

Dependence of Ultrasound Absorption (Amplification) on Electron Scattering Mechanism in Semiconductors

Jurabek Abdiyev¹, Shakhboz Khasanov^{2,3}

¹Samarkand State University, Samarkand 140104, Uzbekistan

²Institute of Modern Physics, Lanzhou 730000, China

³University of Chinese Academy of Sciences, Beijing 100049, China

Corresponding author: shakhboz.khasanov@list.ru (Sh. Khasanov)

Abstract: The absorption and amplification of ultrasound in semiconductors with high mobility in the presence of an external electric field E satisfying the condition $eE \ll q\bar{\epsilon}$, $eE\tau_r \ll \bar{p}$ (q - wave sound vector, τ_r - pulse relaxation time, $\bar{\epsilon}$, \bar{p} - is the characteristic value of the electron energy and momentum). It is shown that the absorption and amplification coefficients of sound essentially from the mechanisms of electron scattering.

Keywords—ultrasound; electric field; absorption; amplification; momentum; scattering; electron; semiconductor; wavelength; wave vector.

1. INTRODUCTION

So far, many theoretical and experimental scientific studies [1-5] have emerged in which the absorption and amplification of a sound wave in piezoelectric semiconductors with a frequency of ω and a wave vector q satisfy the following conditions

$$ql \ll 1, \omega\tau_\epsilon \gg 1 \quad (1)$$

here l - is the free path length of the electrons and τ_ϵ - is the energy relaxation time. The condition ensuring the fulfillment of this inequality can be observed in semiconductors of n-type InSb100, where charge carriers have high mobility. In studies [6-9], it was found that sound absorption and amplification under such conditions could be significantly different from those in the hydrodynamic regime. This difference is especially noticeable in cases where the electron energy increases with increasing the energy relaxation time of electron. In such cases, it was observed that the absorption of sound might be significantly higher than the value of White's in the hydrodynamic theory, as well as the frequency dependence of the absorption coefficient may be completely different [7-9]

Here we show that even with the fulfillment of inequalities

$$\omega\tau_\epsilon \ll 1, q^2 D\tau_\epsilon \gg 1 \quad (2)$$

there are specific frequency dependencies in the sound absorption coefficient. Here $q^2 D = q_i q_m D_{im}$, D_{im} is the electron diffusion tensor.

2. FREQUENCY DEPENDENCIES OF THE SOUND ABSORPTION COEFFICIENT

The conditions satisfying the inequality (2) can be fulfilled experimentally, but this has not been considered in theoretical studies [5,7,9]. In work [8], formulas are given that are appropriate for any size of $\omega\tau_\epsilon$ parameters, but they are derived for the presence of electron temperature.

At the same time, on the one hand, for the existence of these waves, the fulfillment of the following inequalities is required.

$$\omega\tau_{ee} \ll 1, q^2 D\tau_{ee} \ll 1 \quad (3)$$

Here τ_{ee} - is the characteristic time of the interelectron collision. On the other hand, in the field of the existence of wave electron temperature, the specific connections of absorption and amplification disappear. We assume that the opposite of one of the (3) inequalities is fulfilled. We also consider its intensity to be small when we look at sound absorption and amplification. Then we can use the usual Fourier analysis and assume that all quantities are proportional to $\exp[i(\vec{q}\vec{r} - \omega t)]$. Using the elasticity equation and the Poisson equation and considering the interaction of sound with electrons to be piezoelectric, the law of dispersion for a sound wave can be written as [6]:

$$\frac{\omega^2 - \omega_0^2(q)}{\omega_0^2(q)} = \chi \left[1 + \frac{4\pi e^2}{\epsilon_0 q^2} K_{\vec{q}}(\omega) \right]^{-1} \quad (4)$$

Here $\chi = \frac{4\pi\beta^2}{\varepsilon_0 c}$ $\chi = \frac{4\pi\beta^2}{\varepsilon_0 c}$ -is the dimensionless constant of the electromechanical coupling, c -is the modulus of elasticity,

β -is the piezo-module, ε_0 -is the electrical absorption, $\omega_0^2(q) = cq^2/\rho$, ρ -is the crystal density. The coefficient of sound absorption $\Gamma(4)$ is determined by the law of dispersion as $\Gamma = 2ImK_{\vec{q}}(\omega)$. Where $K_{\vec{q}}(\omega)$ -is the response function of the electron concentration n_q to the variable electric field potential φ_q :

$$n_q(\omega) = -e\varphi_q K_{\vec{q}}(\omega) \quad (5)$$

To determine this response function, we need to solve the kinetic equation for the electron distribution function $f_{\vec{p}}$:

$$f_{\vec{p}} = f_0(\varepsilon_{\vec{p}}, T) + f_{\vec{p}}^{(1)},$$

here f_0 -is the equilibrium distribution function. This equation has the following appearance:

$$(-i\omega + i\vec{q}\vec{v})f_{\vec{p}}^{(1)} + \hat{I}f_{\vec{p}}^{(1)} = i(\vec{q}\vec{v})e\varphi_q \frac{df_0}{d\varepsilon_{\vec{p}}}, \quad (6)$$

3. DISEQUILIBRIUM SURFACE DISTRIBUTION FOR CONSTANT ENERGY

In (6) \hat{I} -is a linear collision operator. Fourier component of concentration $n_{\vec{k}}$

$$n_q(\omega) = \frac{2}{(2\pi\hbar)^3} = \int d^3\vec{p} f_{\vec{p}}^{(1)} = \int_0^\infty d\varepsilon \rho(\varepsilon) \tilde{f}(\varepsilon) \quad (7)$$

Here $\rho(\varepsilon) = \frac{2}{(2\pi\hbar)^3} \int d^3\vec{p} \delta(\varepsilon - \varepsilon_{\vec{p}})$ -is the density of states, and the function $f(\varepsilon)$ -is the disequilibrium part of the distribution function averaged over the constant energy surface of

$$\tilde{f}(\varepsilon) = \frac{2}{(2\pi\hbar)^3 \rho(\varepsilon)} \int d^3\vec{p} f_{\vec{p}}^{(1)} \delta(\varepsilon - \varepsilon_{\vec{p}}) \quad (8)$$

It can easily be seen that the part of the distribution function that is symmetric on the momentum \vec{p} depends only on the energy ε and corresponds to the function $\tilde{f}(\varepsilon)$ [11]. The equation (6) for $\tilde{f}(\varepsilon)$ can be found by averaging over the constant energy and has the following appearance [7,9]:

$$[-i\omega + q^2 D(\varepsilon_{\vec{p}})]\tilde{f} + \langle \hat{I}f \rangle = q^2 D(\varepsilon_{\vec{p}}) e\varphi_{\vec{k}} \frac{df_0}{d\varepsilon_{\vec{p}}} \quad (9)$$

here

$$D(\varepsilon_{\vec{p}}) = \frac{q_i q_m}{q^2} D_{im}(\varepsilon_{\vec{p}}), D_{im}(\varepsilon_{\vec{p}}) = \langle v_i \hat{I}^{-1} v_m \rangle$$

Here the magnitude $D(\varepsilon_{\vec{p}})$ has the meaning of the diffusion coefficient of the given energy electrons in \vec{k} direction of wave vector. Let's assume that electrons obey Boltzmann's statistics, and that the leading mechanism of energy relaxation is scattering in acoustic phonons. In this case [11]

$$\langle \hat{I}f \rangle = -\frac{\pi}{2\tau_\varepsilon} \frac{1}{\sqrt{x}} x^2 \left(1 + \frac{d}{dx}\right) \tilde{f}(x) \quad (10)$$

Here $x = \frac{\varepsilon}{T}$, T -is the temperature per unit of energy,

$$\tau_\varepsilon = \frac{\pi^{3/2} \hbar^4 \rho}{4\sqrt{2} \Lambda m^{3/2} \sqrt{T}} \quad (11)$$

-relaxation time of electron energy, m - its effective mass, Λ - deformation potential constant. If the energy dependence of $D(\varepsilon)$ assuming as the power function of $D(\varepsilon)$ the equation (9) can be written as:

$$\left[-i\omega \tau_\varepsilon \frac{2}{\sqrt{\pi}} x + q^2 D \tau_\varepsilon \frac{x^u}{\Gamma(u+1)} - \frac{d}{dx} x^2 \left(1 + \frac{d}{dx}\right)\right] \tilde{f}(x) = -\frac{e\varphi_q}{T} q^2 D \tau_\varepsilon \frac{x^u}{\Gamma(u+1)} e^{-x} \quad (13)$$

The solution of this equation can be easily found in the cases $q^2 D\tau_\epsilon \gg 1$, $\omega\tau_\epsilon \ll 1$. In this case, $\tilde{f}(x) = Ce^{-x}$, in which C is a constant, it is found by integrating (13) by x , and we get the usual hydrodynamic expression for $K_q(\omega)$

$$K_q(\omega) = \frac{n_0 q^2}{T \gamma^2} [-i\omega\tau_M + q^2/\gamma^2]^{-1} \quad (14)$$

Here n_0 -is the total concentration of electrons, $\gamma^2 = 4\pi e^2 n_0 / \epsilon_0 T$ -is the inverse of the square of the Debye radius; $\tau_M = \frac{\epsilon_0}{4\pi\sigma}$ - Maxwell time of relaxation; $\sigma = \frac{en_0 q_i q_m \mu_{im}}{q^2}$; μ_{im} -is the velocity tensor of electrons. When condition (2) is satisfied, the abstract part of f is much smaller than its actual part. Therefore, when calculating the actual part of it, we can say that $\omega = 0$, and it is possible not to take into account the energy relaxation. As a result,

$$ReK_q(\omega) = \frac{n_0}{T} \quad (15)$$

If at this time it is $u < 2$, it is possible to find the abstract part of $K_q(\omega)$ (13) by integrating ω without taking into account the relaxation of energy in phonons. In this case

$$ImK_q(\omega) = \frac{n_0}{T} \frac{\omega\tau_M}{q^2/\gamma^2} \frac{\Gamma(2-u)\Gamma(1+u)}{\Gamma^2(3/2)} \quad (16)$$

as a result, we obtain the formula (11) of the work for the sound absorption coefficient [4]. If $u \geq 2$ ($u = 3$ when scattering occurs in mixtures without a magnetic field, and $u = 3$ when scattering in acoustic phonons in a strong magnetic field) is written for $ImK_q(\omega)$ the integral in the expression becomes distant, excluding the relaxation of energy. If we express $Im\tilde{f}(x)$ as

$$Im\tilde{f}(x) = \theta(x)e^{-x} \quad (17)$$

we can obtain the following equation from (13)

$$x\dot{\theta} + (2x - x^2)\dot{\theta} - \alpha x^u \theta = \beta\sqrt{x} \quad (18)$$

Here

$$\alpha = \frac{q^2 D\tau_\epsilon}{\Gamma(1+u)}, \quad \beta = \frac{2}{\sqrt{\pi}} \frac{e\varphi q}{T} \omega\tau_\epsilon$$

Since the differential operator is significant only at small values of x (parameter $\alpha \gg 1$), the coefficient in front of $\dot{\theta}$ can be omitted x^2 relative to $2x$. Then, by substituting $\theta = y$ the equation (18) can be reduced to following

$$\ddot{y} - \alpha x^{u-2} y = \beta x^{-1/2} \quad (19)$$

The boundary conditions for this equation are determined by $\sum_p \hat{I}f_p = 0$ and

$$\lim_{x \rightarrow \infty} [xye^{-x}] = \lim_{x \rightarrow 0} [xye^{-x}] = 0 \quad (20)$$

The fundamental solutions of the homogeneous equation in (19) are the $\sqrt{x}I_{1/u}(2x^{u/2}\sqrt{\alpha}/u)$ and $\sqrt{x}K_{1/u}(2x^{u/2}\sqrt{\alpha}/u)$ functions, where I, K are the modified functions of Bessel. Accordingly, the solution of (19) corresponding to the boundary conditions (20) can be written as follows:

$$\theta(x) = \frac{u\beta}{2\sqrt{x}} \left[I_{1/u}(2x^{u/2}\sqrt{\alpha}/u) \int_{\infty}^x dt K_{1/u}(2x^{u/2}\sqrt{\alpha}/u) - K_{1/u}(2x^{u/2}\sqrt{\alpha}/u) \int_{\infty}^x dt I_{1/u}(2x^{u/2}\sqrt{\alpha}/u) \right] \quad (21)$$

Since we have $\alpha \gg 1$, it is $ImK_q(\omega)$ that connects $\theta(x)$ with

$$ImK_q(\omega) = -\frac{2n_0}{T\sqrt{\pi}} \int_0^{\infty} \sqrt{x} \theta(x) e^{-x} dx \quad (22)$$

in the expression it will be possible to replace e^{-x} with A_1 , then it will be possible to find the connection of $ImK_q(\omega)$ from the parameter α by bringing the corresponding integral to it in a measurable form. As a result, we will have

$$ImK_q(\omega) = \frac{n_0}{T} \frac{\omega\tau_M}{q^2/\gamma^2} [q^2 D\tau_\epsilon]^{(u-2)/2} A_1 \quad (23)$$

here

$$A_1 = \frac{8}{\pi} \left(\frac{u}{2\Gamma(u+1)} \right)^{2/u} \int_0^\infty \xi^{\frac{2-u}{u}} K_{\frac{1}{u}}(\xi) d\xi \int_0^\xi \eta^{\frac{2-u}{u}} I_{\frac{1}{u}}(\eta) d\eta \quad (24)$$

4. THE INCREASE IN SOUND AT THE EXTERNAL ELECTRIC FIELD

Now let us look at the increase in sound at the external electric field E. In this work, we do not take into account the phenomenon of statistical heating of conductive electrons in such a field, and assume that the condition $(qv^d)\tau_\epsilon \ll 1$ is fulfilled. Where v^d – is the drift velocity of the electrons in the external electric field $v_i^d = \mu_{ik}E_k$. The external field can be taken into account in addition to the right side of equation (9) if the given condition is satisfied

$$\frac{ieq_i E_k}{\sqrt{\epsilon}} \frac{df_0}{d\epsilon} \frac{d}{d\epsilon} (\sqrt{\epsilon} D_{ik}(\epsilon)) \quad (25)$$

The results obtained depend significantly on whether the energy bonds of the combinations $q_i E_k D_{ik}(\epsilon)$ and $q^2 D(\epsilon)$ are the same or different. In the absence of a magnetic field, as well as in the presence of a strong magnet ($\vec{v}^d \parallel \vec{q} \parallel \vec{H}$ and $\vec{v}^d \parallel \vec{q} \perp \vec{H}$ when the Hall contacts are closed), such connections are the same. In practice, this case is considered in [4]. The value of the electric field added to $ImK_q(\omega)$ will be equal to the following

$$-\frac{n_0}{T} (\vec{q} \cdot \vec{v}^d) \cdot A_2, A_2 = 2u. \quad (26)$$

In a strong magnetic field ($\vec{v}^d \perp \vec{q} \parallel \vec{H}$ and when the Hall contacts are disconnected) $q_i E_k D_{ik}(\epsilon)$ gives the main contribution to the combination the nondiagonal part of the diffusion tensor proportional to ϵ . In this case, except for scattering in acoustic phonons, the small range of energy values is not highlighted. The percentage of electric field is determined by the expression (26), in which

$$A_2 = \frac{3}{2} \Gamma(1+u)\Gamma(2-u) \quad (27)$$

However, during the relaxation of electron pulses in deformation-acoustic phonons, the strong magnetic field is $u = 2$ ($s = -1/2$) and a small energy field is highlighted. In this case, in order to calculate the contribution of the electric field to the abstract part of the response function, the same must be taken into account here as the relaxation of energy in sound absorption. The contribution of the electric field to $ImK_q(\omega)$ is carried out by replacing ω with $(-\vec{q}\vec{v}^d) \cdot \frac{3\sqrt{\pi}}{4}$ in the expression (23).

5. CONCLUSION

In the summary section of our work we present the formulas for the coefficient of sound absorption (amplification) for the cases that are not considered in the articles [3-5].

$u \geq 2$, $H = 0$, as well as a strong magnetic field (in the case of $\vec{H} \perp \vec{q} \parallel \vec{v}^d$ and $\vec{H} \parallel \vec{q} \parallel \vec{v}^d$ when the Hall contacts are closed)

$$\Gamma = \chi q (1 + q^2/\gamma^2)^{-2} [\omega A_1 (q^2 D \tau_\epsilon)] \tau_M \quad (28)$$

When $u = 2$ (acoustic scattering in a strong magnetic field $\vec{H} \perp \vec{q} \parallel \vec{v}^d$ and when Hall contacts are disconnected)

$$\Gamma = \chi q \left(1 + \frac{q^2}{\gamma^2} \right)^{-2} A_1 \left[\omega - \frac{3\sqrt{\pi}}{4} (\vec{q}\vec{v}^d) \right] \tau_M \quad (29)$$

When $u < 2$ (in the case of a strong magnetic field $\vec{H} \parallel \vec{q} \parallel \vec{v}^d$ and when the Hall contacts are disconnected)

$$\Gamma = \chi q \frac{4}{\pi} \left(1 + \frac{q^2}{\gamma^2} \right)^{-2} \Gamma(u+1)\Gamma(2-u) \left[\omega - \frac{3\sqrt{\pi}}{4} (\vec{q}\vec{v}^d) \right] \tau_M \quad (30)$$

The formulas given in (28-30) are drastically different from the formulas in hydrodynamic theory. It can be seen from them that the threshold of amplification at the frequencies seen in the study is generally frequency-dependent, the appearance of such a coupling is determined by the predominant mechanism of electron pulse relaxation. The study of the absorption and amplification of ultrasound in the frequencies we are looking at in transmitters is, in our view, a very effective method of studying kinetic phenomena in such materials. It should be noted that the results of this study, together with the results of the study in [6-9], fully cover the experimental cases that can be carried out in high-speed semiconductors.

6. REFERENCES

- [1] Parmenter, R. H. (1953). The acousto-electric effect. *Physical Review*, 89(5), 990.
- [2] Weinreich, G., Sanders Jr, T. M., & White, H. G. (1959). Acoustoelectric effect in n-type germanium. *Physical Review*, 114(1), 33.
- [3] Hutson, A. R., & White, D. L. (1962). Elastic wave propagation in piezoelectric semiconductors. *Journal of Applied Physics*, 33(1), 40-47.
- [4] White, D. L. (1962). Amplification of ultrasonic waves in piezoelectric semiconductors. *Journal of Applied Physics*, 33(8), 2547-2554.
- [5] Pustovoit, V.I. (2016). Direct detection of gravitational waves. *Advances in Physical Sciences*, 186 (10), 1133-1152 (In Russian)
- [6] Gurevich, V. L. (1969). Theory of acoustic properties of piezoelectric semiconductors (Acoustic effects in piezoelectric semiconductors due to sound-electrons interaction including sound absorption, sound velocity changes, electroacoustic effect, etc.). *SOVIET PHYSICS-SEMICONDUCTORS*, 2, 1299-1325.
- [7] Yu.M. Galperin, I.L. Drichko, B.D. Leichtman. Nonlinear effects of sound amplification in n-InSb in a strong magnetic field. // *FTT*, v.12, v.5, pp. 1437-1442 (1970) (In Russian)
- [8] Bugaev A. S. Gulyaev Yu.V., *Phys.Stat.Sol.*v.60, 1973, p. 467
- [9] Bugaev A.S., Gulyaev Yu.V., Shkerdin G.N., *FTT*, v. 14,1972, p. 2655 (In Russian)
- [10] Pardaev A., Musaev P. - *FTT*, v. 15, 1973, p. 2536
- [11] Shodiyev B. Master's dissertation, SamDU, 2013, pp. 50-65