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Constructing the Shadows of Polyhedrons

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Abstract— *The article discusses the use of multi-parameter functions in the construction of shadows and developed computer-aided design of building shadows. Using Auto CAD.*

Keywords— *Themes, polyhedrons, construction of shadows, own shadow, falling shadow, gran, edge.*

1. INTRODUCTION

The rays of sunlight can be considered to a large extent parallel and rectilinear.

It is known [1] that, falling on objects around us, sunlight makes them visible. Thanks to chiaroscuro that revives the surface of illuminated objects, their shape and plasticity are revealed.

The surface of an object, when illuminated from one side, does not receive light from the other. The unlit part of the surface is called its own shadow. If an object is located in front of the illuminated surface of another, then it blocks part of it from the rays of light. This part is called the falling shadow from the subject.

2. MAIN PART

Let's consider the geometric essence of shading.

Figure 1 shows a ball illuminated by parallel rays and a shadow falling from it onto a plane. The side of the ball facing the plane is in its own shadow. The difference between its own shadow and the incident one is that the first is formed on the surface directed away from the light, the second on the surface facing the light.

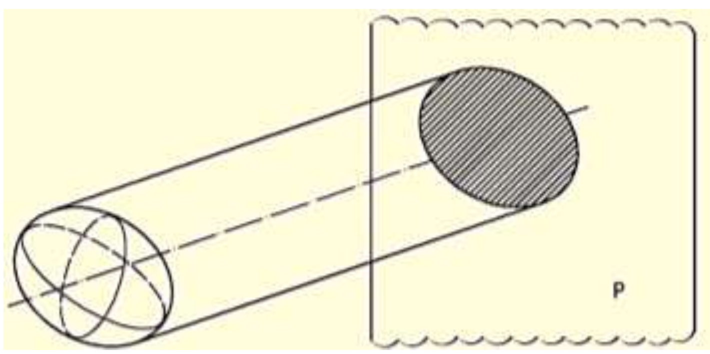

Figure: 1. Falling shadow of the ball on the plane

Depicting the facades of buildings in orthogonal projections, the architect seeks to reveal their plasticity; in reality, buildings are illuminated in different directions of sunlight. The pointed direction of the light has the advantages over the variable one, that the eye, getting used to it, more easily catches the relief, filled with shadows with the familiar directional light, and quickly orientates itself in the drawing.

The constant direction of light adopted for architectural images is determined by the position of the vertical (frontal) and horizontal projections of the light beam, which with the OX projection axis are equal to 450 (Fig. 2).

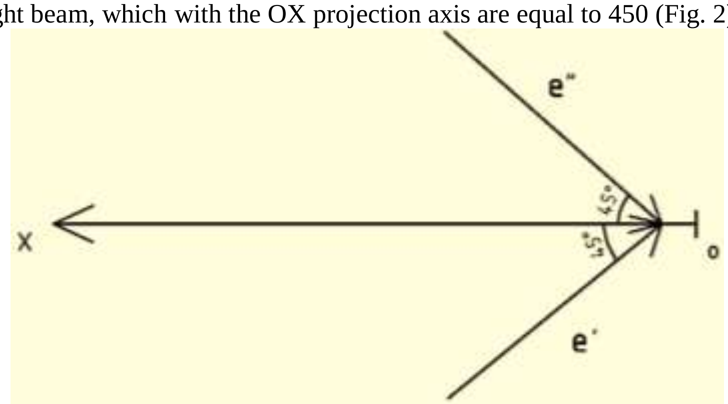

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Figure: 2. Direction of light

This direction of light corresponds to the position of the diagonal of the cube, the edge of which is vertical (frontal), horizontal and profile - the projecting lines. A cube built on a segment of a light ray, as on a diagonal, will be called light.

In fig. 3 shows a light cube, depicted on the frontal plane of the projections by a square equal to its face: the projection of the trimming of the light beam, which is the basis for the formation of the light cube, serves as the diagonal of this square. From the geometric properties of the cube, we establish the following dependence: the edge of the cube is the diagonal of its face (base), corresponding to the projection of the light ray and the diagonal in space, i.e. the ray itself, relate to each other as $1{:}\sqrt{2}:\sqrt{3}$

These elements are located in the frontal projection plane passing through the diagonal of the cube. They form a right-angled triangle, which we will agree to call light.

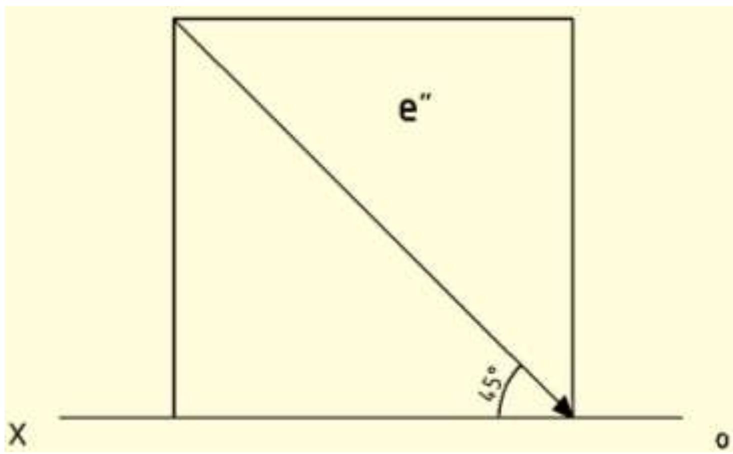

Figure: 3. Projection of the light beam

Bringing the plane of the light triangle to the frontal position, we obtain its true value Fig. 4.

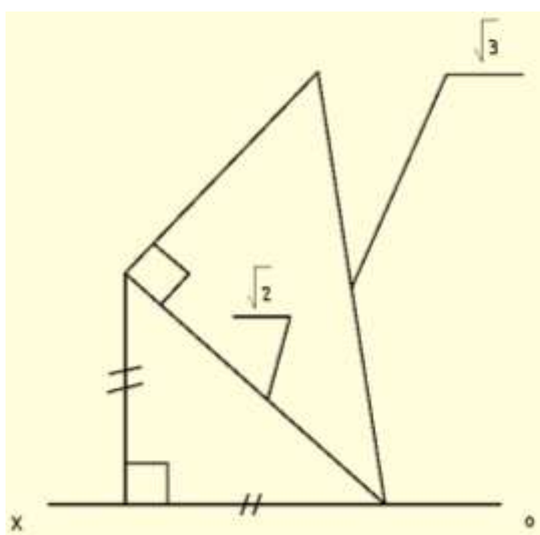

Figure: 4. Construction of the actual size of the light triangle

The direction of light accepted for architectural images is determined by the position of the frontal and horizontal projections of the light beam, which with the projection axis make up angles equal to α and β (Fig-5).

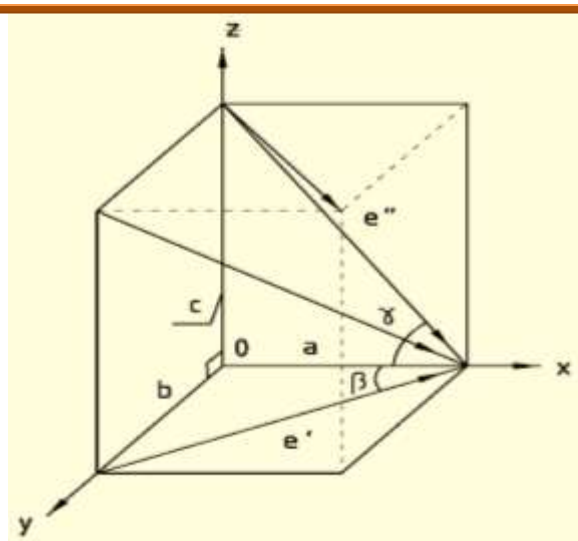

Figure: 5. Light parallelepiped

This direction of light corresponds to the position of the diagonal of a parallelepiped with dimensions (a x in x c) whose edges are frontal, horizontal and profile projecting lines. A parallelepiped built into segments, a, b, c of a light ray, as on a diagonal, will be called light.

In fig. 6 shows a light rectangle depicted on the frontal plane of projections by a rectangle equal to its edge;

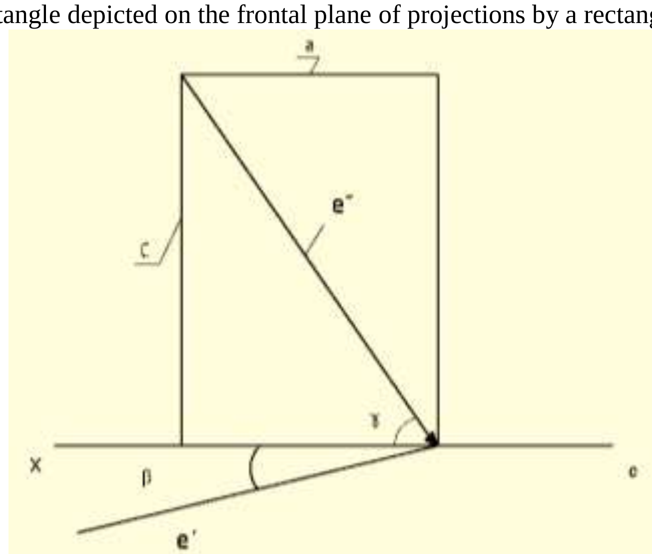

Figure: 6. Projection of the light beam in general position

Below is the analytical relationship of the edge of the parallelepiped light:

$|\ell''| = \sqrt{a^2 + c^2}$

$|\ell'| = \sqrt{a^2 + b^2}$

$|\vec{\ell}| = \sqrt{a^2 + b^2 + c^2}$

$|\vec{\ell}| = \sqrt{(\ell')^2 + c^2} = \sqrt{(\ell'')^2 + b^2}$

$sin\,\alpha = \frac{c}{\ell''}$; $\ell'' = \frac{c}{sin\,\alpha}$ (1) здесь $0 < \alpha < 90^0$ $0 < \beta < 90^0$

$cos\,\alpha = \frac{a}{\ell''} \rightarrow \ell'' = \frac{c}{cos\,\alpha}$ (2)

(1), (2) $=> \frac{c}{sin\,\alpha} = \frac{a}{cos\,\alpha} => c = \frac{sin\,\alpha}{cos\,\alpha} = tg\alpha$

Итак $\ell'' = (c^2 + a^2 tg^2\alpha)^{0,5}$ $\frac{c}{a} = tg\alpha \rightarrow c = a\, tg\alpha$; (3)

analytically define b $= a\ \ tg\beta$; (4)

Calculating the vertical edge of a parallelepiped (3), (4) $\rightarrow \frac{c}{tg\alpha} = \frac{b}{tg\beta}$;

$$c = \frac{b \cdot tg\alpha}{tg\beta}; \tag{5}$$

or

$$tg\alpha = \frac{c \cdot tg\beta}{b}; \tag{6}$$

$\left|\vec{\ell}\right| = \sqrt{a^2 + b^2 + c^2} = \sqrt{a^2 tg^2\alpha + a^2 tg^2\beta + a^2} = a\sqrt{1 + tg^2\alpha + tg^2\beta}$;

formula for calculating the diagonal of the light parallelepiped.

Thus $\left|\vec{\ell}\right| = \sqrt{1 + tg^2\alpha + tg^2\beta}$ (7)

Let a straight prism be given, standing on the plane H. It is required to construct its own and falling shadows on the projection plane H and V. (Fig. 7). Let's analyze the illumination of the faces. For a given direction of light flux, the top and left front faces will be illuminated. The rest of the faces (including the bottom one) are in shadow. To build a falling shadow of a volumetric body, it is necessary to identify the contour of its own shadow, which in this case will be a spatial polyline. The elements of this line are the edges of the prism located at the boundaries of the illuminated and unlit planes. The same figure shows an isometric image of a closed contour of its own shadow, from which a falling shadow is constructed.

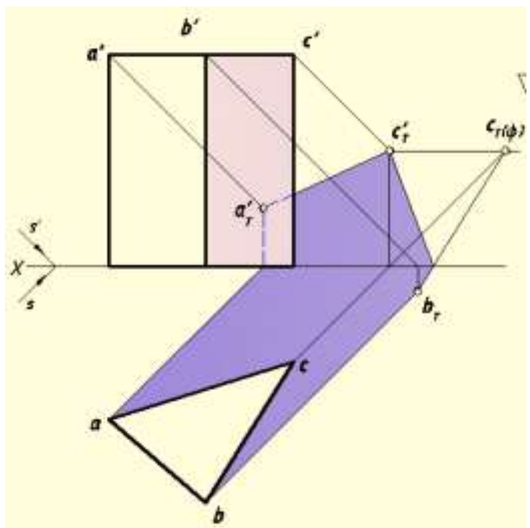

Figure: 7. Construction of own and falling shadows of the prism

Let us present problems related to the construction of shadows for polyhedra.

Build your own and falling shadows of the SABC pyramid; determine which part of the segment [MN] has cast a shadow on the surface of this polyhedron (Fig. 8).

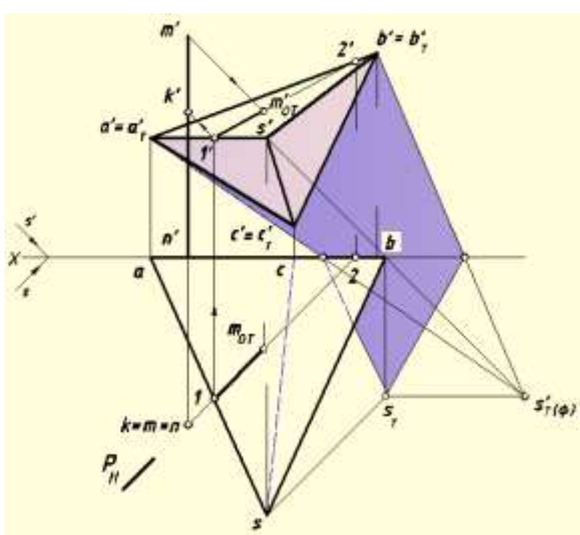

Figure: 8. The shadow of the pillar on the surface of the pyramid

For a given direction of light flux, the only face of the pyramid (ASB) will be illuminated, the rest are in their own shadow. The object's own shadow contour is the sides of the ASB triangle.

To determine the falling shadow of the pillar on the surface of the pyramid, we enclose the segment [MN] in the horizontal projection plane P parallel to the direction of the light flux. This plane will intersect the illuminated face of the pyramid in a straight line, the projections of which 12 and 1'2 'are shown in the drawing. Since the ray passing through point M is in the same plane, it is possible to define the shadow of point M on the ASB face. We mark the point (1, 1') on the edge [AS], along which, using the return ray, we define the point K (k, k') on the segment [MN].

Determine the illumination of the visible faces of a regular six-angle pyramid (Fig. 9).

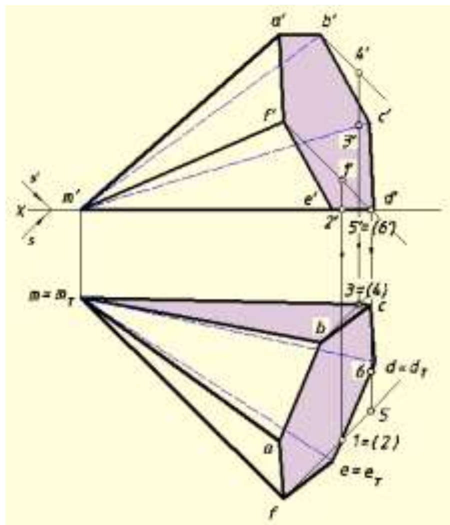

Figure: 9. Determination of illumination of visible edges

Note that the construction of a graphical condition for this problem is already a problem, in the solution of which it is advisable to apply the transformation of the drawing (these constructions are not shown in Fig. 9). The visibility of the edges on the projections of the polyhedron is established using competing points.

To find the contour of the own shadow of polyhedra

in educational sources the following recommendation is given: for a polyhedral body, it is enough to draw rays only through the vertices and find the falling shadows from these points.

In fact, it is proposed to first build a falling shadow, and then find your own.

In our opinion, such an approach is possible, but not always acceptable, since if a polyhedron has a large number of vertices, then many falling shadows from the latter may end up inside the contour of the falling shadow of the polyhedron and a number of constructions will turn out to be inappropriate. Unfortunately, in the educational literature on determining the contour of the own shadow of polyhedrons, errors are often encountered.

In the tasks considered earlier, the determination of the illumination of the faces does not cause difficulties. If the number of polyhedron faces visible on the diagram is large or their illumination is not obvious, we recommend using the method of competing points to determine the illumination of polyhedron faces. This will allow you to avoid mistakes when setting the contour of the object's own shadow and at the same time perform the minimum number of constructions.

Let's draw a light ray through the point F (f, f ') and consider the competing points belonging to this ray and the edge [DE]. From the applica ons of the frontal projections of the points, we make a conclusion about the visibility of the points 1 = (2). Since point 2, located on the edge [DE], is covered by point 1 of the light beam, it is invisible, therefore, the entire 6-sided face of the pyramid is in shadow. From this we can conclude about the face illumination (AFM).

The part of the ray passing through the vertex B (b, b ') is above the face (BMC), which is determined using the competing points 3 and 4 belonging to the ray and edge [MC]. Set that 3 = (4) and conclude that this face is in its own shadow, and the face (ABM) is lit. Analyze the illumination of the remaining faces in the same way. It often turns out that when setting the shadow edge, there is no need to check the vertices of the polyhedron, the shadow of which falls into the area of the contour of the falling shadow.

Such an approach to determining the visibility would allow avoiding an error in a similar problem in determining the illumination of edges, made in one of the textbooks. Build your own and falling shadows of a regular six-angle pyramid (fig. 10).

The analysis of visible illuminated faces is given in the previous task. The face (DEM) located in the plane H will be in its own shadow, therefore, e = eT; d = dT and m = mT, i.e. the three vertices of the pyramid and their actual shadows on the H plane coincide.

The [ME] edge is part of the pyramid's own shadow contour, because face (EFM) is lit.

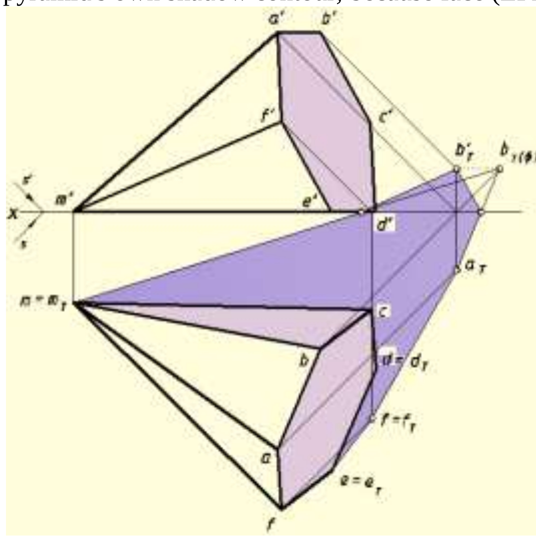

Figure: 10. Building the falling shadows of the pyramid

3. Conclusion

Reasoning in a similar way, we define the entire closed contour of its own shadow - this is the cumulative sequence of edges [ME] - [EF] - [FA] - [AB] - [BM]. From these edges we define the contour of the falling shadow of this pyramid. The solution to the problem is shown in Fig. ten.

4. References

[1] Kolotov S.M. Questions of the theory of images - K.: KSU, 1972, - 161 p.
[2] Murodov Sh.K., va boshgalar, Chizma gemetria kursi. - T.: 1988.375 p.
[3] Khorunov R. Chizma geometry of the course. - T.: 1974. - 268 p.