## Study the Energy Levels and Probability of Reduced Electric Quadruple Transition for <sup>34</sup>S Isotope by Applying the Nuclear Shell Model

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Abstract : in this paper, the OXBASH computer software has been used for apply the nuclear shell model to the sulfur isotope 34S in order to calculate the excitation energies and the reduced probability of electric quadruple transition B(E2) and by depending each of effective interactions w and hbusd that describe the interaction between a nucleon-nucleon in an sd space for the nucleus under study, which contains 18 nucleons in the model space and by considering the isotope of oxygen <sup>16</sup>O as the closed core, our theoretical calculations have been compared with the experimental data available for this isotope and we have noted a good match with it.

**Keywords:** Nuclear shell model, <sup>34</sup>S sulfur isotope, excitation energies, the reduced probability of electric quadruple transition B(E2), sd-Shell, effective interactions w and hbusd, OXBASH program.

### 1. Introduction

The study of the structure of unstable and rich-neutrons nuclei is the most important aims of modern physics because of its importance in many applications in the field of astronomy in the study of the great explosions called (Supernova) and in other fields [1], and the information available on this region is very little due to the difficulty of generating these nuclei [2, 3]. There are several nuclear models to explain the nuclear structure, all of them had a major effect for determine the path that has been followed for explain the problem of nuclear structure [4]. And from the most important basic models proposed to describe the interaction between nucleons is the nuclear shell model, also called the independent particle model, and is based on several suppositions through which it succeeds in explaining some nuclear phenomena and fails in others when compared with experimental data [5]. The first to introduce the idea of closed nuclear shells is the scientist (W. Elasser) in 1934, as some studies on the average of binding energy as well as the properties of the nuclei led to that the nucleons inside the nucleus move within orbits like those in which electrons move in the atom, it has been called shell structure or levels structure in which the nucleus is stable [6]. The importance of the shell model is mainly shape in its ability to give a correct approximation of the energies of the levels in which nucleons can be found with different values of orbital angular momentum. It has been observed that many nuclear properties showed discontinuities at certain even values of numbers of neutrons or protons. Experiments have shown that stable nuclei are characterized by the number of their protons Z or the number of their neutrons N equal to one of the following numbers 126,82,50,20,8,2 [5,4], which was called (Magic Numbers). Magic numbers have been explained as closed shells or covers of neutrons or protons, and that the covers of neutrons and protons appear to be independent of each other according to the nuclear shell model, and for this reason this model is considered the cornerstone of many nuclear studies [7].

### 2. Theoretical part

The nuclear properties of different states can be calculated by the binding energies, the excitation energies of the nuclei [8], and the binding energy of the nuclei is defined as the negative value of the energy needed to divide the nucleus into its component's neutrons and free protons [8, 9], and the indication that there is a direct relationship between the binding energy and the excitation energy with the of expectation value of Hamilton for nuclear system, the binding energy of the ground state is largest that possible while the excitation energy is defined as the difference between the binding energy of the state (n) and the binding energy of the ground state (0) according to the following relationship [8]:

$$E_x(n) = E^b(n) - E^b(0)$$

(1)

where  $E_x(n)$ : the excitation energy of state,  $E^b(n)$ : the binding energy of state and  $E^b(0)$ : the binding energy of ground state.

We assume that the nucleus has a closed core, and two additional particles, outside the closed core within the orbit  $\rho$  [8], if the two particles are located within the specified orbit  $\rho$ , it is called a pure configuration. But if the two particles fill more than one orbit, they share the values of the total momentum (J) and the isospin (T), however, it gets from connected them a state of a mixing and admixture for energy levels, this is called a mixed configuration [4, 5]. The distributions of the different limits of the total binding energy are given as [9]:

$$E_n^b(core + \rho^2) = E^b(core) + 2e_p + E_n^{(1)}(\rho^2)$$
(2)

each of the terms of the equation has a physical explanation, and it is:

 $E_n^b(core)$ : it represents the binding energy of the particles in closed core and this term is a constant amount.

 $E_{\Gamma}^{(1)}(\rho)$ : it represents the contribution to the binding energy from the actual nuclear interaction of the two particles outside the closed core, this term depends not only on the orbit but also on the total momentum (J) and the isospin (T) for two particles system.  $2e_{\rho}$ : It represents the negative value of the energy needed to remove two particles from the low voltage and make them move independently in orbit ( $\rho$ ) and assumes that the voltage does not depend on the number of particles outside the core. However, if we have (n) particles in orbit ( $\rho$ ) and (m) particles in orbit ( $\lambda$ ) outside the closed core, then the total binding energy

equation is given by the following relationship [8]:

$$E_{\Gamma}^{b}(core + \rho^{\lambda} + \lambda^{m}) = E_{c} + E^{b}(core) + ne_{\rho} + me_{\lambda} + E_{\Gamma}^{(1)}(\rho^{n}\lambda^{m})$$
(3)

The term  $E_c$  represents Coulomb's energy,  $e_{\rho}$  and  $e_{\lambda}$  it represents the energy of a single particle in orbits ( $\rho$ ) and ( $\lambda$ ), and the last term it represents the residual reaction energy (matrix of the interacting elements) and is given by the following relationship [8]:

$$E_{\Gamma}^{(1)}(\rho^n \lambda^m) = \langle \rho^n \lambda^m | V(1,2) | \rho^n \lambda^m \rangle_{\Gamma}$$
(4)

It is possible to obtain information about the cores by studying the electromagnetic transitions and by using the harmonic oscillator Potential HO (b > 0) for each transmission in the beam where b (a parameter) represents the size of the harmonic oscillator and its value is equal to [10,11]

$$\mathbf{b}^{2} = \frac{(\hbar \mathbf{c})^{2}}{(\mathbf{mc}^{2})(\hbar \omega)} = \frac{41.4 \text{ MeV fm}^{2}}{\hbar \omega}$$
(5)  
Where  $\hbar \mathbf{c} = 197.33 \text{ MeV}$ . fm,  $\mathbf{mc}^{2} = 940 \text{ MeV}$ ,  $\hbar \mathbf{w} = 41 \text{ A}^{-1/3}$   
 $\mathbf{h} = 1.005 A^{1/6}$ (6)

Transition B (E2) was chosen, which is the reduced quadruple electrode transmission using interactions w and hbusd, for sulfur isotope  $^{34}$ S by OXBASH code The default values for the effective charge of a proton and neutron have been changed in proportion to the practical value of the ground state.

#### 3. Calculations and discussion

Theoretical calculations in this study include the following:

### 3.1 Energy levels

depending on the energy equations included in the shell model that were mentioned in the previous item and programmed using an OXBASH computer software, the excitation energies and the reduced probability for electric quadrupole transition B (E2) for <sup>34</sup>S nucleus containing 18 nucleons outside the closed core <sup>18</sup>O and in the model space sd of orbits  $(1d_{5/2}, 2s_{1/2}, 1d_{3/2})$  and by depending all of the effective interactions w, hbusd, and the results were obtained listed in Tables (1) and (2):

Table (1): The theoretical values of the excitation energies in the <sup>34</sup> S nucleus computed using the hbusd effective interac	ction
with available experiment results [12].	

Theoretical values for (hbusd) interaction		Experimental values [12]	
$\mathbf{J}^+$	E(MeV)	E(MeV)	$\mathbf{J}^{\pi}$
01	0.000	0	0+
23	4.267	4.114	2+
$2_4$	4.412	4.889	2+
31	4.457	4.876	3+
41	4.681	4.688	4+
12	5.395	5.380	1+
03	5.411	5.228	0+
25	5.830	5.998	2+
42	6.378	6.89	(3,4)+

2 ( (25	6.605	6.731	2(+)
$Z_6$	0.025	6.428	(2+)
43	6.697	6.639	4(-)
27	6.975	6.828	2+
2	7 200	7.112	2+
28	7.309	7.219	(2+)
13	7.672	7.974	(1,2+)
2	7 880	7.75	2+
29	7.009	7.974	(1,2+)
210	8.445	8.702	(2-)
35	8.552	8.615	(2-,3+)
16	8.908	8.805	(1-)
1	0.400	9.026	(1,2+)
17	9.409	9.208	(1,2+)
1	0.550	9.479	(1)+
18	9.550	9.546	(1,2+)
1	0.605	9.64	(1,2+)
19	9.095	9.868	(1)+
310	10.022	10.179	(1,2,3)
1	10 155	10.17	(1)+
110	10.155	10.179	(1,2,3)
76	16.907		
83	17.002		
77	17.796		
84	17.974		
<b>9</b> <sub>1</sub>	18.045		
78	18.294		
79	18.378		
85	18.827		
710	19.097		
86	19.200		
87	19.907		
88	21.504		
101	21.651		
89	21.822		
810	21.908		
92	22.080		
<u>9</u> <sub>3</sub>	23.484		
93	24.130		
95	24.785		
102	25.239		
9 <sub>6</sub>	26.749		
<u>9</u> 7	27.097		
98	28.776		
<u> </u>	29.248		
103	29.540		
9 <sub>10</sub>	29.763		
104	30.409		
105	34.617		
106	38.235		

By comparing our theoretical results using the hbusd interaction with the experiment results of this isotope in the above table, the following was found:

- 1. The total angular momentum and the symmetry for ground state of  $0_1^+$  level had been compared when compared with the available experiment values.
- **2.** A good agreement of experiment energy values was obtained, (4.114, 4.889, 4.876, 4.688, 5.380, 5.228, 5.998, 6.828, 7.112, 7.75) MeV Corresponding to the angular momentums and symmetry  $(2_3^+, 2_4^+, 3_1^+, 4_1^+, 1_2^+, 0_3^+, 2_5^+, 2_7^+, 2_8^+, 2_9^+)$  when compared with the available experiment values.
- **3.** The total angular momentum and symmetry of the experiment energies (6.428, 7.219, 7.974, 8.615) MeV had been confirmed, experimentally not confirmed that corresponds to the angular momentums  $(2_6^+, 2_8^+, 2_9^+, 3_5^+)$  when compared with the available experiment values.
- 4. The total angular momentum has been confirmed only for experiment energies values (6.89, 9.479, 9.868, 10.17) MeV, experimentally not confirmed that corresponds to the angular momentums  $(4_2^+, 1_8^+, 1_9^+, 1_{10}^+)$ , when compared with the available experiment values.
- 5. The symmetry has been confirmed only for the experiment energy value (6.625), it isn't confirmed experimentally, that corresponds to the angular momentum  $(2_6^+)$  when compared with the available experiment value.
- 6. The total angular momentum has been confirmed for experiment energies values for which did not determined her symmetry (7.974, 9.026, 9.208, 9.546, 9.64, 10.179, 10.179) MeV, that corresponds to the angular momentums  $(1_3^+, 1_7^+, 1_7^+, 1_8^+, 1_9^+, 3_{10}^+, 1_{10}^+)$  when compared with the available experiment values.
- 7. The total angular momentum has been confirmed for experiment energies values (8.702, 8.805) MeV, that corresponds to the angular momentums  $(1_6^+, 2_{10}^+)$ , but with a different symmetry when compared with the available experiment values.
- 8. We noticed that there are experiment energies values with angular momentum and symmetry that were not compared with our theoretical calculations because they do not converge with them.
- **9.** We noticed through our calculations that there are (Twentynine) levels with total angular momentum and symmetry that were not matched by any available experiment value as well. We also noted that the highest value of the calculated energy theoretically is (38.235) MeV while the highest experiment value of energy is (16.649) MeV meaning that we have obtained (Twentynine) new energy levels above the experiment value.

# Table (2) : The theoretical values of the excitation energies in the ${}^{34}$ S nucleus computed using the effective interaction W with available experiment results [12].

Theoretical values for		Experimental values [12]	
(w) interaction			
$\mathbf{J}^+$	E(MeV)	E(MeV)	$\mathbf{J}^{\pi}$
$0_1$	0.000	0.000	0+
$2_1$	2.201	2.128	2+
$2_{2}$	3.138	3.304	2+
02	3.905	3.916	0+
11	3.951	4.075	1+
23	4.302	4.115	2+
31	4.773	4.877	3+
24	4.851	4.890	2+
41	4.896	4.689	4+
03	5.172	5.228	0+
12	5.557	5.381	1+
2.	6.683	6.731	2(+)
26		6.428	(2+)
42	6.819	6.890	(3,4)+
43	6.987	6.251	4+
2	7.072	6.829	2+
27		7.219	(2+)
$0_4$	7.116	7.164	(0)+
$2_{8}$	7.590	7.112	2+
13	7.662	7.974	(1,2+)
2	7.679	7.750	2+
$\angle 9$		7.974	(1,2+)
14	8.172	8.175	(1,2+)
210	8.755	8.702	(2-)

35	8.823	8.874	(1-,2,3+)
1	0.102	9.026	(1,2+)
16	9.103	9.158	(1,2+)
		9.479	(1)+
17	9.368	9.208	(1,2+)
		9.546	(1,2+)
1	0.046	9.801	(1,2+)
18	9.946	9.706	(1,2+)
39	10.070	10.179	(1,2,3)
1	10 202	10.170	(1)+
19	10.202	10.179	(1,2,3)
110	10.639	10.803	(1,2+)
75	16.792		
76	17.348		
83	17.468		
77	17.753		
91	18.165		
78	18.447		
84	18.673		
79	19.116		
85	19.311		
86	19.421		
7 <sub>10</sub>	19.566		
87	19.804		
$8_{8}$	21.222		
101	21.731		
89	22.076		
$8_{10}$	22.264		
9 <sub>2</sub>	22.656		
93	23.510		
94	24.450		
9 <sub>5</sub>	24.807		
102	25.747		
9 <sub>6</sub>	26.984		
97	27.311		
98	28.633		
103	29.646		
99	30.066		
104	30.533		
9 <sub>10</sub>	30.709		
105	35.372		
106	38.305		

By comparing our theoretical results using the interaction w with the experiment results of this isotope in the above table, the following was found:

- 1. The total angular momentum and the symmetry had been confirmed for ground state  $0_1^+$  level when it compared with the available experiment values.
- **2.** A good agreement of experiment energy values was obtained, (2.128, 3.304, 3.916, 4.075, 4.115, 4.877, 4.890, 4.689, 5.228, 5.381, 6.251, 6.829, 7.112, 7.750) MeV Corresponding to the angular momentums and symmetry  $(2_1^+, 2_2^+, 0_2^+, 1_1^+, 2_3^+, 3_1^+, 2_4^+, 4_1^+, 0_3^+, 1_2^+, 4_3^+, 2_7^+, 2_8^+, 2_9^+)$  when compared with the available experiment values.
- 3. The total angular momentum and symmetry of the experiment energies values (6.428, 7.219, 7.974, 8.874) MeV had been confirmed, experimentally not confirmed that corresponds to the angular momentums  $(2_6^+, 2_7^+, 2_9^+, 3_5^+)$  when compared with the available experiment values.

- **4.** The total angular momentum has been confirmed only for experiment energies values (6.890, 7.164, 8.874, 9.479, 10.170) MeV, experimentally not confirmed that corresponds to the angular momentums  $(4_2^+, 0_4^+, 3_5^+, 1_7^+, 1_9^+)$ , when compared with the available experiment values.
- 5. The symmetry has been confirmed only for the experiment energy value (6.731), it wasn't confirmed experimentally, that corresponds to the angular momentum  $(2_6^+)$  when compared with the available experiment value.
- 6. The total angular momentum has been confirmed for experiment energies values for which do not determined her symmetry (7.974, 8.175, 9.026, 9.158, 9.208, 9.546, 9.801, 9.706, 10.179, 10.803) MeV, that corresponds to the angular momentums  $(1_3^+, 1_4^+, 1_6^+, 1_7^+, 1_7^+, 1_8^+, 1_8^+, 3_9^+, 1_9^+, 1_{10}^+)$  when compared with the available experiment values.
- 7. We noticed that there are experiment energies values with angular momentum and symmetry that were not compared with our theoretical calculations because they do not converge with them.
- **8.** We noticed through our calculations that there are (thirty) levels with total angular momentum and symmetry that were not matched by any available experiment value as well. We also noted that the highest value of the calculated energy theoretically is (38.305) MeV while the highest experiment value of energy is (16.649) MeV meaning that we have obtained (thirty) new energy levels above the experiment value.

### 2.3 The probability of reduced electric quadrupole transition B(E2)

The reduced probability of transition is from very important quantities in nuclear studies after the excitation energies, therefore The reduced probability of electric quadrupole transition B(E2) was calculated in the nucleus of the sulfur isotope <sup>34</sup>S for the interactions w and hbusd, which corresponds to the experiment values available for this isotope as shown in the table (3):

### Table (3) : The reduced probability of electric quadrupole transition B(E2) in the <sup>34</sup>S nucleus computed using the twoeffective interaction hbusd and w with available experiment results [12].

	Theoretical va		
$J_{j}^{+} \rightarrow J_{i}^{+}$	w interaction $e_p = 1.36 e$ $e_n = 0.45 e$	hbusd interaction $e_p = 1.50 e$ $e_n = 0.50 e$	Experimental data B(E2) e <sup>2</sup> fm <sup>4</sup> [12]
$2_1 \longrightarrow 0_1$	39.2600	54.2500	40.8266
$0_2 \longrightarrow 2_1$	22.9100	20.9700	27.4794
$1_2 \longrightarrow 2_1$	3.6760	4.1140	1.4394
$1_1 \longrightarrow 2_1$	2.3110	2.1290	170.1107
$2_4 \longrightarrow 0_1$	2.1560	1.0300	2.2900
$2_3 \longrightarrow 0_1$	1.7030	4.5140	3.7294
$2_2 \longrightarrow 0_1$	4.1680	2.0310	4.9070
$2_3 \longrightarrow 2_1$	16.6800	12.7700	15.0483
$2_2 \longrightarrow 2_1$	45.0600	75.0400	24.8623
$3_1 \longrightarrow 2_1$	1.6030	1.2250	0.5888
$3_1 \longrightarrow 2_2$	82.9700	118.6000	5.2342
$4_1 \longrightarrow 2_1$	54.0200	70.9500	53.6503
$4_3 \longrightarrow 3_1$	11.3800	5.0960	78.5126
$8_1 \longrightarrow 6_1$	26.9400	33.9100	176.6534

We are notice through the above table and after comparing the theoretical results for the values B(E2) for interaction (w) with the experiment results, we found a good agreement with the values of transitions B (E2;  $1_2^+ \rightarrow 2_1^+$ ), B (E2;  $2_4^+ \rightarrow 0_1^+$ ), B (E2;  $2_3^+ \rightarrow 0_1^+$ ), B (E2;  $2_2^+ \rightarrow 0_1^+$ ), B (E2;  $2_3^+ \rightarrow 0_1^+$ ), B (E2; 2\_3^+ \rightarrow 0\_1^+), B (E2; E2; E2; E2; E2; E2; E2; E2; E2

As for comparing the theoretical results B(E2) for interaction (hbusd) with the experiment results, we found a good agreement with the values of transitions B (E2;  $1_2^+ \rightarrow 2_1^+$ ), B (E2;  $2_4^+ \rightarrow 0_1^+$ ), B (E2;  $2_3^+ \rightarrow 0_1^+$ ), B (E2;  $2_2^+ \rightarrow 0_1^+$ ), B (E2;  $2_3^+ \rightarrow 2_1^+$ ) and B (E2;  $3_1^+ \rightarrow 2_1^+$ ) while for the remain of the transitions was acceptable with the experiment results.

### 4. Conclusions:

By application nuclear shell model and using an OXBASH computer software for the two interactions w and hbusd, the energy levels and the reduced probability of electric quadrupole transition B(E2) for the <sup>34</sup>S isotope had been studied. From the values we obtained for the energy levels, we conclude the following:

- The angular momentum and the symmetry for the ground state had been obtained agreement between the values of theoretical and experiment energies for both hbusd and W interactions.
- An acceptable agreement was reached between the theoretical and experiment values for both hbusd and W interactions
- Total angular momentum and symmetry levels have been confirmed for some uncertain energy angular momentum and symmetry experimentally for both HBUSD and W interactions.
- Total angular momentum is confirmed only for some uncertain energy levels by angular momentum experimentally for both hbusd and W interactions.
- Symmetry is confirmed only for some practically uncertain energy levels for both hbusd and W interactions.
- The total angular momentum is confirmed only for some values of experiment energies for which do not determined her symmetry for both HBUSD and W interactions.
- The total angular momentum has been confirmed for some experiment energies values but with a different symmetry for the hbusd interaction.
- Higher energy levels were obtained from the higher values for the experiment levels of both hbusd and W interactions.

from studying the probability of reduced electric quadrupole transition B(E2) for the <sup>34</sup>S isotope using hbusd and w interactions, we found acceptable agreement compatibility between theoretical and experiment results.

Through the results, we observed that the two hbusd and W interactions were appropriate for calculating energy levels and the reduced probability of electric quadrupole transition B(E2) for the studied isotope, and we did not notice significant differences between them.

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