* is normal space.

The *Graphic Topology*

$$S_G = \{\varphi, \{v_2\}, \{v_1, v_3\}, \{v_5\}, \{v_4, v_6\}\}$$

$$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_2\}, \{v_1, v_3\}, \{v_5\}, \{v_4, v_6\}, \{v_1, v_2, v_3\}, \{v_2, v_5\}, \{v_2, v_4, v_6\}, \{v_1, v_3, v_5\}, \{v_1, v_3, v_4, v_6\}, \{v_4, v_5, v_6\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_4, v_6\}, \{v_2, v_4, v_5, v_6\}\}$$

$\{v_1\}$ *and* $\{v_6\}$ are closed sets such that $\{v_1\} \cap \{v_6\} = \varphi$, there is an open sets $\{v_1, v_3\}$ *and* $\{v_4, v_6\}$ such that $\{v_1\} \subseteq \{v_1, v_3\}$, $\{v_6\} \subseteq \{v_4, v_6\}$ and $\{v_1, v_3\} \cap \{v_4, v_6\} = \varphi$. Similarly the other closed sets.

Then the *Graphic Topology* is a normal space.

The *Incidence Topology*

$$S_I = \{\varphi, \{v_2\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_4, v_5\}, \{v_5, v_6\}\}$$

$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_2\}, \{v_5\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_4, v_5\}, \{v_5, v_6\}, \{v_2, v_5\}, \{v_1, v_2, v_3\}, \{v_2, v_4, v_5\}, \{v_2, v_5, v_6\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_4, v_5, v_6\}, \{v_2, v_3, v_4, v_5\}, \{v_2, v_3, v_5, v_6\}\}$$

$\{v_1\}$ *and* $\{v_4\}$ are closed sets such that $\{v_1\} \cap \{v_4\} = \varphi$, there is an open sets $\{v_1, v_2\}$ *and* $\{v_4, v_5\}$ such that $\{v_1\} \subseteq \{v_1, v_2\}$, $\{v_4\} \subseteq \{v_4, v_5\}$ and $\{v_1, v_2\} \cap \{v_4, v_5\} = \varphi$. Similarly the other closed sets. Then the *Incidence Topology* is normal space.

The $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, then IVIG-induced topology is:
$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}\}$

$\{v_1\}$ *and* $\{v_4\}$ are IVIG -closed sets such that $\{v_1\} \cap \{v_4\} = \varphi$, there is an IVIG -open sets $\{v_1, v_2, v_3\}$ *and* $\{v_4, v_5, v_6\}$ such that $\{v_1\} \subseteq \{v_1, v_2, v_3\}$, $\{v_4\} \subseteq \{v_4, v_5, v_6\}$ and $\{v_1, v_2, v_3\} \cap \{v_4, v_5, v_6\} = \varphi$ **.** Similarly, the other closed sets.

Thus $(V, \mathcal{T}_{\mathrm{IVIG}})$ is IVIG-normal space.

Definition 4.7. Let $(V, \mathcal{T}_{\mathrm{IVIG}})$ be an IVIG-induced topological space from tritopological space $(V, \mathcal{T}_{\mathrm{IV}}, \mathcal{T}_{\mathrm{I}}, \mathcal{T}_{\mathrm{G}})$ over V, then $(V, \mathcal{T}_{\mathrm{IVIG}})$ is said to be IVIG -T_4-space iff it is an IVIG -normal and IVIG -T_1-space.

Example 4.8. This example deal with a three topologies on a three different graphs as shown in the following figures. Let $V = \{v_1, v_2, v_3\}$, then:

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 4, Issue 8, August – 2020, Pages: 5-19

The *Independent Topology*

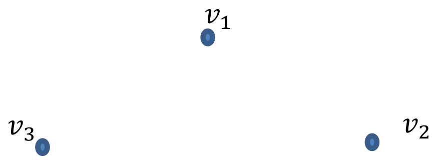

Fig (18)

$$S_{IV} = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{IV}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Clearly The *Independent Topology* is a T_1-space and normal, thus it is T_4-space.

The *Incidence Topology*

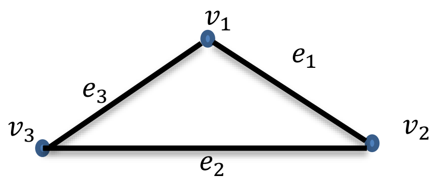

Fig (17)

$$S_I = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{I}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Clearly the *Incidence Topology* is a T_1-space and normal, thus it is T_4-space.

The *Graphic Topology*

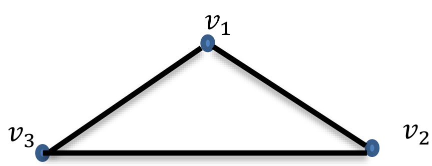

Fig (19)

$$S_G = \{\varphi, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$
$$\mathcal{T}_{\mathrm{G}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Clearly the *Graphic Topology* is a T_1-space and normal, thus it is T_4-space.

Then, $\mathcal{T}_{\mathrm{IVIG}} = \mathcal{T}_{\mathrm{IV}} \cap \mathcal{T}_{\mathrm{I}} \cap \mathcal{T}_{\mathrm{G}}$, and the IVIG-induced topology is:

$$\mathcal{T}_{\mathrm{IVIG}} = \{\varphi, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}\}$$

Also, clearly the IVIG-induced topological space $(V, \mathcal{T}_{\mathrm{IVIG}})$ is an IVIG -T_1-space and IVIG -normal, therefore it is an IVIG -T_4-space.

Conclusions

In the present paper a new kind of separation axioms of IVIG-induced topology (named IVIG-separation axioms) was introduced, which induced from a tritopological space associated with undirected graphs the *Independent Topology*, the *Incident Topology* and the *Graphic Topology*. Some properties of IVIG-separation axioms are studied and some examples are displayed.

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