International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

On Artin's characters table of the group $(Q_{2m} \times C_2)$ when $m=2^h$, $h \in Z^+$

Rajaa Hasan Abbas
University of Kufa, College of Education for Girls, Department of Mathematics
rajaah.alabidy@uokufa.edu.iq

Abstract—*The main objective of this paper is to find the general form of Artin's characters table of the group* $(Q_{2m} \times C_2)$ *When* $m=2^h$ *,* $h \in Z^+$ *where* Q_{2m} *is the Quaternion group of order 4m and* C_2 *is the Cyclic group of order 2 this table depends on Artin's characters table of a quaternion group of order 4m when* $m=2^h$ *,* $h\in Z^+$*. which is denoted by* $Ar(Q_{2^{h+1}} \times C_2)$*.*

Keywords— group; Cyclic group; quaternion group; Artin; Artin's characters.

INTRODUCTION

The square matrix whose rows correspond to Artin's characters and columns correspond to the Γ- classes of G is called Artin's characters table . this matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G).In 1967 T.Y. lam [9] studied A(G) extensively for many groups.In 1970 K.Yamauchi[6] studied 2- part A(G).In 1976 G.David [3] studied A(G) of arbitrary characters of the cyclic subgroups. In 1996 K.K Nwabuez [5] studied A(G) of p-groups. In 2009 S.J.Mahmood [8] studied the general from of Artin's characters table $Ar(Q_{2m})$ when m is an even number. The aim of this paper is to find the general from of the Artin's characters table of the group $(Q_{2m} \times C_2)$ when $m=2^h$,$h \in Z^+$.

1.Preliminaries

This section introduce some important definitions and basic concepts of the Artin's characters tables, the Artin's characters table of C_{p^s} , the Artin's characters table of the Quaternion group Q_{2m} when m is an even number, the Artin's characters table of the Quaternion group Q_{2m} when $m=2^h$,$h\in Z^+$ and the Group $(Q_{2m}\times C_2)$.

1.1 Definition: [7]

Two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G , this defines an equivalence relation on G. It is classes are called Γ- classes.

1.2 Example:

Consider a cyclic group $C_4=\langle x \rangle$ such that:

1 is Γ- conjugate 1

Then the Γ- class [1] = {1}

$\langle x \rangle = \langle x^3 \rangle$

Then x and x^3 are Γ- conjugate , and $[x] = \{ x , x^3\}$

There is another Γ- class $[x^2] = \{x^2\}$

So that there are three Γ- classes of C_4 :[1] , [x] and $[x^2]$

In general for C_{p^s} *where p is any prime number, so that are s+1 distinct*

Γ- classes Which are $[1], [x], [x^p], \dots, [x^{p^{s-1}}]$.

1.3 Definition: [5]

Let H be a subgroup of G and let ϕ be a class function on H, ***the induced class function on G***, is given by :

$\phi'(g) = \frac{1}{|H|}\sum_{x\in G}\phi^{\circ}(xgx^{-1})$

where ϕ° is defined by:

$$\phi^{\circ}(h) = \begin{cases}\phi(h) & if \quad h\in H \\ 0 & if \quad h\notin H\end{cases}$$

1.4 Proposition: [3]

Let H be a subgroup of G and ϕ be a character of H, then ϕ' is a character of G and it is called ***induced character*** on G

1.5 Example:

Take H=C_4 as acyclic subgroup of Q_4 the character φ on C_4 is defined as follows : $\phi(1) = 1$, $\phi(x) = \omega$, $\phi(x^2) = \omega^2$, $\phi(x^3) = \omega^3$

Where $\omega = e^{2\pi i/4}$

$\phi'(1) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.1.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(1)$

$= \frac{1}{4}(1+1+1+1+1+1+1+1) = \frac{1}{4}.8 = 2$

$\phi'(x) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x)$

$= \frac{1}{|H|}[\phi(x)+\phi(x)+\phi(x)+\phi(x)+\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x^3)] = (1/4).4(\phi(x)+\phi(x^3)) = \omega+\omega^3$

$\phi'(x^2) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x^2.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x^2)$

$= \frac{1}{|H|}[\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)] = (1/4).8\phi(x^2) = 2\omega^2$

$\phi'(x^3) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x^3.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x^3)$

$= \frac{1}{|H|}[\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x)+\phi(x)+\phi(x)+\phi(x)] = (1/4).4(\phi(x^3)+\phi(x)) = \omega^3+\omega$

Since $y, xy, x^2y, x^3y \notin C_4$ then $\phi'(y) = \phi'(xy) = \phi'(x^2y) = \phi'(x^3y) = 0$

Hence ϕ' is induced characters of Q_4.

1.6 Theorem:[4]

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen representative for m-conjugate classes, then :

1- $\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$

1.7 Example:

To find the Artin's character of C_4 ,there are three cyclic subgroups of C_4 , which are {1}, < x > and < x^2 >, there are three Γ-classes which are [1]={1},[x^2]={x^2} and [x]={x,x^3}

So we have three distinct Artin's characters, then by using theorem (1.6)

$$\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) \quad \text{if } h_i \in H \cap CL(g)$$

$$\varphi'(g) = 0 \quad \text{if } H \cap CL(g) = \phi .$$

(i) if H = {1} and G = C_4

since $H \cap CL(1) = \{1\}$, then

$$\varphi_1'(1) = \frac{2^2}{1} . \varphi(1) = 2^2.1 = 2^2$$

since $H \cap CL(x) = \phi$, then $\varphi_1'(x) = 0$

since $H \cap CL(x^2) = \phi$, then $\varphi_1'(x^2) = 0$

(ii) if H = < x^2 > ={1,x^2}

$\varphi_2'(1) = \frac{2^2}{2} . \varphi(1) = 2.1 = 2$, since $H \cap CL(1) = \{1\}$

$\varphi_2'(x^2) = \frac{2^2}{2} . \varphi(1) = 2.1 = 2$, since $H \cap CL(x^2) = \{x^2\}$

since $H \cap CL(x) = \phi$, then $\varphi_1'(x) = 0$

(iii) if H = < x > ={1,x,x^2,x^3}

$\varphi_3'(1) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(1) = \{1\}$

$\varphi_3'(x^2) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(x^2) = \{x^2\}$

$\varphi_3'(x) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(x) = \{x\}$

Then we get three Artin's characters φ_1', φ_2' and φ_3'.

1.8 Definition:[9]

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called ***Artin's characters of G.***

In theorem (1.6) , if φ is the principal character , then $\varphi(h_i)=\varphi(1)=1$, where $h_i \in H$

1.9 Proposition:[2]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.

Furthermore , Artin's characters are constant on each Γ-classes.

1.10 Definition: [1]

Artin's characters of finite group G can be displayed in table ***called Artin's characters table of G*** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest row contain the values of Artin's characters.

1.11Example:

In the Artin's character table of C_4 there are three Γ- classes, [1], $[x^2]$ and [x] then, from proposition (1.9) they obtain three distinct Artin's characters

And From example (1.7) we obtain the values of Artin's characters, then the table of it as follows:

$Ar(C_4)=$

Γ- classes	[1]	$[x^2]$	[x]
$\|CL_\alpha\|$	1	1	1
$\|C_{C_4}(CL_\alpha)\|$	2^2	2^2	2^2
φ_1'	2^2	0	0
φ_2'	2	2	0
φ_3'	1	1	1

Table (1)

1.12 Theorem:[1]

The general form of Artin's character table of C_{p^s} when p is a prime number and s is an integer number is given by:

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

Ar(C_{p^s})=

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^p]$	$[x]$
$\lvert CL_\alpha \rvert$	1	1	1	1	…	1	1
$\lvert C_{p^s}(CL_\alpha)\rvert$	P^s	p^s	P^s	p^s	…	p^s	P^s
φ'_1	p^s	0	0	0	…	0	0
φ'_2	P^{s-1}	P^{s-1}	0	0	…	0	0
φ'_3	P^{s-2}	P^{s-2}	P^{s-2}	0	…	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮
φ'_s	P	P	P	P	…	P	0
φ'_{s+1}	1	1	1	1	…	1	1

Table (2)

1.13 Example:

Consider the cyclic group C_{128},To find the Artin's character table we use theorem (1.12) as follows : The group $C_{128} = C_{2^7}$

then Ar(C_{2^7})=

Γ- classes	[1]	$[x^{2^6}]$	$[x^{2^5}]$	$[x^{2^4}]$	$[x^{2^3}]$	$[x^{2^2}]$	$[x^2]$	[x]
$\lvert CL_\alpha \rvert$	1	1	1	1	1	1	1	1
$\lvert C_{C_{2^7}}(CL_\alpha)\rvert$	2^7	2^7	2^7	2^7	2^7	2^7	2^7	2^7
φ'_1	2^7	0	0	0	0	0	0	0
φ'_2	2^6	2^6	0	0	0	0	0	0
φ'_3	2^5	2^5	2^5	0	0	0	0	0
φ'_4	2^4	2^4	2^4	2^4	0	0	0	0
φ'_5	2^3	2^3	2^3	2^3	2^3	0	0	0
φ'_6	2^2	2^2	2^2	2^2	2^2	2^2	0	0
φ'_7	2	2	2	2	2	2	2	0
φ'_8	1	1	1	1	1	1	1	1

Table (3)

1.14 Theorem: [8]

The Artin's characters table of the Quaternion group Q_{2m} when m is an even number is given as follows :

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

Ar (Q_{2m}) =

Γ- classes	Γ- classes of C_{2m}							
	[1]	$[x^m]$					[y]	[xy]
$\lvert CL_\alpha \rvert$	1	1	2	2	…	2	m	m
$\lvert C_{Q_{2m}}(CL_\alpha) \rvert$	4m	4m	2m	2m	…	2m	4	4
Φ_1	**2Ar(C_{2m})**						0	0
Φ_2							0	0
⋮							⋮	⋮
Φ_l							0	0
Φ_{l+1}	m	m	0	0	…	0	2	0
Φ_{l+2}	m	m	0	0	…	0	0	2

Table(4)

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group Q_{2m}.

Let $m=2^h$, $h \in Z^+$ then $Ar(Q_{2m})=Ar(Q_{2^{h+1}})$ and it is given by:

$Ar(Q_{2^{h+1}})$=

Γ- classes	Γ- classes of C_{2m}							
	[1]	$[x^{2^h}]$					$[y]$	$[xy]$
$\lvert CL_\alpha \rvert$	1	1	2	2	…	2	2^h	2^h
$\lvert C_{Q_{2^{h+1}}}(CL_\alpha) \rvert$	2^{h+2}	2^{h+2}	2^{h+1}	2^{h+1}	…	2^{h+1}	4	4
Φ_1	**2Ar($C_{2^{h+1}}$)**						0	0
Φ_2							0	0
⋮							⋮	⋮
Φ_l							0	0
Φ_{l+1}	2^h	2^h	0	0	…	0	2	0
Φ_{l+2}	2^h	2^h	0	0	…	0	0	2

Table (5)

1.15 Example:

To construct $Ar(Q_{128})$ by using theorem (1.14) we get the following table :

$Ar(Q_{128})=Ar(Q_{2^7})$=

Γ- classes	[1]	$[x^{2^6}]$	$[x^{2^5}]$	$[x^{2^4}]$	$[x^{2^3}]$	$[x^{2^2}]$	$[x^2]$	[x]	[y]	[xy]

$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	64	64
$\lvert C_{Q_{2^7}}(CL_\alpha) \rvert$	256	256	128	128	128	128	128	128	4	4
Φ_1	2^8	0	0	0	0	0	0	0	0	0
Φ_2	2^7	2^7	0	0	0	0	0	0	0	0
Φ_3	2^6	2^6	2^6	0	0	0	0	0	0	0
Φ_4	2^5	2^5	2^5	2^5	0	0	0	0	0	0
Φ_5	2^4	2^4	2^4	2^4	2^4	0	0	0	0	0
Φ_6	2^3	2^3	2^3	2^3	2^3	2^3	0	0	0	0
Φ_7	2^2	2^2	2^2	2^2	2^2	2^2	2^2	0	0	0
Φ_8	2	2	2	2	2	2	2	2	0	0
Φ_9	2^6	2^6	0	0	0	0	0	0	2	0
Φ_{10}	2^6	2^6	0	0	0	0	0	0	0	2

Table (6)

1.16The Group ($Q_{2m} \times C_2$) [10]

The direct product group ($Q_{2m} \times C_2$) where Q_{2m} is Quaternion group of order 4m with tow generators x and y is denoted by

$$Q_{2m} = \{x^k y^j : x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}, 0 \leq k \leq 2m-1, j=0,1\}$$

and C_2 is acyclic group of order 2 consisting of elements {I,z} .the generalized the group ($Q_{2m} \times C_2$) is denoted by

$$(Q_{2m} \times C_2) = \{(q,c) : q \in Q_{2m}, c \in C_2\} \text{ and } |Q_{2m} \times C_2| = |Q_{2m}|.|C_2| = 4m.2 = 8m$$

2. The main results

In this section is to find the general form of Artin's characters table of the group ($Q_{2m} \times C_2$) When $m=2^h$,$h \in Z^+$

2.1 Proposition:

The general from of the Artin's characters table of the group ($Q_{2^{h+1}} \times C_2$) when $m=2^h, h \in Z^+$ is give as follows:
$Ar(Q_{2^{h+1}} \times C_2)=$

Γ- classes of $(Q_2^{h+1})\times\{I\}$							Γ- classes of $(Q_2^{h+1})\times\{z\}$					
Γ-classes	**[1,I]**	$[x^{2^h},I]$	**…**	**[x,I]**	**[y,I]**	**[xy,I]**	**[1,z]**	$[x^{2^h},z]$	**…**	**[x,z]**	**[y,z]**	**[xy,z]**
$\lvert CL_\alpha\rvert$	**1**	**1**	**…**	**2**	**2^h**	**2^h**	**1**	**1**	**…**	**2**	**2^h**	**2^h**
$\lvert C_{Q_{2^{h+1}}\times C_2}(CL_\alpha)\rvert$	**2^{h+3}**	**2^{h+3}**	**…**	**2^{h+2}**	**8**	**8**	**2^{h+3}**	**2^{h+3}**	**...**	**2^{h+2}**	**8**	**8**
$\Phi_{(1,1)}$												
$\Phi_{(2,1)}$	**$2Ar(Q_2^{h+1})$**											
⋮							**0**					
$\Phi_{(l,1)}$												
$\Phi_{(l+1,1)}$												
$\Phi_{(l+2,1)}$												
$\Phi_{(1,2)}$												
$\Phi_{(2,2)}$	**$Ar(Q_2^{h+1})$**						**$Ar(Q_2^{h+1})$**					
⋮												
$\Phi_{(l,2)}$												
$\Phi_{(l+1,2)}$												
$\Phi_{(l+2,2)}$												

Table (7)

Proof :

Let $g \in (Q_2^{h+1}\times C_2)$;g=(q,I) or g=(q,z),q $\in Q_2^{h+1}$,I,z $\in$ C_2
Case (I):
If H is a cyclic subgroup of $(Q_2^{h+1}\times \{I\})$,then:
1-H=$\langle (x,I)\rangle$ 2- H=$\langle (y,I)\rangle$ 3- H=$\langle (xy,I)\rangle$
And φ the principal character of H, Φ_j Artin characters of Q_2^{h+1} $1\leq j\leq l+2$ then by using theorem (1.6)

1- $\Phi_j(g)=\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H\cap CL(g)$

2- $\Phi_j(g)=0$ if $H\cap CL(g)=\phi$

1- IF H=$\langle (x,I)\rangle$

(i) If g=(1,I)

$$\Phi_{(j,1)}((1,I))=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(I,1)|}.1=\frac{2.2^{h+2}}{|C_H(I,1)|}.1=\frac{2|C_{Q_{2^{h+1}}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1)=2.\Phi_j(1)$$

since H $\cap$CL(1,I)={(1,I)}

(ii) if g=($(x^{2^h},I),g\in H$

$$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(g)|}.1=\frac{2.2^{h+2}}{|C_H(g)|}.1=\frac{2|C_{Q_2^{h+1}}(x^{2^h})|}{|C_{\langle x\rangle}(x^{2^h})|}.\varphi(g)=2.\Phi_j(x^{2^h})$$

since H $\cap$CL(g)={g},φ(g)=1

(iii) if g$\neq(x^{2^h},I),g\in H$

$$\Phi_{(j,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{2^{h+2}}{\left|C_H(g)\right|}.(1+1)=$$

$$\frac{2.2^{h+1}}{\left|C_H(g)\right|}.(1+1)=\frac{2\left|C_{Q_{2^{h+1}}}(q)\right|}{\left|C_{\langle x\rangle}(q)\right|}.(\varphi(g)+\varphi(g^{-1}))=2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_2^{h+1} and q≠ x^{2^h}
(iv) if g∉H

$\Phi_{(j,1)}(g)=2.0=2.\Phi_j(q)$ Since H∩CL(g)= ϕ

2- IF H=$\langle(y,I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)=\frac{8.2^h}{4}.1=2.2^h=2.\Phi_{l+1}(1)$$

(ii) If g= (x^{2^h},I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)=\frac{8.2^h}{4}.1=2.2^h=2.\Phi_{l+1}(x^{2^h})$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g≠ (x^{2^h},I) and g∈ H,i.e.{g=(y,I) or g=(y^3,I)}

$$\Phi_{(l+1,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{4}.(1+1)=2.2=2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1
Otherwise

$\Phi_{(l+1,1)}(g)=0$ since H∩CL(g)=∅

3-IF H=$\langle(xy,I)\rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)=\frac{8.2^h}{4}.1=2.2^h=2.\Phi_{l+2}(1)$$

(ii) If g= (x^{2^h},I)=($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)=\frac{8.2^h}{4}.1=2.2^h=2.\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ (g)=1

(iii) If g≠ (x^{2^h},I) and g∈ H,i.e.{g=(xy,I) or g=($(xy)^3$,I)}

$$\Phi_{(l+2,1)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_2}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{4}.(1+1)=2.2=2.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1
Otherwise

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

$\Phi_{(l+2,1)}(g)=0$ since H∩CL(g)=ϕ

Case (II):

If H is a cyclic subgroup of ($Q_2^{h+1}\times$ {z}),then:

1-H=$\langle(x,z)\rangle$ 2- H=$\langle(y,z)\rangle$ 3- H=$\langle(xy,z)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_2^{h+1} $1\le j\le l+2$,

then by using theorem (1.6)

1- $\Phi_j(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i\in H\cap CL(g)$

2- $\Phi_j(g)=0$ if $H\cap CL(g)=\phi$

1- IF H=$\langle(x,z)\rangle$

(i) If g=(1,I) or g=(1,z)

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(1,I)|}.1=\frac{2.2^{h+2}}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{2|C_{Q_{2^{h+1}}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z)}

(ii) if g=$(1,I)or\quad g=(x^{2^h},I)or\quad g=(x^{2^h},z)or\quad g=(1,z),g\in H$

if g=(1,I) or g=(1,z)

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=$$

$$\frac{2^{h+3}}{|C_H(g)|}.1=\frac{2.2^{h+2}}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{2|C_{Q_{2^{h+1}}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$ since H ∩CL(g)={g}, φ (g)=1

if g=($(x^{2^h},I)or\quad g=(x^{2^h},z),g\in H$

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(g)|}.1=\frac{2.2^{h+2}}{|C_H(g)|}.1=\frac{2|C_{Q_2^{h+1}}(x^{2^h})|}{2|C_{\langle x\rangle}(x^{2^h})|}.\varphi(x^{2^h})=\Phi_j(x^{2^h})$$

since H ∩CL(g)={g}, φ (g)=1

(iii) if {g≠$(x^{2^h},I)or\quad g\neq(x^{2^h},z)$ },$g\in H$

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{2^{h+2}}{|C_H(g)|}(1+1)=$$

$$\frac{2.2^{h+1}}{|C_H(g)|}.(1+1)=\frac{2|C_{Q_2^{h+1}}(q)|}{2|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z),q∈ Q_2^{h+1} and q≠ x^{2^h}

(iv) if g∉H

$\Phi_{(j,2)}(g)=0$ Since H∩CL(g)= ϕ

2- IF H=$\langle(y,I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z)}

(i) If g=(1,I) or g=(1,z) H∩CL(g)={(1,I),(1,z)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=.\Phi_{l+1}(1)$$

(ii) If g= (x^{2^h},I)=(y^2,I),(y^2,z) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+1}(x^{2^h})$$

Since H∩CL(g)={g} , φ(g)=1

(iii) If g≠ (x^{2^h},I) and g∈ H,i.e.{g=(y,I),(y,z) or g=(y^3,I),(y^3,z)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{8}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,2)}(g)=0$$ since H∩CL(g)=ϕ

3.IF H=$\langle(xy,I)\rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I),(1,z),(xy,z),($(xy)^2$,z), ($(xy)^3$,z)}

(i) If g=(1,I) or g=(1,z) H∩CL(g)={g}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+2}(1)$$

(ii) If g= (x^{2^h},I)=($(xy)^2$,I)=(y^2,I) ,($(xy)^2$,z) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ(g)=1

(iii) If g≠ (x^{2^h},I) and g∈ H ,i.e.g={(xy,I),($(xy)^3$,I),(xy,z),($(xy)^3$,z)}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{8}.(1+1)=2=\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+2,2)}(g)=0$$ since H∩CL(g)=ϕ

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 4, Issue 8, August – 2020, Pages: 297-309

2.2 Example:

To construct $Ar(Q_{128}\times C_2)$ by using proposition (2.1) then

$Ar(Q_{2^7}\times C_2)$=

Γ-classes	[1,I]	$[x^{64},I]$	$[x^{32},I]$	$[x^{16},I]$	$[x^8,I]$	$[x^4,I]$	$[x^2,I]$	[x,I]	[y,I]	[xy,I]	[1,z]	$[x^{64},z]$	$[x^{32},z]$	$[x^{16},z]$	$[x^8,z]$	$[x^4,z]$	$[x^2,z]$	[x,z]	[y,z]	[xy,z]
$\|CL_\alpha\|$	1	1	2	2	2	2	2	2	64	64	1	1	2	2	2	2	2	2	64	64
$\|C_{Q_{2^7}}(CL_\alpha)\|$	512	512	256	256	256	256	256	256	8	8	512	512	256	256	256	256	256	256	8	8
$\Phi_{(1,1)}$	512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	256	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	128	128	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	64	64	64	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	32	32	32	32	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	16	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	128	128	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,1)}$	128	128	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	256	0	0	0	0	0	0	0	0	0	256	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	128	128	0	0	0	0	0	0	0	0	128	128	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	64	64	64	0	0	0	0	0	0	0	64	64	64	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	32	32	32	32	0	0	0	0	0	0	32	32	32	32	0	0	0	0	0	0
$\Phi_{(5,2)}$	16	16	16	16	16	0	0	0	0	0	16	16	16	16	16	0	0	0	0	0
$\Phi_{(6,2)}$	8	8	8	8	8	8	0	0	0	0	8	8	8	8	8	8	0	0	0	0
$\Phi_{(7,2)}$	4	4	4	4	4	4	4	0	0	0	4	4	4	4	4	4	4	0	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	0	2	2	2	2	2	2	2	2	0	0
$\Phi_{(9,2)}$	64	64	0	0	0	0	0	0	2	0	64	64	0	0	0	0	0	0	2	0
$\Phi_{(10,2)}$	64	64	0	0	0	0	0	0	0	2	64	64	0	0	0	0	0	0	0	2

Table (8)

References

[1] A.S.Abid, "Artin's Characters Table of Dihedral Group for Odd Number ",M.Sc. thesis, University of kufa,2006.
[2] C.Curits and I. Reiner, " Methods of Representation Theory with Application to Finite Groups and Order ", John wily & sons, New York, 1981.
[3] G. David, " Artin's Exponent of Arbitrary Characters of Cyclic Sub groups ", Journal of Algebra,61, pp.58-76, 1976.
[4] I. M.Isaacs, "on Character Theory of Finite Groups ",Academic press, New York, 1976.
[5] K.Knwabusz, " Some Definitions of Artin's Exponent of Finit Group ", USA.National foundation Math.GR.1996.
[6] K. Yamauchi, "On The 2 – Parts Artin's Exponent of Finite Groups ", ci.Kep.Tokyo, Daigaken seet Alo (234 – 240),1970.
[7] L. E. Sigler, " Algebra ", Springer- verlage, 1976.
[8] S.J. Mahmood "On Artin cokernel of the Quaternion Group Q_{2m} when m is an Even Number" M.Sc. thesis, University of Kufa,2009.
[9] T.Y. Lam," Artin Exponent of Finite Groups ", Columbia University, New York, 1967.
[10] Z.F. Abdul - Hussein, " The Artin Cokernel of The Group $Q_{2m} \times C_2$ When m is an Odd Number ", M.Sc. thesis , University of Kufa ,2012.