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Artin's Characters Table of the Group $Q_{2m}\times D_4$ when m is an Odd Number

Naba Hasoon Jaber

Department of Mathematics,Faculty of eduction for Girl,University of Kufa,Najaf, Iraq

Abstract: *The main purpose of this paper is to find Artin's characters table of the group $Q_{2m}\times D_4$ when m is an odd number, which is denoted by $Ar(Q_{2m}\times D_4)$, where Q_{2m} is denoted to quaternion group of order 4m, such that for each positive integer m, there are two generators x and y for Q_{2m} satisfies $Q_{2m}=\{x^h y^k, 0\leq \square \leq 2m-1, k=0,1\}$which has the properties $x^{2m}=y^4=I$, $yx^r y^{-1}=x^{-r}$ and D_4 is the dihedral group of order 8 is generate by a rotation r of order 4 and reflection s of order 2. The eight elements of D_4 can be written as: $\{I^*,r,r^2, r^3,s, sr, sr^2,sr^3\}$with properties $sr^k s = r^{-k}$, $k = 0,1,2,3$.*

Keywords: Characters, Artin, group, Q_{2m}, D_4,odd number.

1. Introduction:

let G be a finite group, two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes [1].

Let H be a subgroup of G and let ϕ be a class function on H, the induced class function on G, is given by:

$$\phi'(g) = \frac{1}{|H|}\sum_{h\in G}\phi^{\circ}(hgh^{-1}),\ \forall g\in G, \text{ where } \phi^{\circ} \text{ is defined by:}$$

$$\phi^{\circ}(x) = \begin{cases}\phi(x) & \text{if } x\in H\\ 0 & \text{if } x\notin H\end{cases} \quad [2].$$

Let H be a subgroup of G and ϕ be a character of H, then ϕ′ is a character of G, and it is called the induced character on G[3].

In 1976 ,David.G[4] studied "Artin Exponent of arbitrary characters of cyclic subgroup ", Journal of Algebra,61,p p.58-76.

In 1996, Knwabusz .K[3] studied "Some Definitions of Artin's Exponent of finite Group", USA, National foundation Math,GR.

In this work we find Artin's characters table of the Group $Q_{2m}\times D_4$ when m is an odd number.

2. Preliminaries

In this section we find Artin's characters table of the group Q_{2m} when m is an odd number and Artin's characters table of the group D_4 .

Theorem(2.1): [2]

Let H be a cyclic subgroup of G and $h_1, h_2,\ldots, h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G,then: $\varphi'(g) = \begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$

2.2 Artin's Characters Tables:

Definition(2.2.1) [2] :

Let G be a finite group, all the characters of G induced from a principal character of cyclic subgroups of G are called Artin characters of G.

Proposition(2.2.2) [1]:

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on G. Furthermore, Artin characters are constant on each Γ-classes.

Definition(2.2.3) [5]:

Artin characters of the finite group G can be displayed in a table called Artin characters table of G which is denoted by Ar(G). The first row is the Γ-conjugate classes, the second row is the number of elements in each conjugate class, the third row is the size of the centralized $|C_G(CL_\alpha)|$ and other rows contains the values of Artin characters.

Theorem(2.2.4) [5]:

The general form of Artin characters table of C_{p^s} when p is a prime number and s is a positive integer number is given by:

Ar(C_{p^s}) =

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	[x]
$\|CL_\alpha\|$	1	1	1	1	…	1
$\|C_{p^s}(CL_\alpha)\|$	p^s	p^s	p^s	p^s	…	p^s
φ_1'	p^s	0	0	0	…	0
φ_2'	p^{s-1}	p^{s-1}	0	0	…	0
φ_3'	p^{s-2}	p^{s-2}	p^{s-2}	0	…	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮
φ_s'	p	p	p	p	…	0
φ_{s+1}'	1	1	1	1	…	1

Table(1)

Corollary(2.2.5) [5]:

Let m = $p_1^{\alpha_1}.p_2^{\alpha_2}\dots p_n^{\alpha_n}$ where g.c.d(p_i,p_j) =1, if $i\neq$j and p_i's are prime numbers, and α_i any positive integers for all 1≤ $i \leq n$, then : Ar(C_m) = Ar($C_{p_1^{\alpha_1}}$) ⊗ Ar($C_{p_2^{\alpha_2}}$) ⊗…⊗ Ar($C_{p_n^{\alpha_n}}$).

Theorem(2.2.6) [6]:

The Artin characters table of the quaternion group Q_{2m} when m is an odd number is given as follows:

Ar(Q_{2m}) =

	Γ-classes of C_{2m}								
Γ-classes	[1]	$[X^{2r}]$			$[x^m]$	$[X^{2r+1}]$			[y]
$\|CL_\alpha\|$	1	2	⋯	2	1	2	⋯	2	2m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	⋯	2m	4m	2m	⋯	2m	2
Φ_1	2Ar(C_{2m})								0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	⋯	0	m	0	⋯	0	1

Table(2)

where 0 ≤ r ≤ m-1, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the Quaternion group Q_{2m} , for all 1≤ j ≤ l+1.

Example(2.2.7) :

To find Ar(Q_{22}) by using theorem(2.2.6) we get the following table: Ar(Q_{22}) = Ar($Q_{2\cdot 11}$) =

Ar(Q_{22}) =

Γ-classes	[1]	$[x^2]$	$[x^{11}]$	[x]	[y]
$\|CL_\alpha\|$	1	2	1	2	2p
$\|C_{Q_{2p}}(CL_\alpha)\|$	44	22	44	22	2
Φ_1	44	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	22	0	22	0	0
Φ_4	2	2	2	2	0
Φ_5	11	0	11	0	1

Table(3)

Theorem(2.2.8):[6]

The Artin's character table of the dihedral group D_n when n is an even number is given as follows:

Ar(D_n) =

	$[I]$	$[r^{\frac{n}{2}}]$	Γ – Classes of C_n		$[s]$	$[sr]$
$\lvert CL_\alpha \rvert$	1	1	2 2 …	2	$n/2$	$n/2$
$\lvert C_{D_n}(CL_\alpha) \rvert$	$2n$	$2n$	n n …	n	2^2	2^2
Φ_1	2.Ar(C_n)				0	0
⋮					⋮	⋮
Φ_l					0	0
Φ_{l+1}	n	0	…	0	0	2
Φ_{l+2}	n	0	…	0	2	0

Table (4)

3. The main results:

Propostion(3.1)

If $m = p_1^{\alpha_1} . p_2^{\alpha_2} \ldots . p_n^{\alpha_n}$ where g.c.d(p_i,p_j) =1, if i ≠j and for all i, $p_i \neq 2$ are prime numbers and α_i any positive integers, then The Artin's character table of the group ($Q_{2m}\times D_4$) when m is an odd number is given as:

Ar($Q_{2m}\times D_4$) =

	Γ-classes of $Q_{2m}\times\{I\}$	Γ-classes of $Q_{2m}\times\{r^2\}$	Γ-classes of $Q_{2m}\times\{r\}$	Γ-classes of $Q_{2m}\times\{s\}$	Γ-classes of $Q_{2m}\times\{sr\}$
Γ-classes	$[1,I][x^2,I][x^m,I][x,I][y,I]$	$[1,r^2][x^2,r^2][x^m,r^2][x,r^2][y,r^2]$	$[1,r][x^2,r][x^m,r][x,r][y,r]$	$[1,s][x^2,s][x^m,s][x,s][y,s]$	$[1,sr][x^2,sr][x^m,sr][x,sr][y,sr]$
$\lvert CL_\alpha \rvert$	1 2 1 2 2m	1 2 1 2 2m	1 2 1 2 2m	2 4 2 4 4m	2 4 2 4 4m
$\lvert C_{Q_{2m}\times D_4}(CL_\alpha) \rvert$	32m 16m 32m 16m 16	32m 16m 32m 16m 16	32m 16m 32m 16m 16	16m 8m 16m 8m 8	16m 8m 16m 8m 8
$\Phi_{(1,1)}$ $\Phi_{(2,1)}$ $\Phi_{(3,1)}$ $\Phi_{(4,1)}$ $\Phi_{(5,1)}$	8Ar(Q_{2m})	0	0	0	0
$\Phi_{(1,2)}$ $\Phi_{(2,2)}$ $\Phi_{(3,2)}$ $\Phi_{(4,2)}$ $\Phi_{(5,2)}$	4Ar(Q_{2m})	4Ar(Q_{2m})	0	0	0
$\Phi_{(1,3)}$ $\Phi_{(2,3)}$ $\Phi_{(3,3)}$ $\Phi_{(4,3)}$ $\Phi_{(5,3)}$	2Ar(Q_{2m})	2Ar(Q_{2m})	2Ar(Q_{2m})	0	0
$\Phi_{(1,4)}$ $\Phi_{(2,4)}$ $\Phi_{(3,4)}$ $\Phi_{(4,4)}$ $\Phi_{(5,4)}$	4Ar(Q_{2m})	0	0	0	2Ar(Q_{2m})
$\Phi_{(1,5)}$ $\Phi_{(2,5)}$ $\Phi_{(3,5)}$ $\Phi_{(4,5)}$ $\Phi_{(5,5)}$	4Ar(Q_{2m})	0	0	2Ar(Q_{2m})	0

Table(5)

which is (25 × 25) square matrix .

Proof: Let g∈ $(Q_{2m}\times D_4)$; g=(q,d),q∈Q_{2m},d∈D_4

Case (I):

Consider the group G=$(Q_{2m}\times D_4)$ and if H is a cyclic subgroup of $(Q_{2m}\times\{I\})$,then 1-H =<(x,I)>2- H= <(y,I)> and φ the principle character of H, φ_j Artin's characters of Q_{2p}, $1\le j\le l+1$, then by using theorem (3.1):

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- H = <(x,I)>

(i) If g =(1,I) then

$\Phi_{(j,1)}(1,I)=\frac{|C_G(g)|}{|C_H(g)|}.\varphi(g)=\frac{32m}{|C_H(g)|}.1=\frac{8.4m}{|C_H(g)|}.1$

$=\frac{8|C_{Q_{2m}}(1)|}{|C_{<x>}(1)|}.\varphi(1)=8.\Phi_j(1)$ (since H∩CL(1,I)={(1,I)})

(ii) If g =(x^m,I) , g∈H then

$\Phi_{(j,1)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(g)|}.1$

$=\frac{8|C_{Q_{2m}}(x^m)|}{|C_{<x>}(x^m)|}\varphi(x^m)=8.\Phi_j(x^m)$ (since H∩ $CL(g)=\{g\},\varphi(g)=1$)

(iii) If g≠(x^m,I) , g∈H then

$\Phi_{(j,1)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16m}{|C_H(g)|}(1+1)=\frac{4.4m}{|C_H(g)|}.2$

$=\frac{4|C_{Q2m(q)}|}{|C_{H(q)}|}.2=8.\ \Phi_j(q)$ (Since H∩ CL(g) =$\{g,g^{-1}\}$and $\varphi(g)=\varphi(g^{-1})=1$ and since g =(q,I),q∈Q_{2m} , q≠x^m)

If g∉H then

$\Phi_{(j,1)}(g)=0=8.0=8.\Phi_j(q)$ (since H∩$CL(g)=\phi$)

2-If H=<(y,I)>={(1,I),(y,I),(y^2,I),(y^3,I)} then

(i) If g=(1,I) then

$\Phi_{(l+1,1)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{4}.1=8.p=8.\Phi_{l+1}(1)$ (since H∩ CL(1,I)={(1,I)})

(ii)If g=$(x^m,I)=(y^2,I)$ and g∈ H then

$\Phi_{(5,1)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{4}.1=8.m$ (since m=$\frac{|C_{Q_{2m}}(g)|}{|C_H(g)|}$)

$=8.\Phi_5(x^m)$ (since H∩$CL(g)=\{g\},\varphi(g)=1$)

(iii) If g≠(x^m,I) and g∈H ,i.e.{g=(y,I) or g=(y^3,I)} then

$\Phi_{(5,1)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16}{4}(1+1)$

= 4.2 = 8 since H∩$CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise

$\Phi_{(5,1)}(g)=0$ (since H∩CL(g)=ϕ)

Case (II):

If H is a cyclic subgroup of $(Q_{2m}\times\{r^2\})$ then:

1-H = $<(x,r^2)>$ 2- H = $<(y,r^2)>$

and φ the principle character of H, then by using theorem (3.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1-H=$<(x,r^2)>$={(1,I),$(1,r^2)$,(x,r^2),…,(x^m,r^2)}

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If g=(1,I) then

$$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(1,I)|}.1=\frac{8.4m}{|C_H(1,I)|}.1$$

$$=\frac{8|C_{Q2m}(1)|}{2|C_{<x>}(1)|}\varphi(1)=4.\Phi_j(1) \quad \text{since } H\cap CL(g)=\{(1,I),(1,r^2)\}$$

If g=(1,r^2) then

$$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(1,I)|}.1=\frac{8.4m}{|C_H(1,I)|}.1$$

$$=\frac{8|C_{Q2m}(1)|}{2|C_{<x>}(1)|}\varphi(1)=4.\Phi_j(1) \quad \text{since } H\cap CL(g)=\{(1,I),(1,r^2)\}$$

(i)If g =(x^m,I); g∈ H then

$$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(g)|}.1=\frac{8.4m}{|C_H(g)|}.1$$

$$=\frac{8|C_{Q2m}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=4\Phi_j(x^m)(\text{since } H\cap CL(g)=\{g\}\text{and } \varphi(g)=1)$$

If g =(x^m,r^2); g∈ H then

$$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(g)|}.1=\frac{8.4m}{|C_H(g)|}.1 = \frac{8|C_{Q2m}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=4\Phi_j(x^m)(\text{since } H\cap CL(g)=\{g\}\text{and } \varphi(g)=1)$$

(iii) If g≠ (x^m,I),(x^m,r^2) and g∈ H then

$$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16m}{|C_H(g)|}(1+1)$$

$$=\frac{4|C_{Q2m}(q)|}{2|C_{<x>}(q)|}.2 = 4\Phi_j(q) \ (\text{since} H\cap CL(g)=\{g,g^{-1}\}\text{and } \varphi(g)=\varphi(g^{-1})=1)$$

(iv) If g ∉ H then

$$\Phi_{(j,2)}(g) =0 = \Phi_j(q) \qquad (\text{ since } H\cap CL(g)=\phi)$$

2-If H =<(y,r^2)>={(1,I),(y,I),(y^2,I),(y^3,I),(1,r^2), (y,r^2),(y^2,r^2),(y^3,r^2)}

(i)If g =(1,I) then

$$\Phi_{(5,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1= 4p = 4\Phi_5(g)$$

If g=(1,r^2) then

$$\Phi_{(5,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1= 4m = 4\Phi_5(g)$$

If g=(y^2,I^*)=(x^m,I^*) and g∈ H then

$$\Phi_{(l+1,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1$$

$$=4p=4\Phi_5(g) \ (\text{since } H\cap CL(g)=\{g\} \text{ and } \varphi(g)=1)$$

If g =(y^2,r^2)=(x^m,r^2) and g∈ H then

$$\Phi_{(5,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1= 4m = 4\Phi_5(g)$$

If g≠(x^m,I) and g∈ H i.e. g={(y,I),(y,r^2)}or g={ (y,r^2),(y^3,r^2)} then

If g = (y,I) then

$$\Phi_{(5,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{8}.(1+1) = 4 \quad (\text{since } H\cap CL(g)=\{g,g^{-1}\} \text{ and } \varphi(g)=\varphi(g^{-1})=1)$$

If g = (y,r^2) then

$\Phi_{(5,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16}{8}.(1+1)=4$ (since H∩CL(g)={g,g^{-1}} and φ (g)= φ (g^{-1})=1)

(since H∩CL(g)={ (y,r^2),(y^3,r^2) }and φ (g)= φ (g^{-1})=1)

otherwise $\Phi_{(5,2)}(g)=0$ since H∩CL(g)= ϕ

Case (III):

If H is a cyclic subgroup of ($Q_{2m}\times\{r\}$) then:

1-H = <(x,r)> 2- H = <(y,r)>

and φ the principle character of H, then by using theorem (3.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1-H=<(x,r)>={(1,I),(1,r),(1,r^2),(x,r),…,(x^{2p-1},r),(x,r^2),…,(x^{2p-1},r^2)}

If g=(1,I) then

$\Phi_{(j,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(1,I)|}.1=\frac{8.4m}{|C_H(1,I)|}.1$

$=\frac{8|C_{Q_{2m}}(1)|}{4|C_{<x>}(1)|}\varphi(1)=2.\Phi_j(1)$ since H∩CL(g)={(1,I),(1,r),(1,r^2)}

If g=(1,r) then

$\Phi_{(j,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16.m}{|C_H(g)|}(1+1)=\frac{4.4m}{|C_H(g)|}.2$

$=\frac{4|C_{Q_{2m}}(1)|}{4|C_{<x>}(1)|}.2=2.\Phi_j(q)$ sinceH∩CL(g)={((1,r),(1,r^2)}

(i)If g =(x^m,I); g∈ H then

$\Phi_{(j,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32p}{|C_H(g)|}.1=\frac{8.4p}{|C_H(g)|}.1$

$=\frac{8|C_{Q2m}(x^m)|}{4|C_{<x>}(x^p)|}\varphi(1)=2\Phi_j(x^m)$ (since H∩CL(g)={g}and φ(g)=1)

If g =(x^m,r); g∈ H then

$\Phi_{(j,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16m}{|C_H(g)|}(1+1)$(sinceH∩$CL$(g)={g,g^{-1}}and φ(g)=φ(g^{-1})=1)

$=\frac{4|C_{Q2m}(q)|}{4|C_{<x>}(q)|}.2=2\Phi_j(q)$

(iii) If g≠ (x^m,I),(x^m,r) and g∈ H then

$\Phi_{(j,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)=\frac{8m}{4|C_H(g)|}.(1+1+1+1)$

$=\frac{32m}{4|C_H(g)|}=\frac{8.4m}{4|C_H(g)|}$ (sinceH∩CL(g)={(x,r),(x^{2m-1},r^2),(x, r^2),(x^{2m-1},r) }and φ(g)= φ(g^{-1})=1)

$=\frac{8|C_{Q2m}(q)|}{4|C_{<x>}(q)|}=2\Phi_j(q)$ (Since g=(q,r),q∈Q_{2p},q≠x^p)

(iv) If g ∉ H then

$\Phi_{(j,3)}(g)=0=\Phi_j(q)$ (since H∩CL(g)=ϕ)

2-If H =<(y,r)>={(1,I),(y,I),(y^2,I),(y^3,I),(1,r),(y,r),(y^2,r),(y^3,r),(1,r^2),
(y,r^2),(y^2,r^2),(y^3,r^2)}

(i)If g =(1,I) then

$\Phi_{(l+1,2)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{16}.1=2m=2\Phi_5(g)$

If g=(1,r) then $\Phi_{(5,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16p}{16}.(1+1)=2m=2\Phi_5(g)$

If $g=(y^2,I)=(x^m,I)$ and $g\in H$ then

$\Phi_{(5,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{16}.1$

$=2.p=2\Phi_5(g)$ (since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$)

If $g=(y^2,r)=(x^m,r)$ or $g=(y^2,r^2)=(x^m,r^2)$ and $g\in H$ then

$\Phi_{(5,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16m}{16}.(1+1)=2m=2\Phi_5(g)$

(ii) If $g\neq(x^m,I)$ and $g\in H$ i.e. $g=\{(y,I),(y,r)\}$ or $g=\{(y^3,I),(y^3,r),(y,r^2),(y^3,r^2)\}$ then

If g = (y,I) then

$\Phi_{(5,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16}{16}.(1+1)=2$ (since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

If g = (y,r) then

$\Phi_{(5,3)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{p}\varphi(h_i)=\frac{8}{16}.(1+1+1+1)$

$=\frac{32}{16}=2$

(since $H\cap CL(g)=\{(y,r),(y^3,r^2),(y,r^2),(y^3,r)\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

otherwise $\Phi_{(5,3)}(g)=0$ since $H\cap CL(g)=\phi$

Case (IV):

If H is a cyclic subgroup of $(Q_{2m}\times\{s\})$ then

1- H= <(x,s)> , 2- H=<(y,s)>

and φ the principle character of H, then by using theorem (3.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- H = <(x,s)>

(i) If g =(1,I) then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(1,I)|}.1=\frac{8.4m}{|C_H(1,I)|}.1=\frac{8|C_{Q2m}(1)|}{2|C_{<x>}(1)|}.1=4\Phi_j(1)$ (since $H\cap CL(g)=\{(1,I)\}$)

If g = (1,sr) then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{16m}{|C_H(1,s)|}.1=\frac{4.4m}{|C_H(1,s)|}.1$

$=\frac{4|C_{Q2m}(1)|}{2|C_{<x>}(1)|}.1=2\Phi_j(1)$ since $H\cap CL(g)=\{(1,s)\}$

(ii) $g=(x^p,I)$; $g\in H$

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(g)|}.1=\frac{8.4m}{|C_H(g)|}.1=\frac{8|C_{Q_{2p}}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=4\Phi_j(x^m)$

(iii) If $g=(x^m,sr)$ then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\ \varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2p}}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=2\Phi_j(x^m)$

(iv)If $g\neq$ $(x^m,I),(x^m,sr)$ and $g\in H$

If $g\neq$ (x^m,I) and $g\epsilon$ $(Q_{2p}\times\{I\})$ then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16m}{|C_H(g)|}(1+1)$ (since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

$=\frac{4.4m}{|C_H(g)|}.2=\frac{4|C_{Q_{2m}}(q)|}{2|C_{<x>}(q)|}2=4\Phi_j(q)$ Since $g=(q,I),q\in Q_{2p},q\neq x^m$ If $g\neq$ (x^p,sr) and $g\epsilon$ $(Q_{2m}\times\{sr\})$ then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}.\varphi(g)=\frac{8p}{|C_H(g)|}.(1+1)=\frac{2.4m}{|C_H(g)|}.2=\frac{2|C_{Q_{2m}}(q)|}{2|C_{<x>}(q)|}.2=2\Phi_j(q)$ Since $g=(q,s),q\in Q_{2m},q\neq x^m$ (since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

(v) If $g\notin H$ then

$\Phi_{(j,4)}(g)=0=\Phi_j(q)$ since $H\cap CL(g)=\phi$

2-If $H=<(y,s)>=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,s),(y,s),(y^2,s),(y^3,s)\}$

(i) If $g=(1,I)$ then

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1=4.m=4\Phi_5(g)$

If $g=(1,s)$ then

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}.\varphi(g)=\frac{16m}{8}.1=4p=\Phi_5(g)$

(ii)If $g=(y^2,I)=(x^m,I)$ and $g\in H$

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32p}{8}.1=4.m=4\Phi_5(g)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(y^2,s)$ and $g\in H$

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{16p}{8}.1=4p=\Phi_5(g)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

(iii) If $g\neq$ (x^p,I) and $g\in H$ i.e. $g=\{(y,I),(y,s)\}$ or $g=\{(y^3,I),(y^3,s)\}$

If $g=(y,I)$ then

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16}{8}(1+1)=4$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

(iv)If $g=(y,s)$ then

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{8}{|C_H(g)|}.(1+1)=\frac{8}{8}.2=2$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(5,4)}(g)=0$ since $H\cap CL(g)=\phi$

Case (V):

If H is a cyclic subgroup of $(Q_{2m}\times\{s\})$ then

1- $H=<(x,sr)>$, 2- $H=<(y,sr)>$

and φ the principle character of H, then by using theorem (3.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- $H=<(x,sr)>$

(i) If g =(1,I) then

$\Phi_{(j,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(1,I)|}.1=\frac{8.4m}{|C_H(1,I)|}.1=$

$\frac{8|C_{Q2m}(1)|}{2|C_{<x>}(1)|}.1=4\Phi_j(1)$ (since $H\cap CL(g)=\{(1,I)\}$)

If g = (1,s) then

$\Phi_{(j,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{16m}{|C_H(1,s)|}.1=\frac{4.4m}{|C_H(1,s)|}.1$

$=\frac{4|C_{Q2m}(1)|}{2|C_{<x>}(1)|}.1=2\Phi_j(1)$ since $H\cap CL(g)=\{(1,s)\}$

(ii) g= (x^m,I) ; g∈ H

$\Phi_{(j,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{|C_H(g)|}.1=\frac{8.4m}{|C_H(g)|}.1=\frac{8|C_{Q2p}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=4\Phi_j(x^m)$

(iii) If g= (x^m,s) then

$\Phi_{(j,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q2m}(x^m)|}{2|C_{<x>}(x^m)|}\varphi(1)=2\Phi_j(x^m)$

(iv)If g≠ $(x^m,I),(x^m,s)$ and g∈ H

If g≠ (x^m,I) and gϵ $(Q_{2m}\times\{I\})$ then

$\Phi_{(j,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{16p}{|C_H(g)|}(1+1)$ (since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

$=\frac{4.4m}{|C_H(g)|}.2=\frac{4|C_{Q2m}(q)|}{2|C_{<x>}(q)|}2=4\Phi_j(q)$Since $g=(q,I),q\in Q_{2p},q\neq x^m$ If g≠ (x^p,sr) and gϵ $(Q_{2p}\times\{sr\})$ then

$\Phi_{(j,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}.\varphi(g)=\frac{8m}{|C_H(g)|}.(1+1)=\frac{2.4m}{|C_H(g)|}.2=\frac{2|C_{Q2p}(q)|}{2|C_{<x>}(q)|}.2=2\Phi_j(q)$ Since $g=(q,s),q\in Q_{2m},q\neq x^m$
(since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$)

(v) If g ∉ H then

$\Phi_{(j,5)}(g)=0=\Phi_j(q)$ since $H\cap CL(g)=\phi$

2-If $H=<(y,s)>=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,s),(y,s),(y^2,s),(y^3,s)\}$

(i) If g =(1,I) then

$\Phi_{(5,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1=4.p=4\Phi_5(g)$

If g =(1,s) then

$\Phi_{(5,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}.\varphi(g)=\frac{16m}{8}.1=4p=\Phi_5(g)$

(ii)If g =(y^2,I) =(x^m,I) and g∈ H

$\Phi_{(5,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{32m}{8}.1=4.m=4\Phi_5(g)$ since $H\cap CL(g)=\{g\}$ and φ (g)=1

If g=(y^2,s) and g∈ H

$\Phi_{(5,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}\varphi(g)=\frac{16m}{8}.1=4p=\Phi_5(g)$ since $H\cap CL(g)=\{g\}$ and φ (g)=1

(iii) If g≠ (x^m,I) and g∈ H i.e. g={(y,I),(y,s)} or g={$(y^3,I),(y^3,s)$}

If g=(y,I) then

$\Phi_{(5,4)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{16}{8}(1+1)=4$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

(iv)If g =(y,s) then

$\Phi_{(5,5)}(g)=\frac{|C_G(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{8}{|C_H(g)|}.(1+1)=\frac{8}{8}.2=2$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(5,5)}(g)=0$ since $H\cap CL(g)=\phi$

Example(3.2):

To find Artin's characters table of the group $Q_{150}\times D_4$, we must find Artin's characters table of the group Q_{150} from (2.2.8) we get: $Ar(Q_{150}) = Ar(Q_{2.3.5^2})$

Γ-classes	[1]	[x^{50}]	[x^{30}]	[x^{10}]	[x^6]	[x^2]	[x^{75}]	[x^{25}]	[x^{15}]	[x^5]	[x^3]	[x]	[y]
$\lvert CL_\alpha\rvert$	1	2	2	2	2	2	1	2	2	2	2	2	150
$\lvert C_{Q_{150}}(CL_\alpha)\rvert$	300	150	150	150	150	150	300	150	150	150	150	150	2
Φ_1	300	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	100	100	0	0	0	0	0	0	0	0	0	0	0
Φ_3	60	0	60	0	0	0	0	0	0	0	0	0	0
Φ_4	20	20	20	20	0	0	0	0	0	0	0	0	0
Φ_5	12	0	12	0	12	0	0	0	0	0	0	0	0
Φ_6	4	4	4	4	4	4	0	0	0	0	0	0	0
Φ_7	150	0	0	0	0	0	150	0	0	0	0	0	0
Φ_8	50	50	0	0	0	0	50	50	0	0	0	0	0
Φ_9	30	0	30	0	0	0	30	0	30	0	0	0	0
Φ_{10}	10	10	10	10	0	0	10	10	10	10	0	0	0
Φ_{11}	6	0	6	0	6	0	6	0	6	0	6	0	0
Φ_{12}	2	2	2	2	2	2	2	2	2	2	2	2	0
Φ_{13}	75	0	0	0	0	0	75	0	0	0	0	0	1

Table(6)

and we must find Artin's characters table of the group D_4 from (2.2.9) we get:

$Ar(D_4) =$

Γ-classes	[I]	[r^2]	[r]	[s]	[sr]
$\lvert CL_\alpha\rvert$	1	1	2	2	2
$\lvert C_{D_3}(CL_\alpha)\rvert$	8	8	4	4	4
Φ_1	8	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	2	2	2	0	0
Φ_4	4	0	0	0	2
Φ_5	4	0	0	2	0

Table(7)

Then from theorem (3.1) we get

$Ar(Q_{150}\times D_4) =$

Γ-classes	$[1,I^*]$	$[x^{50},I^*]$	$[x^{30},I^*]$	$[x^{10},I^*]$	$[x^6,I^*]$	$[x^2,I^*]$	$[x^{75},I^*]$	$[x^{25},I^*]$	$[x^{15},I^*]$	$[x^5,I^*]$	$[x^3,I^*]$	$[x,I^*]$	$[y,I^*]$	$[1,r^2]$	$[x^{50},r^2]$	$[x^{30},r^2]$	$[x^{10},r^2]$	$[x^6,r^2]$	$[x^2,r^2]$	$[x^{75},r^2]$	$[x^{25},r^2]$
$\|CL_\alpha\|$	1	2	2	2	2	2	1	2	2	2	2	2	150	2	2	2	2	2	2	1	2
$\|C_{Q_{150}\times D_3}(CL_\alpha)\|$	2400	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	16	1200	1200	1200	1200	1200	1200	2400	1200
$\Phi_{(1,1)}$	2400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	1200	1200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	600	600	600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	1200	0	0	0	600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	1200	0	0	600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	800	0	0	0	0	800	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	400	400	0	0	0	400	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	200	200	200	0	0	200	200	200	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	400	0	0	0	200	400	0	0	0	200	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,1)}$	400	0	0	200	0	400	0	0	200	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(11,1)}$	480	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	480
$\Phi_{(12,1)}$	240	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	024
$\Phi_{(13,1)}$	120	120	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	120

Table(8)

$[x^{15},r^2]$	$[x^5,r^2]$	$[x^3,r^2]$	$[x,r^2]$	$[y,r^2]$	$[1,r]$	$[x^{50},r]$	$[x^{30},r]$	$[x^{10},r]$	$[x^6,r]$	$[x^2,r]$	$[x^{75},r]$	$[x^{25},r]$	$[x^{15},r]$	$[x^5,r]$	$[x^3,r]$	$[x,r]$	$[y,r]$
2	2	2	2	150	3	3	3	3	3	3	3	3	3	3	3	3	150
1200	1200	1200	1200	16	800	800	800	800	800	800	800	800	800	800	800	800	16
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	120	0	0	0	0	0	0	0	0	48	0	0	0	0	0
160	0	0	0	0	160	0	0	0	0	0	0	24	24	0	0	0	0
80	80	0	0	0	80	0	0	0	0	0	0	12	12	12	0	0	0
80	0	0	40	0	0	0	0	0	0	0	0	24	0	0	0	12	0
96	0	0	0	0	0	0	0	0	0	0	0	24	0	0	12	0	0
48	48	0	0	0	0	0	0	0	0	0	0	16	0	0	0	0	16
24	24	24	0	0	0	0	0	0	0	0	0	8	0	0	0	0	8
48	0	0	0	24	0	0	0	0	0	0	0	4	0	0	0	0	4
32	0	0	0	0	32	0	0	0	0	0	0	8	0	0	0	0	0
16	16	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	0
8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

[1,s]	[x^{50},s]	[x^{30},s]	[x^{10},s]	[x^{6},s]	[x^{2},s]	[x^{75},s]	[x^{25},s]	[x^{15},s]	[x^{5},s]	[x^{3},s]	[x,s]	[y,s]	[1,s]	[x^{50},s]	[x^{30},s]	[x^{10},s]	[x^{6},s]	[x^{2},s]	[x^{75},s]	[x^{25},s]	[x^{15},s]	[x^{5},s]	[x^{3},s]	[x,s]	[y,s]
3	3	3	3	3	3	3	3	3	3	3	3	150	3	3	3	3	3	3	3	3	3	3	3	3	150
800	800	800	800	800	800	800	800	800	800	800	800	16	800	800	800	800	800	800	800	800	800	800	800	800	16
240	0	0	0	0	0	0	0	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	120	0	0	0	0	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	0	120	0	0	0	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	120	0	0	0	0	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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80	80	0	0	0	80	0	0	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	40	0	0	0	0	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	48	0	0	0	0	0	0	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	24	24	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	24	0	0	0	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	32	0	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	16	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\Phi_{(1,1)}$	240	0	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	240	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	160	0	0	0	0	160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	80	80	0	0	80	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,2)}$	40	40	40	0	0	40	40	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,2)}$	80	0	0	0	40	80	0	0	0	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,2)}$	80	0	0	40	0	80	0	0	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,2)}$	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,2)}$	48	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,2)}$	24	24	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(11,2)}$	48	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(12,2)}$	48	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(13,2)}$	32	0	0	0	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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200	0	0	0	100	200	0	0	0	100	0	0	0
1200	0	0	0	0	0	0	0	0	0	0	0	0
600	600	0	0	0	0	0	0	0	0	0	0	0
300	300	300	0	0	0	0	0	0	0	0	0	0
600	0	0	0	300	0	0	0	0	0	0	0	0

$\Phi_{(1,3)}$	16	16	0	0	0	16	16	0	0	0	0	0	0
$\Phi_{(2,3)}$	8	8	8	0	0	8	8	8	0	0	0	0	0
$\Phi_{(3,3)}$	16	0	0	8	16	0	0	8	0	0	0	0	0
$\Phi_{(4,3)}$	16	0	0	8	0	16	0	0	16	0	0	0	0
$\Phi_{(5,3)}$	1200	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,3)}$	600	600	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,3)}$	300	300	300	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,3)}$	600	0	0	0	300	0	0	0	0	0	0	0	0
$\Phi_{(9,3)}$	600	0	0	300	0	0	0	0	0	0	0	0	0
$\Phi_{(10,3)}$	400	0	0	0	0	400	0	0	0	0	0	0	0
$\Phi_{(11,3)}$	200	200	0	0	0	200	200	0	0	0	0	0	0
$\Phi_{(12,3)}$	100	100	100	0	0	100	100	100	0	0	0	0	0
$\Phi_{(13,3)}$	200	0	0	0	100	200	0	0	0	100	0	0	0

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0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

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$\Phi_{(1,4)}$	200	0	0	100	0	200	0	0	100	0	0	0	0
$\Phi_{(2,4)}$	240	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,4)}$	120	120	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,4)}$	60	60	60	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,4)}$	120	0	0	0	60	0	0	0	0	0	0	0	0
$\Phi_{(6,4)}$	120	0	0	60	0	0	0	0	0	0	0	0	0
$\Phi_{(7,4)}$	80	0	0	0	0	80	0	0	0	0	0	0	0
$\Phi_{(8,4)}$	40	40	0	0	0	40	40	0	0	0	0	0	0
$\Phi_{(9,4)}$	20	20	20	0	0	20	20	20	0	0	0	0	0
$\Phi_{(10,4)}$	40	0	0	0	20	40	0	0	0	0	0	0	0
$\Phi_{(11,4)}$	40	0	0	20	0	40	0	0	20	0	0	0	0
$\Phi_{(12,4)}$	48	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(13,4)}$	24	24	0	0	0	0	0	0	0	0	0	0	

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

$\Phi_{(1,5)}$	12	12	12	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,5)}$	24	0	0	0	12	0	0	0	0	0	0	0	0
$\Phi_{(3,5)}$	24	0	0	12	0	0	0	0	0	0	0	0	0
$\Phi_{(4,5)}$	16	0	0	0	0	16	0	0	0	0	0	0	0
$\Phi_{(5,5)}$	8	8	0	0	0	8	8	0	0	0	0	0	0
$\Phi_{(6,5)}$	4	4	4	0	0	4	4	4	0	0	0	0	0
$\Phi_{(7,5)}$	8	0	0	0	4	8	0	0	4	0	0	0	0
$\Phi_{(8,5)}$	8	0	0	4	0	8	0	0	0	0	0	0	0
$\Phi_{(9,5)}$	600	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,5)}$	300	300	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(11,5)}$	150	150	150	0	0	0	0	0	0	0	0	0	0
$\Phi_{(12,5)}$	300	0	0	0	150	0	0	0	0	0	0	0	0
$\Phi_{(13,5)}$	300	0	0	150	0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

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